CRITICALITY OF A SYSTEM AND ITS INDIVIDUAL COMPONENTS

V.M. Shmakov

Russian Federal Nuclear Center – Zababakhin Institute of Applied Physics
v.m.shmakov@vniitf.ru

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Introduction

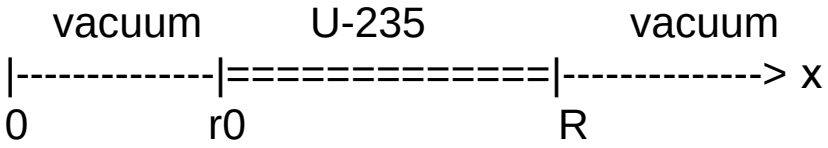
We start with considering the eigenvalue and iegenfunction problem for a uranium sphere with a vacuum cavity in the center. We are searching for neutron density *u(x)* , total neutron current *νs(x),* and time constant *λ*. The exact analytical solution for the problem in the *back-and-forth* approximation can be found in [1, 2].

The basic assumptions of this approximation are 1D geometry, one-group cross-sections, and allowance for neutron transport along only normals to the spherical (planar, cylindrical) surface. Note that the *back-and-forth* approximation for neutron transport is principally different from the Schwarzschild *back-and-forth* approximation for radiation transport.

The solutions to be obtained will be used to derive some equations for calculating *λ* for the entire system and λk's for its individual components. It will be shown that $\lambda k = \lambda$ for any component *k*.

We then give a brief overview of some known and new methods for static and dynamic criticality calculation where we propose using the total neutron reproduction cross-sections.

Sphere with a central cavity
r0=2cm, R=12, λ=2.6520 s-7



In [1, 2], we can find the solution

$\begin{cases} u(x) = C\cosh\left(\frac{\lambda x}{\upsilon}\right) \\ s(x) = -C\sinh\left(\frac{\lambda x}{\upsilon}\right) \end{cases}$		$\begin{cases} u(x) = C*B*\exp\left(-\frac{\lambda x}{\upsilon}\right) \\ s(x) = u(x) \end{cases}$
for **0<x≤ r0** (inner vacuum)		for **x ≥ R** (outer vacuum)

Here $B = \exp\left(\frac{\lambda R}{\upsilon}\right)\left[\cosh\left(\frac{\lambda r_0}{\upsilon}\right)\cdot\cos\left(\rho(R-r_0)\sqrt{b^2-a^2}\right)+\sinh\left(\frac{\lambda r_0}{\upsilon}\right)\cdot\sin\left(\rho(R-r_0)\sqrt{b^2-a^2}\right)\sqrt{\frac{(b+a)}{(b-a)}}\right]$

$$\begin{cases} u(x) = C\cdot\cosh\left(\frac{\lambda r_0}{\upsilon}\right)\cdot\cos\left(\rho(x-r_0)\sqrt{b^2-a^2}\right)+C\cdot\sinh\left(\frac{\lambda r_0}{\upsilon}\right)\cdot\sqrt{\frac{b+a}{b-a}}\cdot\sin\left(\rho(x-r_0)\sqrt{b^2-a^2}\right) \\ s(x) = C\cdot\cosh\left(\frac{\lambda r_0}{\upsilon}\right)\cdot\sqrt{\frac{b-a}{b+a}}\cdot\sin\left(\rho(x-r_0)\sqrt{b^2-a^2}\right)-C\cdot\sinh\left(\frac{\lambda r_0}{\upsilon}\right)\cdot\cos\left(\rho(x-r_0)\sqrt{b^2-a^2}\right) \end{cases}$$

for **r0<x≤ R** (uranium)

Далее C=1

Here λ is found from the transcendental equation for **λ, r0, R,** and one-group cross-sections

$$\mathrm{tg}\left(\rho(R-r_0)\sqrt{b^2-a^2}\right)=\frac{\left(1+\mathrm{tgh}\left(\frac{\lambda r_0}{\upsilon}\right)\right)}{\left(\sqrt{\frac{b-a}{b+a}}-\sqrt{\frac{b+a}{b-a}}\mathrm{tgh}\left(\frac{\lambda r_0}{\upsilon}\right)\right)}$$

The one-group cross-sections are in combinations (*b*+*a*) and (*b*-*a*):

$$b+a=\left[\sigma_t-\sigma_s(1-2q)\right]+\frac{\lambda}{\rho\upsilon} \qquad b-a=\left[\sigma_f\left(\nu_f-1\right)-\sigma_a\right]-\frac{\lambda}{\rho\upsilon}$$

The combinations come from the equations derived in the back-and-forth approximation [1, 2] where interaction with the nucleus is described by three reactions:

<u>scattering</u> $\sigma_s(\mu'\to\mu)=\sigma_s[(1-q)\delta(\mu-\mu')+q\delta(\mu+\mu')]$

σ_s = scattering cross-section,

q = back scatter probability, (1-q) = forth scatter probability

<u>fission:</u> $\sigma_f(\mu'\to\mu)=\nu_f\sigma_f\left[\frac{1}{2}\delta(\mu-\mu')+\frac{1}{2}\delta(\mu+\mu')\right]$

σ_f = fission cross-section,

ν_f = fission neutrons, q_f =1/2 is back or forth direction probability

<u>absorption:</u> σa = absorption cross-section

<u>Total cross-section:</u> σt = σs + σf + σa

Figures 1 and 2 compare the analytical and Monte Carlo (PRIZMA) solutions to demonstrate correctness of the equations we derived. The figures show neutron fluxes in a sphere with **R=12cm , r0=2cm** and **λ=2.6520** [1/s-7].

Figure 2 shows the flows in the central region in more detail.

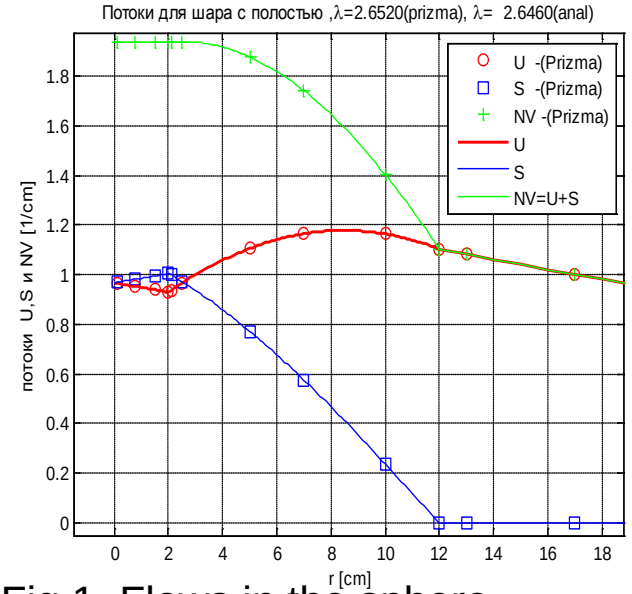


Fig 1. Flows in the sphere

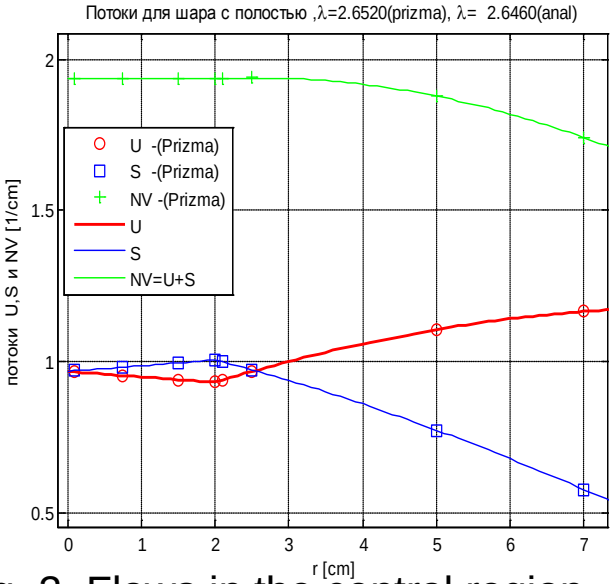


Fig. 2. Flows in the central region

N is the inward flow. **S** is the outward flow. **N+S=u** is the total flow.

We can clearly see how the “time absorption” works in the inner cavity and beyond the sphere. The analytical (solid lines) and Monte Carlo solutions are seen to absolutely agree with each other.

To calculate λ for the entire system and λk's for its individual components we will need <u>three functionals</u>:

***N*, N_k** = total neutrons in the system and in region *k*,
ΔJ, ΔJ_k = balance between neutron inflow and outflow through the system surface and the surface of region k, and
***B*, B_k** = total neutron reproduction in the system and in region k.

In the general case, for $x_{k-1} \le x \le x_k$, they read as

$$N_k = \int_{x_{k-1}}^{x_k} u(x)dx$$

$$\Delta J_k = \upsilon s_{k-1} - \upsilon s_k$$

$$B_k = \rho\upsilon\sigma_t\left(\nu_t - 1\right)\int_{x_{k-1}}^{x_k} u(x)dx = \rho\upsilon\left[\sigma_f\left(\nu_f - 1\right) - \sigma_a\right]N_k = \rho\upsilon\left[(b-a) + \frac{\lambda}{\rho\upsilon}\right]N_k$$

For derivation of B_k we use the <u>total microscopic neutron reproduction cross-section</u>

$$\sigma_t\left(\nu_t - 1\right)$$

where ν_t is the average number of secondary neutrons per collision

$$\nu_t = \frac{\sigma_s + \sigma_f\nu_f}{\sigma_t}$$

We suggest that λ and λk should be calculated with the following formulas for ***N, Nk , ΔJ, ΔJk, B,*** and ***Bk***:

$$\lambda_k = \frac{B_k + \Delta J_k}{N_k} \qquad \text{for region k}$$

$$\lambda = \frac{B + \Delta J}{N} \qquad \text{for the entire system}$$

Let's calculate the functions using the solution we obtained for the hollow sphere.

Region 1 = inner vacuum **0 ≤ x ≤ r0**

Solution

$$\begin{cases} u(x) = \cosh\left(\frac{\lambda x}{\upsilon}\right) \\ s(x) = -\sinh\left(\frac{\lambda x}{\upsilon}\right) \end{cases}$$

Functionals

$$\Delta J_1 = \upsilon s(0) - \upsilon s(r_0) = \upsilon \sinh\left(\frac{\lambda r_0}{\upsilon}\right)$$

$$N_1 = \int_0^{r_0} u(x)dx = \int_0^{r_0} \cosh\left(\frac{\lambda x}{\upsilon}\right)dx = \frac{\upsilon}{\lambda}\sinh\left(\frac{\lambda r_0}{\upsilon}\right)$$

$$B_1 = 0$$

λ1 for the cavity

$$\lambda_1 = \frac{B_1 + \Delta J_1}{N_1} = \frac{0 + \upsilon \sinh\left(\frac{\lambda r_0}{\upsilon}\right)}{\frac{\upsilon}{\lambda}\sinh\left(\frac{\lambda r_0}{\upsilon}\right)} = \lambda$$

Region 2 – uranium **r0 ≤ x ≤ R**

Here we can use the awkward expressions we derived but it seems better to use the second equation from the system for the *back-and-forth* approximation and express *u* in terms of *s*.

The system of equations:

$$\begin{cases} \dfrac{\partial u}{\partial x} = -s\rho(b+a) \\ \dfrac{\partial s}{\partial x} = u\rho(b-a) \end{cases}$$

, i.e., $u = \dfrac{1}{\rho(b-a)}\dfrac{\partial s}{\partial x}$, then

Functionals

$$N_2 = \int_{r_0}^{R} u(x)dx = \int_{r_0}^{R} \frac{1}{\rho(b-a)}\frac{\partial s}{\partial x}dx = \frac{s(R)-s(r_0)}{\rho\sigma_t(\nu_t-1)-\dfrac{\lambda}{\upsilon}} = \frac{\upsilon[s(R)-s(r_0)]}{\rho\upsilon\sigma_t(\nu_t-1)-\lambda}$$

$$\Delta J_2 = \upsilon[s(0)-s(r_0)]$$

$$B_2 = \int_{r_0}^{R} \rho\upsilon\sigma_t(\nu_t-1)u(x)dx = \rho\upsilon\sigma_t(\nu_t-1)N_2$$

***λ2* for the uranium region**

$$\lambda_2 = \frac{B_2+\Delta J_2}{N_2} = \frac{B_2}{N_2}+\frac{\Delta J_2}{N_2} = \frac{\rho\upsilon\sigma_t(\nu_t-1)N_2}{N_2} + \upsilon[s(r_0)-s(R)]\frac{\rho\upsilon\sigma_t(\nu_t-1)-\lambda}{\upsilon[s(R)-s(r_0)]} =$$

$$= \rho\upsilon\sigma_t(\nu_t-1) - [\rho\upsilon\sigma_t(\nu_t-1)-\lambda] = \lambda$$

Region 3 = outer vacuum **R ≤ x ≤ R3**

Solution

$$\begin{cases} u(x) = B * \exp\left(-\dfrac{\lambda x}{\upsilon}\right) \\ s(x) = u(x) \end{cases}$$

Functionals

$$B_3 = 0$$

$$\Delta J_3 = \upsilon\left[s(R) - s(R_3)\right] = B * \upsilon\left[\exp\left(-\frac{\lambda R}{\upsilon}\right) - \exp\left(-\frac{\lambda R_3}{\upsilon}\right)\right]$$

$$N_3 = \int_R^{R_3} u(x)dx = \int_R^{R_3} B * \exp\left(-\frac{\lambda x}{\upsilon}\right)dx = -\frac{B}{\frac{\lambda}{\upsilon}} * \left[\exp\left(-\frac{\lambda R_3}{\upsilon}\right) - \exp\left(-\frac{\lambda R}{\upsilon}\right)\right]$$

***λ3* for vacuum beyond the sphere**

$$\lambda_3 = \frac{0 + \Delta J_3}{N_3} = -\frac{B * \upsilon\left[\exp\left(-\frac{\lambda R}{\upsilon}\right) - \exp\left(-\frac{\lambda R_3}{\upsilon}\right)\right]}{\frac{B}{\left(\frac{\lambda}{\upsilon}\right)} * \left[\exp\left(-\frac{\lambda R_3}{\upsilon}\right) - \exp\left(-\frac{\lambda R}{\upsilon}\right)\right]} = \lambda$$

Now calculate λ_{12} for the entire system.

Hollow sphere 0 < x < R

Note that

$$\begin{cases} N_{12} = N_1 + N_2 \\ \Delta J_{12} = 0 - \upsilon s(R) = \left(0 - \upsilon s(r_0)\right) + \left(\upsilon s(r_0) - \upsilon s(R)\right) = \Delta J_1 + \Delta J_2 \\ B_{12} = B_1 + B_2 \end{cases}$$

Then

$$\lambda_{12} = \frac{B_{12} + \Delta J_{12}}{N_{12}} = \frac{\left(B_1 + \Delta J_1\right) + \left(B_2 + \Delta J_2\right)}{N_{12}} = \frac{N_1}{N_{12}}\left[\frac{\left(B_1 + \Delta J_1\right)}{N_1}\right] + \frac{N_2}{N_{12}}\left[\frac{\left(B_2 + \Delta J_2\right)}{N_2}\right] =$$

$$= \frac{N_1}{N_{12}}\lambda_1 + \frac{N_2}{N_{12}}\lambda_2 = \lambda\left(\frac{N_1 + N_2}{N_{12}}\right) = \lambda$$

So, the time constant of neutron multiplication in the entire system and in each of its region is one and the same.

$$\lambda_{12} = \lambda_1 = \lambda_2 = \lambda$$

Why we choose these functionals

Eigenvalue calculations by the relaxation method start from an arbitrary instantaneous neutron source distribution and then track the neutron flux over time. It is assumed that the neutron density distribution in the generation eventually becomes the eigenfunction (the system relaxes to its fundamental state). By that time the energy, space, angle, and time variables have become separated and the further behavior of the system is described by the simple exponential flow dependence on time

$$N\left(\vec{r},E,\vec{\Omega},t\right)=N_0\left(\vec{r},E,\vec{\Omega}\right)\exp\left[\lambda_0 t\right]$$

3. Dermott E. Cullen, “Static and Dynamic Criticality: Are They Different?”, UCRL-TR-201506 (November 2003).

Why we choose these functionals

There are two methods for λ calculation: static (by Kellog) and dynamic (relaxation) [3]. The first method uses an equation with a trial λ in the time absorption cross-section

$$\vec{\Omega}\vec{\nabla}\upsilon N = -\left(\Sigma_t + \frac{\lambda}{\upsilon}\right)\upsilon N + \iint\left[\frac{\Sigma_f^{'}\nu'_f f'_f}{k} + \Sigma_s^{'} f'_s\right]\upsilon' N' dE' d\vec{\Omega}$$

Iterations in λ are used to find the root of equation $k(\lambda) = 1$

The second method uses the equation

$$\frac{\partial N}{\partial t} + \vec{\Omega}\vec{\nabla}\upsilon N = -\Sigma_t \upsilon N + \iint\left[\Sigma_f^{'}\nu'_f f'_f + \Sigma_s^{'} f'_s\right]\upsilon' N' dE' d\vec{\Omega}$$

to determine the logarithmical time derivative of the total neutrons *No* in the system at the time when the neutron density function relaxes to its fundamental mode:

$$\lambda = \frac{dLn(N_0)}{dt}$$

Why we choose these functionals
Dynamic multiplicity

Here we consider the integrals of the left-hand and right-hand parts with respect to the space of the system (or region V), all energies and all directions.

$$\frac{\partial N}{\partial t} = \iint \left[\Sigma_f' \nu_f' f_f' + \Sigma_s' f_s' \right] \upsilon' N' dE' d\vec{\Omega} - \Sigma_t \upsilon N - \vec{\Omega}\vec{\nabla}\upsilon N$$

Integration yields $\frac{\partial N_0}{\partial t} = B + \Delta J$

$N_0 = \iiint\limits_{V,E,\Omega} N(r,E,\vec{\Omega}) d\vec{r} dE d\vec{\Omega}$ is the total number of neutrons in the system (V)

$\Delta J = \iiint\limits_{V,E,\Omega} \vec{\Omega}\vec{\nabla}\upsilon N d\vec{r} dE d\vec{\Omega}$ is the balance of neutron inflow and outflow through the system surface (V)

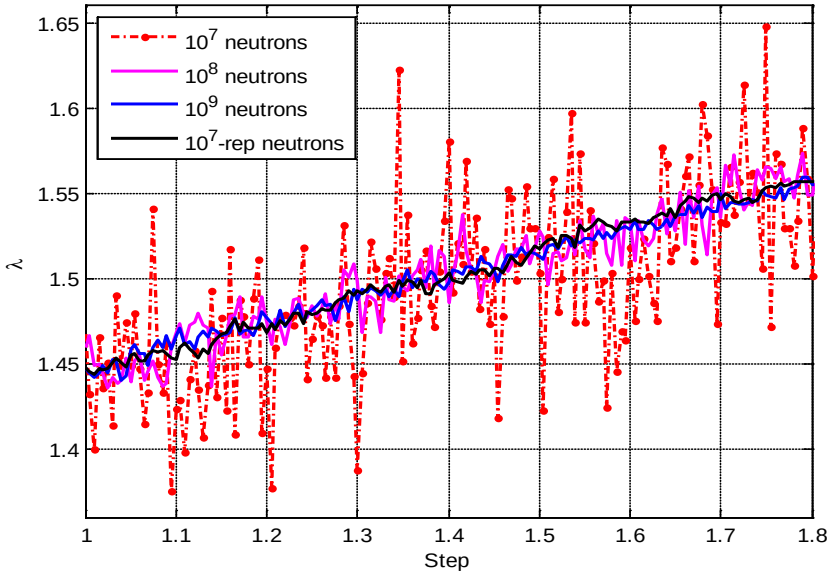
$B = \iiint\limits_{V,E,\Omega} \rho\sigma_t(\nu_t - 1)\upsilon N(\vec{r},E,\vec{\Omega}) d\vec{r} dE d\vec{\Omega}$ is the total neutron reproduction in the system (V)

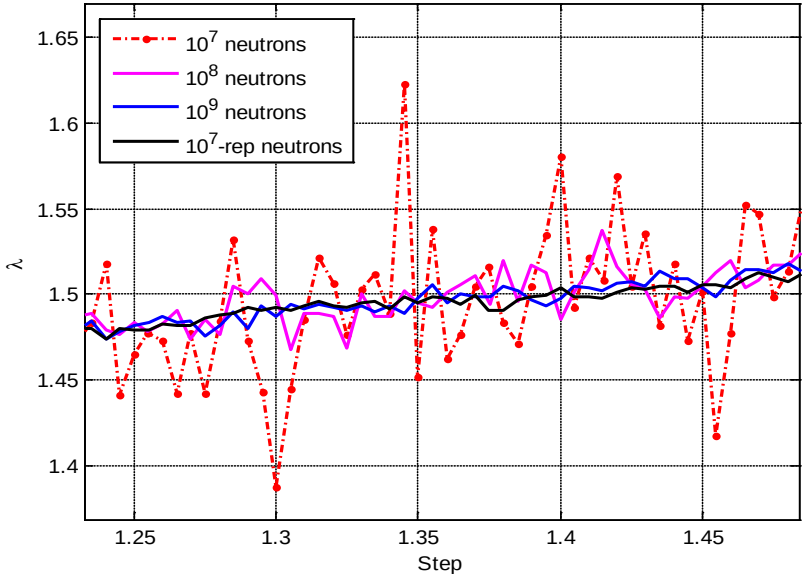
Solving gives that the right-hand and left-hand parts of the equation are equal and then $\lambda = \frac{\partial Ln(N_0)}{\partial t} = \frac{B + \Delta J}{N_0}$

A sample Monte Carlo calculation of layered system compression by the relaxation method with the logarithmic derivative and neutron reproduction cross-section

- The layered system contains U, Pu, Be, Al, C, and other.
- Continuous-energy data from ENDF/B-V in PROM format

Relaxation and balance methods for **10^7, 10^8,** and **10^9** neutron histories



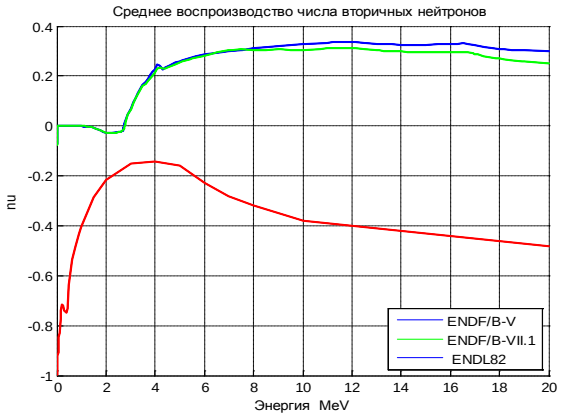


The calculation of the right-hand side with the reproduction cross-section is 100 times more effective that that of the derivative in the left-hand side.

Neutron reproduction data in PROM [4-5] libraries

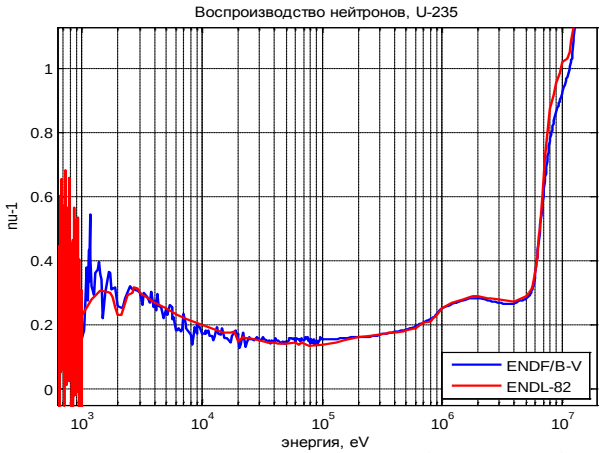
$$(\bar{\nu} - 1) = \sum_i^{Nxs} \frac{\sigma_i(E)}{\sigma_{tot}(E)} \sum_k (\nu_i^{(k)}(E) - 1)$$

Be-9

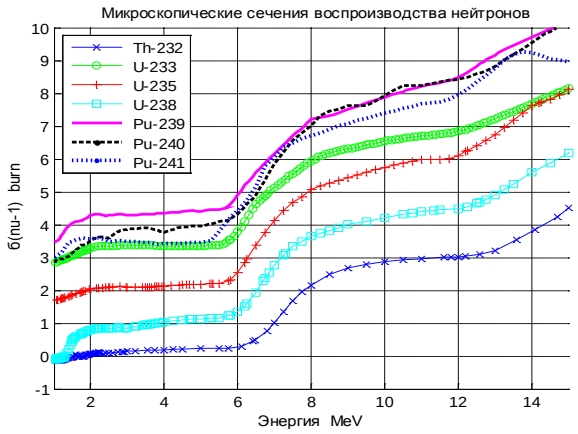


$(\nu_{tot} - 1)$

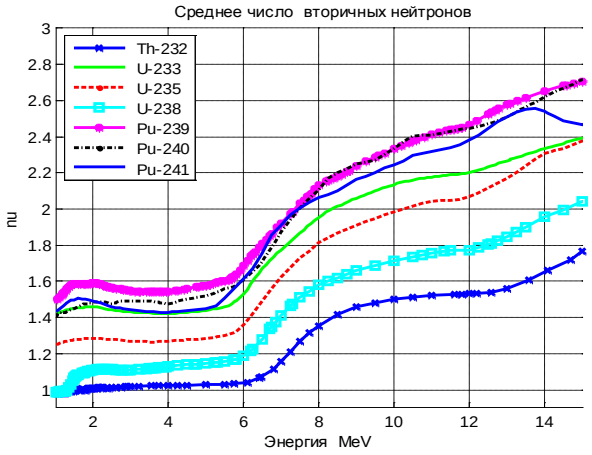
U-235



$(\nu_{tot} - 1)$



$\sigma_{tot}(\nu_{tot} - 1)$



ν_{tot}

Static multiplicity

Consider the equation for static criticality with an addition $\pm\Sigma_f' f_f'$

$$\lambda N = \iint \left[\frac{\Sigma_f' \nu_f' f_f'}{k} + \Sigma_s' f_s' \pm \Sigma_f' f_f' \right] \upsilon' N' dE' d\vec{\Omega}' - \Sigma_t \upsilon N - \vec{\Omega}\vec{\nabla}\upsilon N$$

Integrate its right- and left-hand sides : $\iiint\limits_{V,E,\Omega} \cup d\vec{r}dEd\vec{\Omega}$

The integration gives the functional dependence $\lambda(k)$

$$\lambda N_0 = \left(\frac{1}{k} - 1\right)(\nu N)_f + B + \Delta J$$

where $(\nu N)_f = \iiint\limits_{V,E,\Omega} \rho\sigma_f \nu_f \upsilon N(\vec{r}, E, \vec{\Omega}) d\vec{r}dEd\vec{\Omega}$ is the number of fission neutrons in the system.

This is the fourth functional we will need later. You remember the other three.

The root of $k(\lambda) = 1$ is sought by iterations.

For the trial $\boldsymbol{\lambda_n}$ we solve Keff and calculate the functionals $N_{0,n}$, B_n , ΔJ_n $\Rightarrow$ $\lambda_n = \left[\left(\frac{1}{k} - 1 \right) (\nu N)_{f,n} + B_n + \Delta J_n \right] \Big/ N_{0,n}$

The next iteration is done with a new trial $\boldsymbol{\lambda_{n+1}}$ obtained from $\boldsymbol{\lambda_n}(k=1)$:

$$\lambda_{n+1} = \frac{B_n + \Delta J_n}{N_{0,n}}$$

Introduce new quantities whose physical meaning is more comprehensible to derive more convenient expressions for *λ(k)* and *k(λ)*.

$\lambda_f = (\nu N)_f / N_0$ is fission neutron production rate.

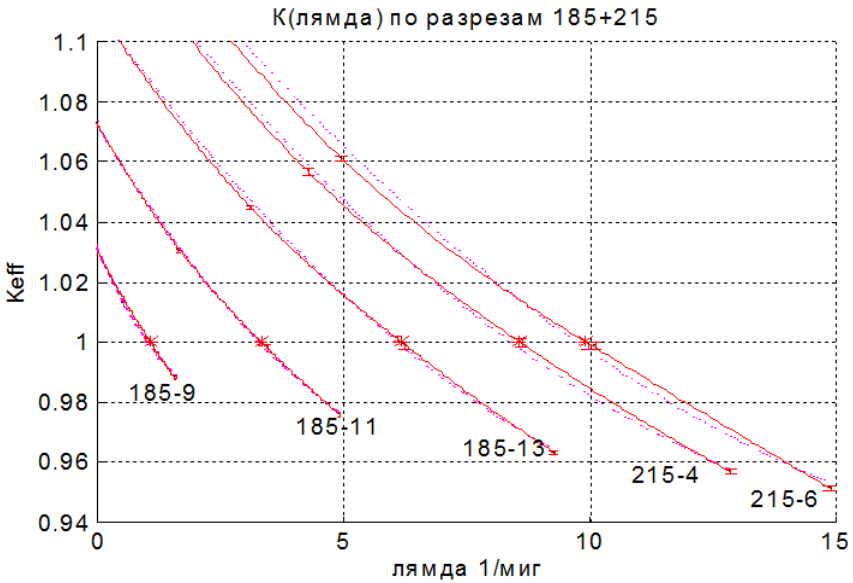
$\lambda_{rep} = B/N_0$ is neutron reproduction rate.

$L = -\Delta J / N_0$ is neutron leakage rate.

Then $\lambda(k) = \left(\frac{1}{k} - 1\right)\lambda_f + \lambda_{rep} - L$ $\qquad k(\lambda) = \frac{\lambda_f}{\lambda + \lambda_f - \lambda_{rep} + L}$

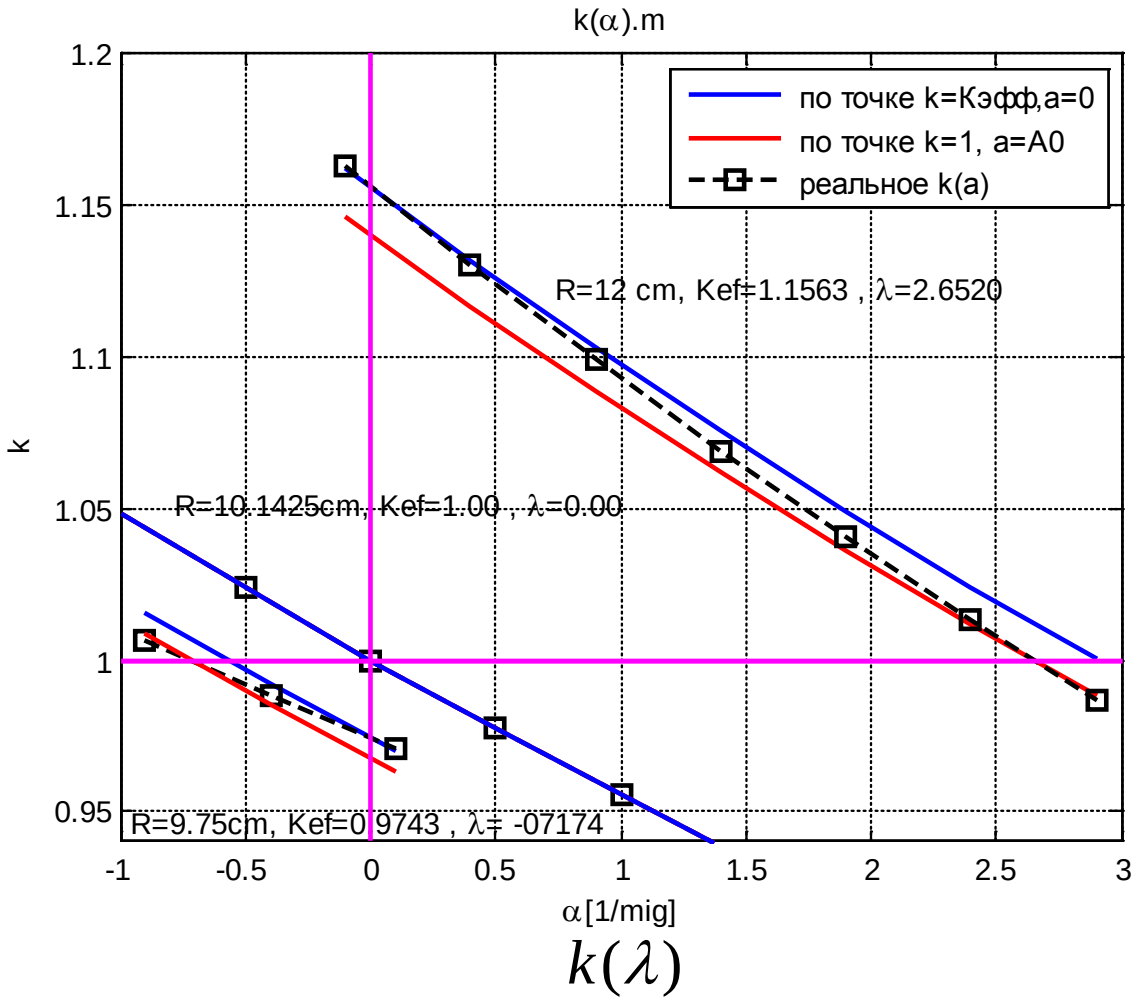
The plots below demonstrate $k(\lambda)$ for spheres of different radii.

Monte Carlo, ENDF/B-V



In the back-and-forth approximation,

$$k(\lambda) = \frac{\lambda_f}{\lambda + \lambda_f - \lambda_{rep} + L}$$ and: $\lambda_f = \rho\upsilon\sigma_f\nu_f$, $\lambda_{rep} = \rho\upsilon\sigma_t(\nu_t - 1)$



$k(\lambda)$

The relaxation method with two λ: $\lambda_0 = \lambda_1 + \lambda_2(\lambda_1)$

Let $$N\left(\vec{r},E,\vec{\Omega},t\right) = N_0\left(\vec{r},E,\vec{\Omega}\right)\exp\left[\lambda_0 t\right] = N_0\left(\vec{r},E,\vec{\Omega}\right)\exp\left[\left(\lambda_1+\lambda_2\right)t\right]$$

Then for **λ1>0**, the transport equation can be written as

$$\frac{\partial N}{\partial t} = \iint\left[\Sigma_f^{'}\nu_f' f_f' + \Sigma_s^{'} f_s'\right]\upsilon' N'\,dE'\,d\vec{\Omega} - \left(\Sigma_t + \frac{\lambda_1}{\upsilon}\right)\upsilon N - \vec{\Omega}\vec{\nabla}\upsilon N$$

The solution is either $\lambda_2 = \dfrac{\partial Ln\left(N_0\right)}{\partial t}$, or $\lambda_2 = \dfrac{B+\Delta J}{N_0} = \lambda_{rep} - L$

with all the three funationals dependent on the “time absorption” $\frac{\lambda_1}{\upsilon}$ added to all macroscopic total cross-sections of system materials including vacuum.

It is clear that the solution is the sum: $\lambda_0 = \lambda_1 + \lambda_2(\lambda_1)$

Here the eigenfunction of the new system will coincide with that of the initial system for any **λ_1** which can be varied when calculating **$\lambda_2(\lambda_1)$**.

Tow lambdas with a negative λ1 : $\lambda_0 = -|\lambda_1| + \lambda_2(\lambda_1)$

Let $N(\vec{r}, E, \vec{\Omega}, t) = N_0(\vec{r}, E, \vec{\Omega}) \exp[\lambda_0 t] = N_0(\vec{r}, E, \vec{\Omega}) \exp\left[\left(-|\lambda_1| + \lambda_2\right) t\right]$

To reduce the problem with a negative λ to a problem where λ is positive, we need to add in the entire system a fictitious δ-fission with the cross-section $|\lambda_1| / \upsilon$ which produces two secondary neutrons: $\delta(n, fis2n)$

In this δ-fission, each of the two secondary neutrons moves in the same direction and with the same energy as the incident neutron.

Then the transport equation can be written as

$$\frac{\partial N}{\partial t} = \iint \left[\Sigma_f' \nu_f' f_f' + \Sigma_s' f_s' + 2\frac{|\lambda_1|}{\upsilon} \right] \upsilon' N' dE' d\vec{\Omega} - \left(\Sigma_t + \frac{|\lambda_1|}{\upsilon} \right) \upsilon N - \vec{\Omega}\vec{\nabla}\upsilon N$$

Note that fictitious δ-fission is added to all macroscopic total cross-sections of system materials including vacuum.

It is clear that the solution is again the sum $\lambda_0 = -|\lambda_1| + \lambda_2(|\lambda_1|)$

Neutron batch combing

It is known that Keff is calculated using the method of generations which estimates the ratio of neutrons produced by the neutrons of the previous generation to the neutrons of the previous generation

$$K_{эф,i} = \frac{N_i}{N_{i-1}}$$

The method requires neutron combing. Each iteration begins with removing or adding some starting neutrons to make them equal to a predefined number.

If Kef >1, some neutrons are not used in the next generation.

If Kef <1, we have to increase neutrons to a predefined number in some way.

In the first case, excessive histories will be tracked otherwise.

In the second one, there arises a problem of proving independence of results for neighbor batches.

To keep the number of neutrons in generations close to the predefined one, we can increase (or decrease) the number of fission neutrons by a value γ and the resulted value of Keff, respectively, decrease (or increase) by γ. It is clear that the iegenfunctions are conserved and the factor γ can be varied in the course of calculation.

Conclusion

We have considered a number of methods for calculating the neutron multiplication rate $N(t)=N0\exp(\lambda t)$ with the time constant λ for systems with stationary geometry. I used a number of known and obvious concepts but blended them and presented in one paper and in one manner.

The method proposed for λ calculation uses continuous-energy microscopic neutron reproduction cross-sections. Results calculated with the balance method are shown to completely agree with calculations by the relaxation (dynamic λ) and Kellog (static λ) methods.

For stationary systems, it is shown that λ for the full system can be obtained by calculating λ for its individual component. For this end we use the total number of neutrons (N), total neutron reproduction (B), the resultant neutron current (ΔJ) through the surface of the selected system component, and the expression $\lambda=(B+\Delta J)/N$.

Conclusion

A method is proposed for calculating dynamic λ2 with the additional “time absorption” λ1/v or the additional “time multiplication” –λ1/v. As a result, the sought neutron multiplication rate λ0 is calculated as λ0 = λ1 + λ2(λ1).

For the Monte Carlo calculation of the effective neutron multiplication factor Kef by the method of generation, an approach is proposed for neutron batch combing which helps keep the number of neutrons in each next generation close to the initial number of neutrons in the batch.

Some statements made in the paper were tested in Monte Carlo calculations. One-group λ calculations by the PRIZMA code are shown to completely agree with the analytical solutions derived in the back-and-forth approximation.

PRIZMA calculations of λ with the ENDF/B–V continuous-energy cross-sections suggest that the use of reproduction cross-sections is quite acceptable both in accuracy and in efficiency.

References

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THANK YOU FOR YOUR TIME.