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STATE ATOMIC ENERGY CORPORATION «ROSATOM»

MHD in Focus 3D code

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Actuality and problems

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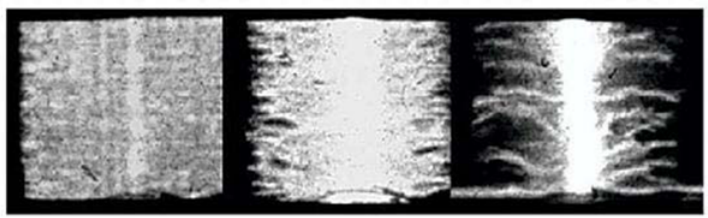
- ✓ The laboratories involved in thermonuclear fusion are actively discussing the use of high-current electro physical facilities to achieve the ignition conditions*.
- ✓ When a substance is compressed in high-current Z-pinch installations, instabilities develop due to manufacturing defects of the compressing liner. Instabilities impede a high degree of pinch compression, preventing them from reaching thermonuclear temperatures.
- ✓ Clarification of the role of MHD instability on the parameters of a compressed plasma.



The experimental image of Z-pinch compression by installation Angara-5

- ✓ To study the development of instabilities in the compression of a substance by a magnetic field, the Focus 3D code is used **.

* Ryutov D.D., Derzon M.S., Matzen M.K. The physics of fast Z pinches // Rev. Mod. Phys., Vol. 72, No. 1, 2000. – Pp. 167 – 223.
** Ershova A.V., Glazyrin I.V., Mikhailov N.A. Accounting of spontaneous magnetic fields in Focus 3D code // IOP Conf. Series: Journal of Physics: Conf. Series 1103 (2018) 012002.

The system of MHD equations

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The system of MHD equations:

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \\ \dfrac{\partial}{\partial t}(\rho \vec{v}) + \rho(\vec{v}\nabla)\vec{v} + \nabla p - \dfrac{1}{4\pi}\left(\nabla \times \vec{B} \times \vec{B}\right) = 0, \\ \dfrac{\partial}{\partial t}(\rho E) + \nabla \cdot \left[\left(\rho E + p^*\right)\vec{v} - \dfrac{\left(\vec{v}\cdot\vec{B}\right)\vec{B}}{4\pi} \right] = 0, \\ \dfrac{\partial \vec{B}}{\partial t} = \vec{v}\cdot\nabla\cdot\vec{B} - \vec{B}\cdot\nabla\cdot\vec{v}, \end{cases}$$

$$\frac{\partial \vec{u}}{\partial t} + \nabla \cdot \overline{\overline{f}}(\vec{u}) = 0,$$

$$\vec{u} = \left(\rho, \rho v_x, \rho v_y, \rho v_z, \rho E, B_x, B_y, B_z, \rho Y_1, \ldots, \rho Y_{N-1}\right)^T,$$

where ρ – density, $\vec{v}$ – velocity,

p – pressure, t – time,

$E = e + \dfrac{v^2}{2} + \dfrac{B^2}{8\pi\rho}$ – specific total energy,

e – specific internal energy,

$p^* = p + \dfrac{B^2}{8\pi}$ – total pressure,

$\vec{B}$ – magnetic induction,

$Y_1, \ldots, Y_N$ – mass fractions of the components of the mixture of substances, $\sum_{n=1}^{N} Y_n = 1.$

$$\overline{\overline{f}}(\vec{u}) = \begin{pmatrix} \rho v_x & \rho v_y & \rho v_z \\ \rho v_x^2 + p^* - \dfrac{B_x^2}{4\pi} & \rho v_x v_y - \dfrac{B_x B_y}{4\pi} & \rho v_x v_z - \dfrac{B_x B_z}{4\pi} \\ \rho v_y v_x - \dfrac{B_y B_x}{4\pi} & \rho v_y^2 + p^* - \dfrac{B_y^2}{4\pi} & \rho v_y v_z - \dfrac{B_y B_z}{4\pi} \\ \rho v_z v_x - \dfrac{B_z B_x}{4\pi} & \rho v_z v_y - \dfrac{B_z B_y}{4\pi} & \rho v_z^2 + p^* - \dfrac{B_z^2}{4\pi} \\ \left(\rho E + p^*\right) v_x - \dfrac{\left(\vec{v}\cdot\vec{B}\right) B_x}{4\pi} & \left(\rho E + p^*\right) v_y - \dfrac{\left(\vec{v}\cdot\vec{B}\right) B_y}{4\pi} & \left(\rho E + p^*\right) v_z - \dfrac{\left(\vec{v}\cdot\vec{B}\right) B_z}{4\pi} \\ 0 & v_y B_x - v_x B_y & v_z B_x - v_x B_z \\ v_x B_y - v_y B_x & 0 & v_z B_y - v_y B_z \\ v_x B_z - v_z B_x & v_y B_z - v_z B_y & 0 \\ \rho Y_1 v_x & \rho Y_1 v_y & \rho Y_1 v_z \\ \ldots & \ldots & \ldots \\ \rho Y_{N-1} v_x & \rho Y_{N-1} v_y & \rho Y_{N-1} v_z \end{pmatrix}.$$

Scheme of computation

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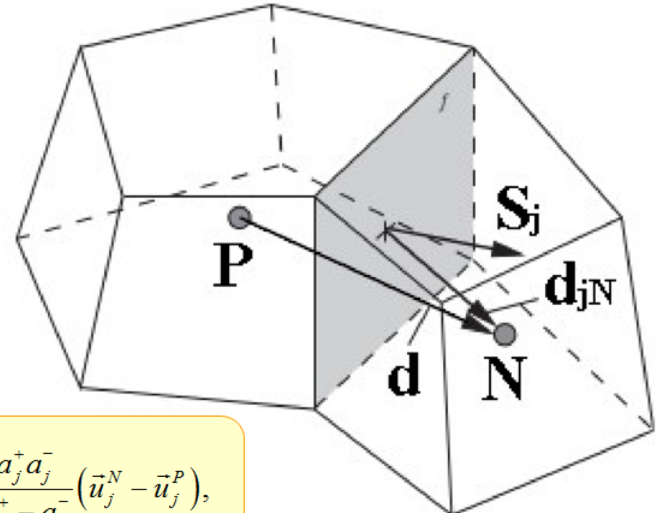
For spatial discretization – the HLL scheme (2-order).

For time sampling – the Runge-Kutta method (2-order).

Reconstruction on the cell faces –

the TVD-slope limiter approach: minmod and vanLeer.



$$\vec{f}_n(\vec{u}) = \overline{\overline{f}}(\vec{u})\cdot\vec{n},\ \vec{n} = (n_x, n_y, n_z),$$

$$\vec{f}_n(\vec{u}) = \begin{pmatrix} \rho v_n \\ \rho v_x v_n + n_x p^* - \dfrac{B_x B_n}{4\pi} \\ \rho v_y v_n + n_y p^* - \dfrac{B_y B_n}{4\pi} \\ \rho v_z v_n + n_z p^* - \dfrac{B_z B_n}{4\pi} \\ (\rho E + p^*) v_n - \dfrac{(\vec{v}\cdot\vec{B}) B_n}{4\pi} \\ B_x v_n - v_x B_n \\ B_y v_n - v_y B_n \\ B_z v_n - v_z B_n \\ \rho Y_1 v_n \\ \dots \\ \rho Y_{N-1} v_n \end{pmatrix},$$

$$\vec{f}_n(\vec{u})_j \approx \frac{a_j^+ \vec{f}_n(\vec{u}_j^P) - a_j^- \vec{f}_n(\vec{u}_j^N)}{a_j^+ a_j^-} + \frac{a_j^+ a_j^-}{a_j^+ - a_j^-}(\vec{u}_j^N - \vec{u}_j^P),$$

$$a_j^+ = \max\left\{\lambda_K\left(\frac{\partial\vec{f}_n}{\partial\vec{u}}(\vec{u}_j^P)\right), \lambda_K\left(\frac{\partial\vec{f}_n}{\partial\vec{u}}(\vec{u}_j^N)\right), 0\right\},$$

$$a_j^- = \min\left\{\lambda_1\left(\frac{\partial\vec{f}_n}{\partial\vec{u}}(\vec{u}_j^P)\right), \lambda_1\left(\frac{\partial\vec{f}_n}{\partial\vec{u}}(\vec{u}_j^N)\right), 0\right\},$$

$$\lambda_1 = v_n - a_f,\ \lambda_K = v_n + a_f,$$

$$a_f = \sqrt{\frac{1}{2}\left(c_s^2 + \frac{B^2}{4\pi\rho} + \sqrt{\left(c_s^2 + \frac{B^2}{4\pi\rho}\right)^2 - c_s^2\cdot\frac{B_n^2}{\pi\rho}}\right)},$$

$$v_n = n_x v_x + n_y v_y + n_z v_z,\ B_n = n_x B_x + n_y B_y + n_z B_z.$$

Condition div(B)=0

In the process of calculation, an error accumulates in the magnitude of the magnetic field and

$$\nabla \cdot \vec{B} \neq 0.$$

A function φ (artificial scalar potential)* is introduced that satisfies the Poisson equation

$$\nabla^2 \varphi = \nabla \cdot \vec{B}.$$

The new value of the magnetic field $\vec{B}'$ is calculated using the found value of the magnetic induction vector $\vec{B}$ taking into account the scalar potential

$$\vec{B}' = \vec{B} - \nabla \varphi.$$

For a new magnitude magnetic field: $\nabla \cdot \vec{B}' = \nabla \cdot \vec{B} - \nabla \cdot \nabla \varphi \equiv 0.$

Correction of specific total energy is also carried out $E' = E + \dfrac{\left(\vec{B}'\right)^2 - \left(\vec{B}\right)^2}{8\pi\rho}.$

* Kulikovskii A.G., Pogorelov N.V., Semenov A.Yu. Mathematical problems of numerical solution of hyperbolic systems. Moscow: Physics and mathematics publishers, 2001.

Tests

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Focus

Spatial scheme – HLL 2-order;

Time scheme – Runge-Kutta 2-order.

Flash*

Spatial and time scheme – MUSCL-Hancock 2-order.

Parameters of computation:

Reconstruction 2-order, slope vanLeer;

Riemann Solver – HLL.

Test 1D Brio-Wu

Rated operating conditions $x \in [0;1]$

Equation of state $p = (\gamma - 1)\rho\varepsilon$, $\gamma = 2$

Initial condition by $t = 0$:

$$v_x = v_y = v_z = 0, \quad B_x = 0.75 \cdot \sqrt{4\pi}$$

$$0 \le x < 0.5 \quad \rho = 1, \quad p = 1, \quad B_y = 1 \cdot \sqrt{4\pi}$$

$$0.5 \le x \le 1 \quad \rho = 0.125, \quad p = 0.1, \quad B_y = -1 \cdot \sqrt{4\pi}$$

End point of time $t = 0.1$

Courant number 0.3

Test 2D Orszag-Tang

Rated operating conditions $(x, y) \in [0;1]^2$

Equation of state $p = (\gamma - 1)\rho\varepsilon$, $\gamma = 5/3$

Initial condition by $t = 0$: $\rho = 1$, $p = 1/\gamma$,

$\vec{v} = (-\sin(2\pi y), \sin(2\pi x), 0)$,

$\vec{B} = B_0 \cdot (-\sin(2\pi y), \sin(4\pi x), 0)$,

$B_0 = \sqrt{4\pi} / \gamma$.

End point of time $t = 0.5$

Courant number 0.125

Test 3D Orszag-Tang

Rated operating conditions $(x, y, z) \in [0;1]^3$

Equation of state $p = (\gamma - 1)\rho\varepsilon$, $\gamma = 5/3$

Initial condition by $t = 0$: $\rho = 1$, $p = 1/\gamma$,

$\vec{v} = (-\sin(2\pi z), \sin(2\pi x), \sin(2\pi y))$,

$\vec{B} = B_0 \cdot (-\sin(2\pi z), \sin(4\pi x), \sin(2\pi y))$,

$B_0 = \sqrt{4\pi} / \gamma$.

End point of time $t = 0.3$

Courant number 0.125

* www.flash.uchicago.edu.

Test 2D Orszag-Tang

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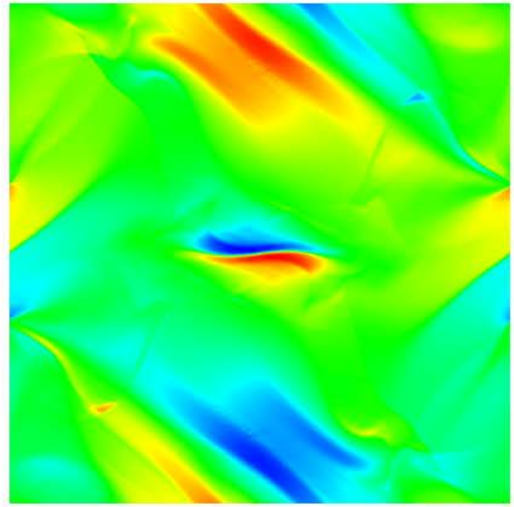
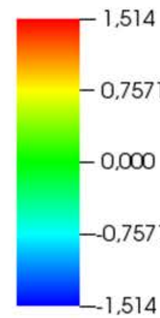
Quantity – B_x

Mesh 800x800 cells

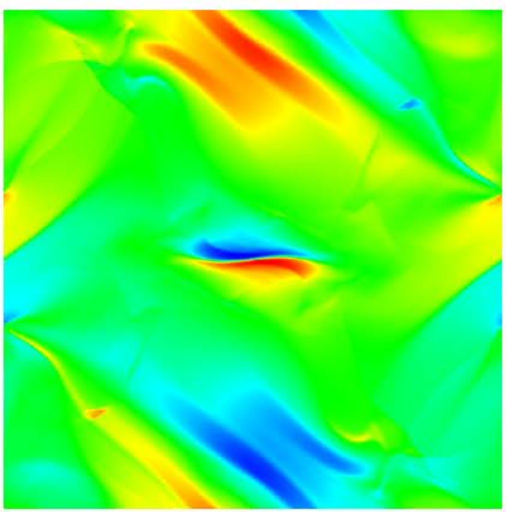
ASP – artificial scalar potential for condition div(B)=0

Focus – without ASP

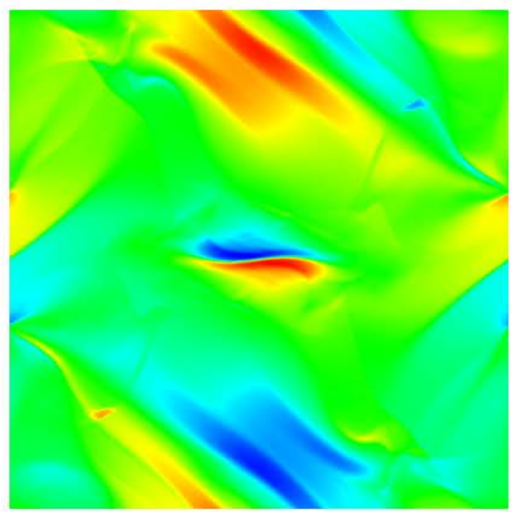




Flash – with ASP



Focus – with ASP



Test 2D Orszag-Tang

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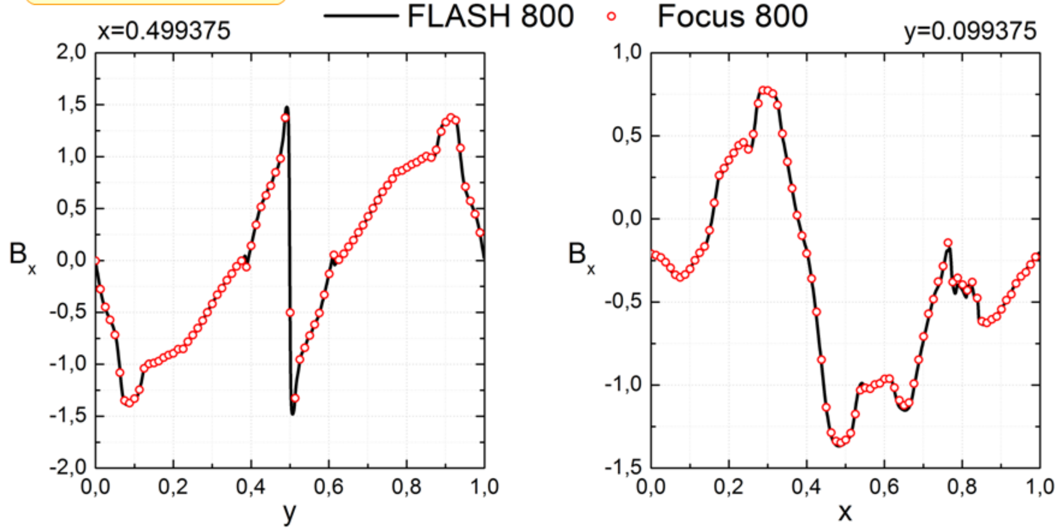
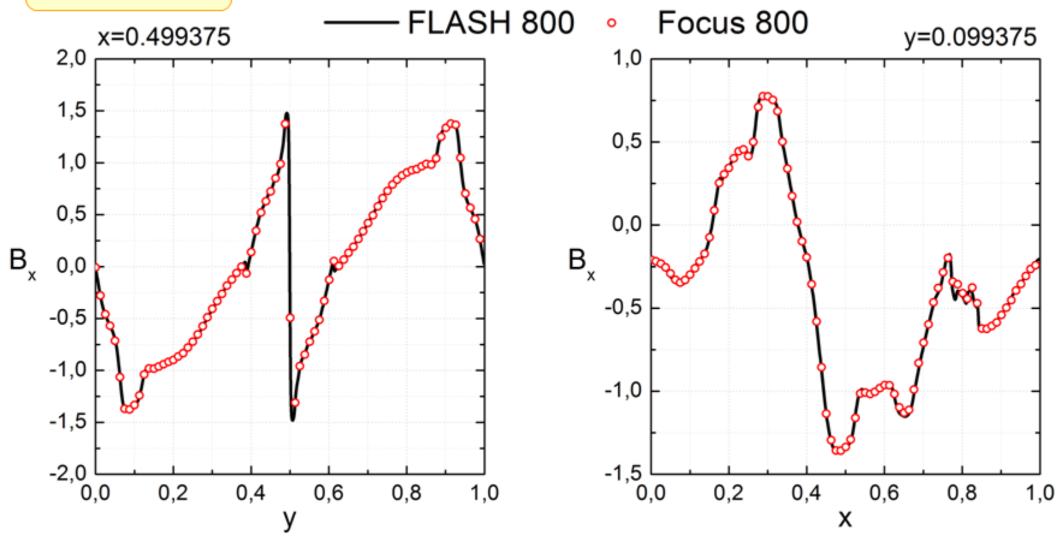
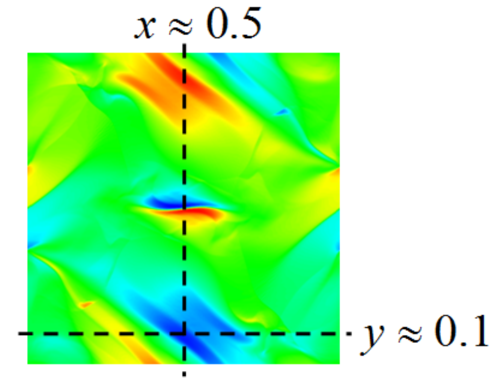
without ASP

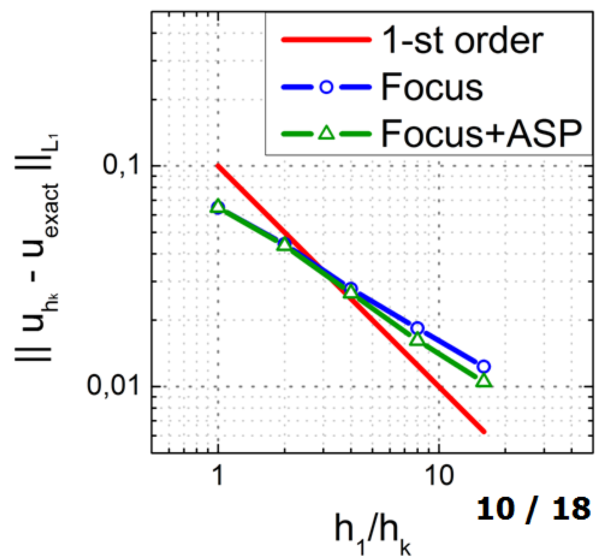


with ASP





u_{exact} – solution by Flash on mesh 1600 cells along each direction, u_{h_k} – solution by Focus on meshes h_k from 50 to 800, $k = 1,...,5$.



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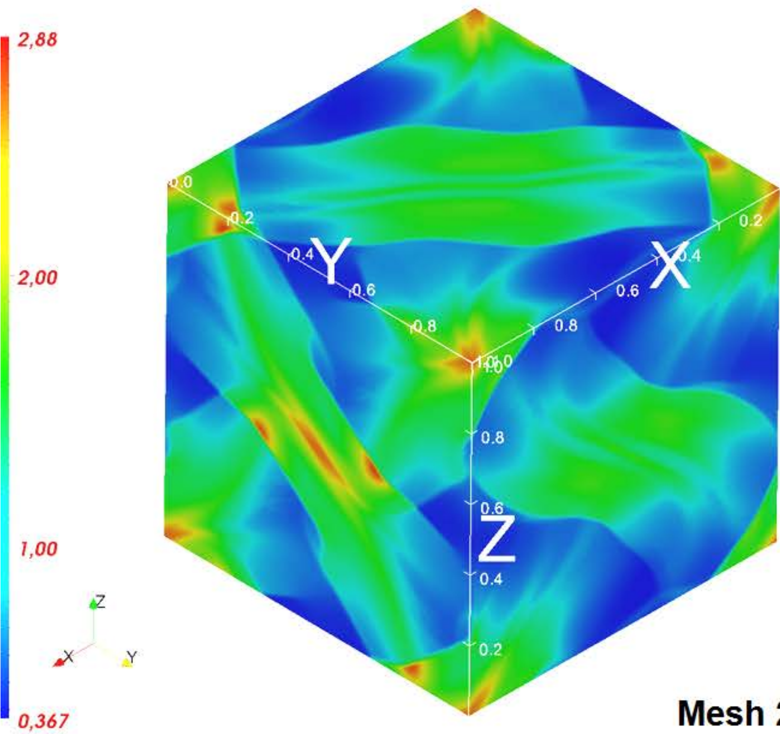
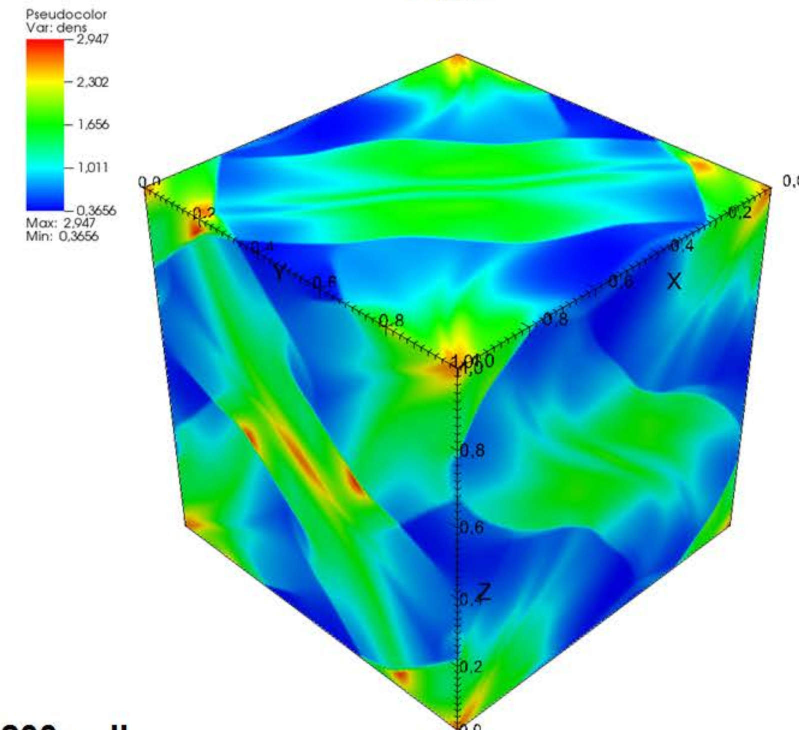
Quantity – density
MHD with ASP

Focus



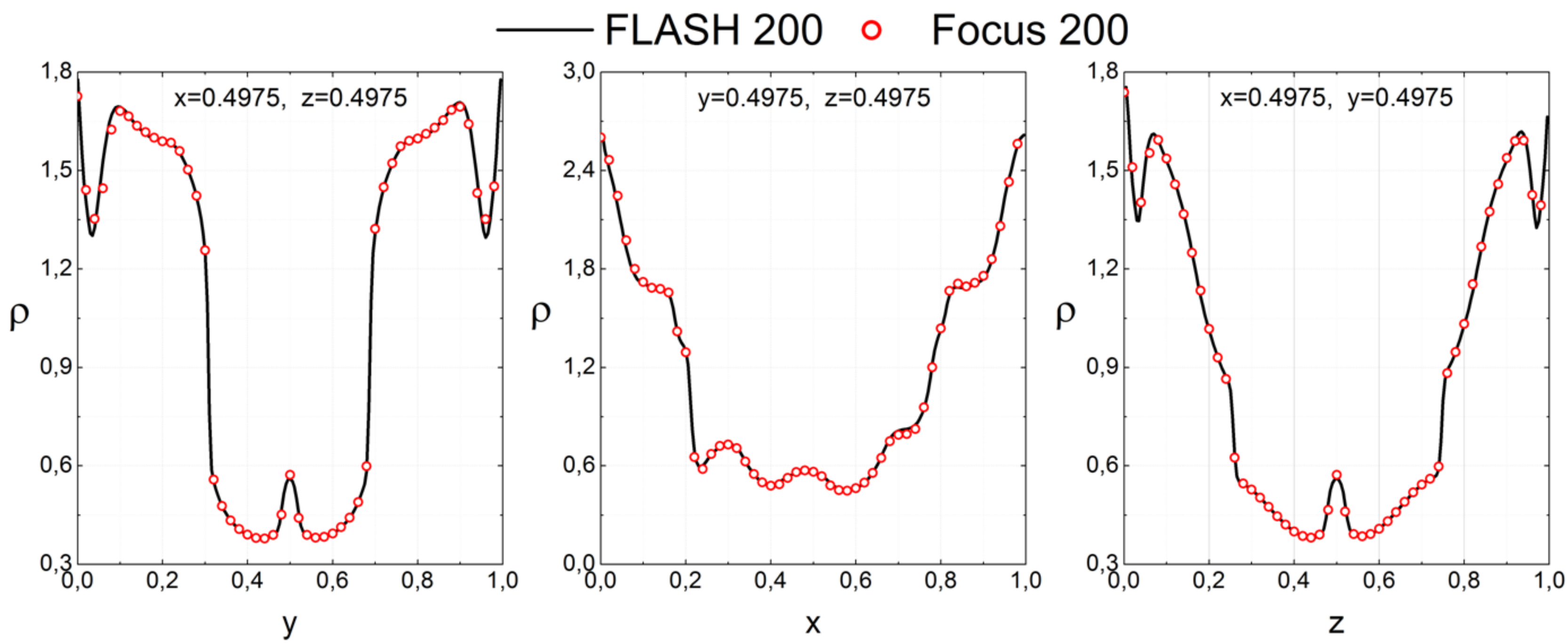
Flash

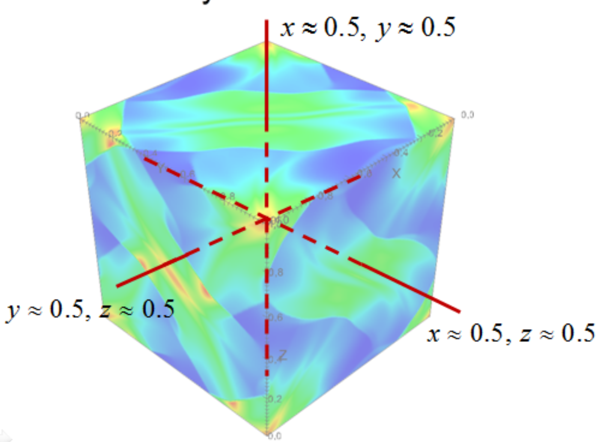


Mesh 200x200x200 cells

Test 3D Orszag-Tang

Mesh 200x200x200 cells, with ASP





$\|\text{Flash} - \text{Focus}\|_{L_1}$

Mesh	$x \approx 0.5,\ z \approx 0.5$	$y \approx 0.5,\ z \approx 0.5$	$x \approx 0.5,\ y \approx 0.5$
50	$3.908 \cdot 10^{-2}$	$3.649 \cdot 10^{-2}$	$3.34 \cdot 10^{-2}$
100	$1.708 \cdot 10^{-2}$	$1.884 \cdot 10^{-2}$	$1.482 \cdot 10^{-2}$
200	$9.52 \cdot 10^{-3}$	$1.339 \cdot 10^{-2}$	$7.32 \cdot 10^{-3}$

Z-pinch compression

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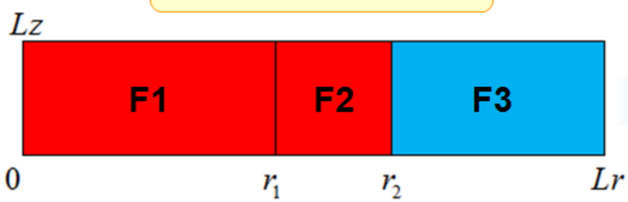
Homogeneous

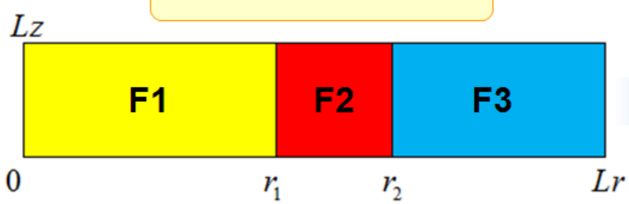


I [MA]	10	30	50	80	100
r_0 [sm]	1	1	1	1	1
m [mg/sm]	2.9	25.7	71.3	182.5	285.1
Δr [mm]	1	1	1	1	1
$\rho\ [\mathrm{g/sm^3}]$	$9.1 \cdot 10^{-4}$	0.0082	0.023	0.058	0.091

$$\frac{m}{2\pi r}\frac{d^2r}{dt^2} = -\frac{B^2}{8\pi} = -\frac{I^2}{2\pi c^2 r^2}$$

$$\rho = \frac{m}{\pi\left(r_0^2 - \left(r_0 - \Delta r\right)^2\right)}$$

One-cascade



Mass of liner is 90% by total mass of matter.

I [MA]	10	30	50	80	100
r_0 [sm]	1	1	1	1	1
m [mg/sm]	2.9	25.7	71.3	182.5	285.1
Δr [mm]	0.1	0.1	0.1	0.1	0.1
$\rho_1\ [\mathrm{g/sm^3}]$	$0.112 \cdot 10^{-3}$	0.001	0.0028	0.0072	0.0112
$\rho_2\ [\mathrm{g/sm^3}]$	$4.29 \cdot 10^{-3}$	0.0387	0.1075	0.2752	0.4299

Rated operating conditions $r \in [0; Lr]$ sm, $z \in [0; Lz]$ sm, $Lr = 1$, $Lz = 1.5$.

Matter – fully ionized deuterium, molar weight $\mu = 2$ g/mole, average charge $\langle z \rangle = 1$.

EOS of ideal gas $p = (\gamma - 1)\rho\varepsilon$, $\varepsilon = c_V T$, $c_V = \frac{R}{(\gamma-1)\mu}\left(\langle z \rangle + 1\right)$, $\gamma = 5/3$.

Initial condition by $t = 0$: $T_{std} = 298.15\,^\circ\mathrm{K}$, $P_{std} = 1.01325 \cdot 10^6\ \mathrm{g \cdot sm^{-1} \cdot s^{-2}}$, $\vec{v} = 0\ \mathrm{sm \cdot s^{-1}}$, $\vec{B} = 0$ Gs.

For F3: $B_z = \frac{I(t)}{5 \cdot r(t)}$, $I(t) = I_0 \cdot \sin\left(\frac{\pi \cdot t}{2 \cdot \tau}\right)$, where front $\tau = 10^{-7}$ s, $[I_0] = \mathrm{A}$, $[r(t)] = \mathrm{cm}$.

End point of time $t_{end} = 2 \cdot 10^{-7}$ s.

Z-pinch + perturbations

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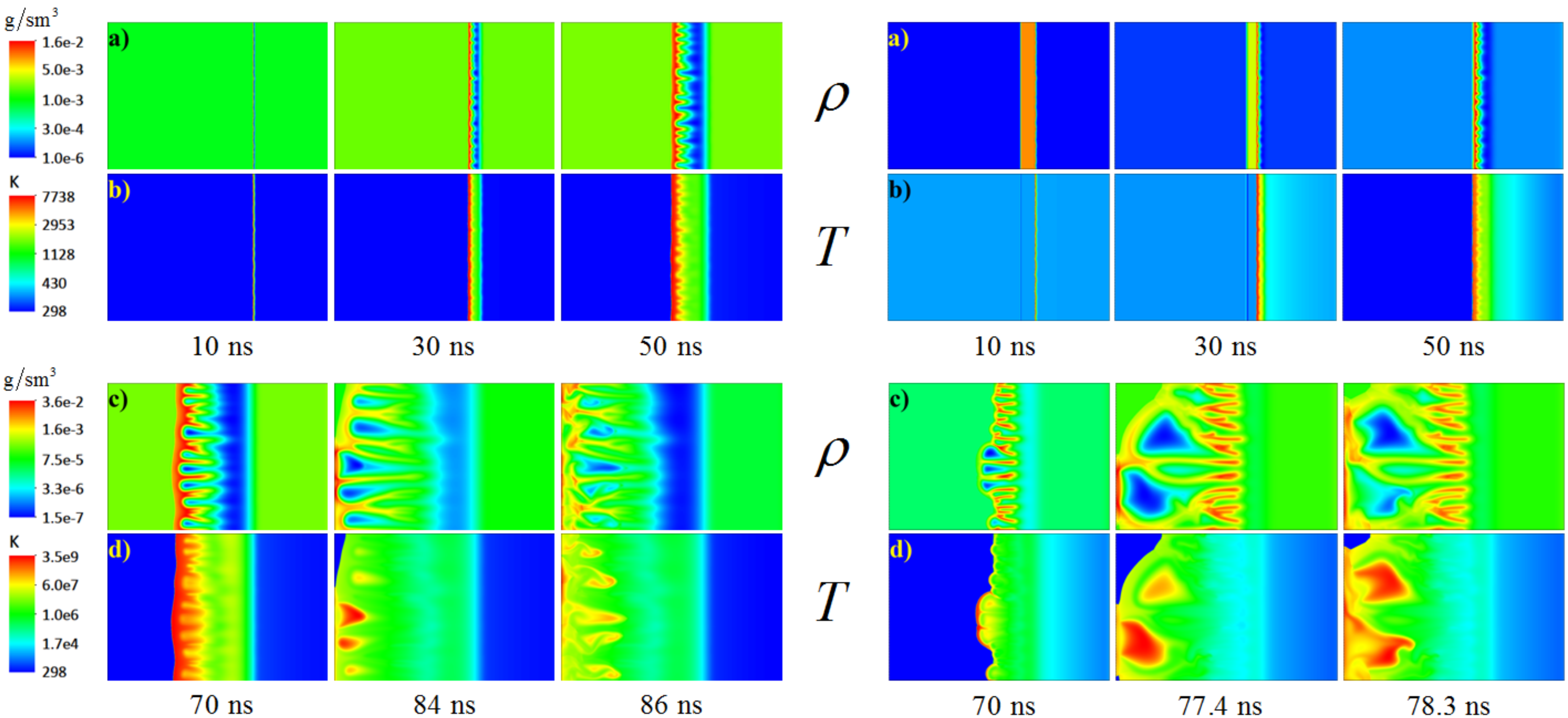
Homogeneous | **One-cascade**



According to Lawson's criterion* for ignition, a combination of high concentration of substance and high temperature is necessary.

When compressing without perturbations, there is a high concentration, but insufficient temperature.

When compressing with perturbations, there is a high temperature, but insufficient concentration due to "necks".

* Lawson J.D. Some criteria for a power producing thermonuclear reactor // Proc. Phys. Soc. London, Vol. 70, 1957. – P. 6.

Conclusion

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- ✓ MHD is realized in Focus 3D code.
- ✓ Testing in comparison with the reference program Flash showed the correctness of the implementation of the MHD section in Focus 3D code.
- ✓ A numerical calculation of the Z-pinch compression problem in a two-dimensional cylindrical formulation for homogeneous and one-cascade liners with and without perturbations has been carried out.
- ✓ It is shown that the development of instabilities during liner compression leads to the appearance of “necks”. Because of the necks, the compression speed is much higher than in 1D, which reduces the mass of the substance in the center.
- ✓ To obtain the conditions for ignition of thermonuclear reactions, a combination of a high concentration of a substance and a high temperature is necessary.
  - In a homogeneous liner it is received a high concentration, but insufficient temperature.
  - In a one-cascade liner it is on the contrary.
- ✓ To stabilize the compression authors propose to use two-cascade liners*. We assume that the second cascade will slow down the development of perturbations from the first cascade.

* Glazyrin I.G., Diyankov O.V., Karlykhanov N.G. et.al. Stability of plasma liner implosion // Laser and Particle Beams (2000), 18, 261-267.

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THANKS FOR YOUR ATTENTION !