SELF-ADJUSTING METHOD OF VELOCITY PROFILE RECONSTRUCTION FROM PDV-DATA

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Fundamental challenges of PDV-Signal Processing :

- *The relation of the PDV-method's resolving time and accuracy*

$$\delta v \cdot \delta \tau \geq \lambda / 8\pi$$

- *The assumption of a single frequency PDV-signal's spectrum allows to increase the accuracy several times at the same resolving time [1]*

- *The Hilbert transform applying allows Doppler phase obtaining formally with any value of resolving time*

[1] D. H. Dolan, REV. OF SCI. INSTRUMENTS **81**, 053905 2010

The Hilbert transform applying to PDV-signal's phase reconstruction.

CHALLENGES

- *Higher Doppler harmonics’ presence in the PDV-signal*
- *The amplitude of the first Doppler harmonic is constantly changing during the experiment*
- *An unknown "center-line" presenting in the signal, and hereinafter referred to as zero-harmonic - the signal component, which frequency is much below of the Doppler's one*
- *The presence of random noise in the signal*

- ***Estimates of uncertainty and resolving time***

An alternating approach. Signal filtering.

- *Let we know the dependence of the signal's phase on time ψ(t), then the signal can be filtered by averaging it over half of it's period*

$$< S >_{\pi} (\psi_0) = \frac{1}{\pi}\int_{-\pi/2}^{+\pi/2} S(\psi_0 + \psi)d\psi$$

- *By averaging the signal over it's period, it is possible to calculate the zero harmonic*

$$S_0 = < S >_{2\pi} (\psi_0) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} S(\psi_0 + \psi)d\psi$$

- *Thus, knowing ψ(t), one can significantly improve signal quality*

$$S_c(\psi_0) = < S >_{\pi} (\psi_0) - < S >_{2\pi} (\psi_0)$$

Alternative approach. Signal filtering with using approximate phase time-dependence $\varphi(t)$

- *Let now know an approximate time dependence of the phase $\varphi(t)$, then one can filter the signal, convoluting it with some EVEN kernel function $F(\varphi)$*

$$< F(\varphi) * S(\varphi_0 + \varphi) >_{2\pi n} = \int_{-\pi n}^{+\pi n} F_n(\varphi) S(\varphi_0 + \varphi) d\varphi$$

- *In this case, one can find such a function $F(\varphi)$ and such an interval [+πn,-πn] which allow cleaning the experimental signal from all Doppler harmonics, including zero one, as well as from the component of random noise spectrally distant from the Doppler frequency*
- *Then the cleaned signal deviation from the true first harmonic will be a value of the second order of smallness from the values of $\frac{\partial A}{\partial \varphi}$ and $\varepsilon = (\psi - \varphi)$, that is, the cleaned signal is almost the same as the first harmonic.*

$$F_n(\varphi) = f_0 \cos\varphi + f_{-1}\cos\left(\varphi - \frac{\varphi}{n}\right) + f_{+1}\cos\left(\varphi + \frac{\varphi}{n}\right)$$

Alternative approach. Signal filtering with using approximate phase time-dependence $\varphi(t)$

- *If to convolute the experimental signal on the interval [+πn,-πn] with ODD kernel function $F_n^{\mathrm{T}}(\varphi)$ of the follow general view :*

$$F_n^{\mathrm{T}}(\varphi) = f_0^{\mathrm{T}} \sin\varphi + f_{-1}^{\mathrm{T}} \sin\left(\varphi - \frac{\varphi}{n}\right) + f_{+1}^{\mathrm{T}} \sin\left(\varphi + \frac{\varphi}{n}\right)$$

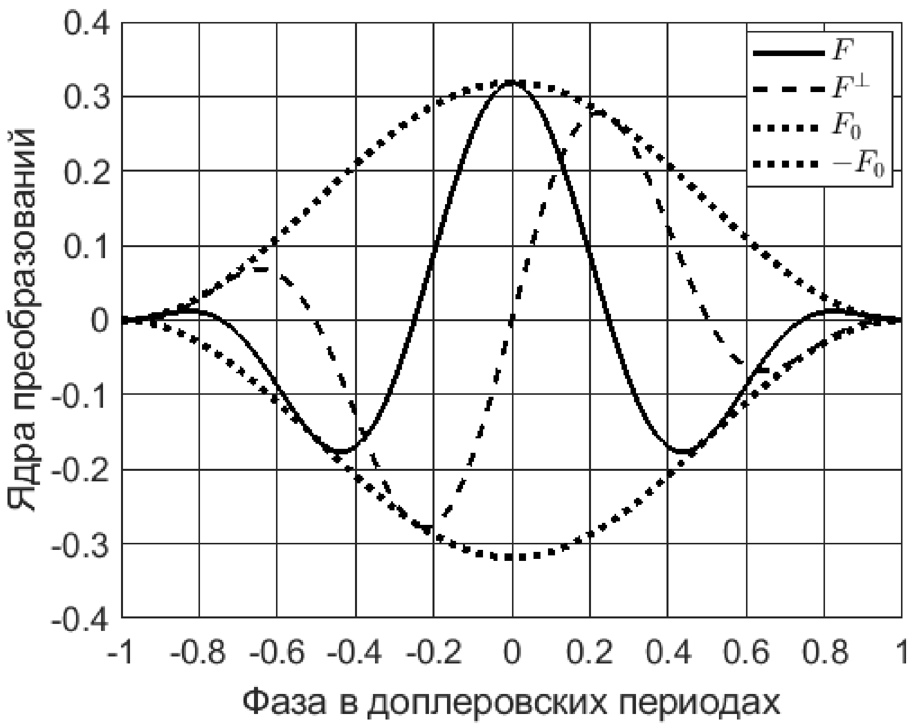
- *one can obtain a signal, which is* quadrature (quarter phase) to the approximate first harmonic:

$$S_1^{\mathrm{T}}(\varphi_0) = < F_n^{\mathrm{T}}(\varphi) * S(\varphi_0 + \varphi) >_{2\pi n} = \int_{-\pi n}^{+\pi n} F_n^{\mathrm{T}}(\varphi) S(\varphi_0 + \varphi) d\varphi$$

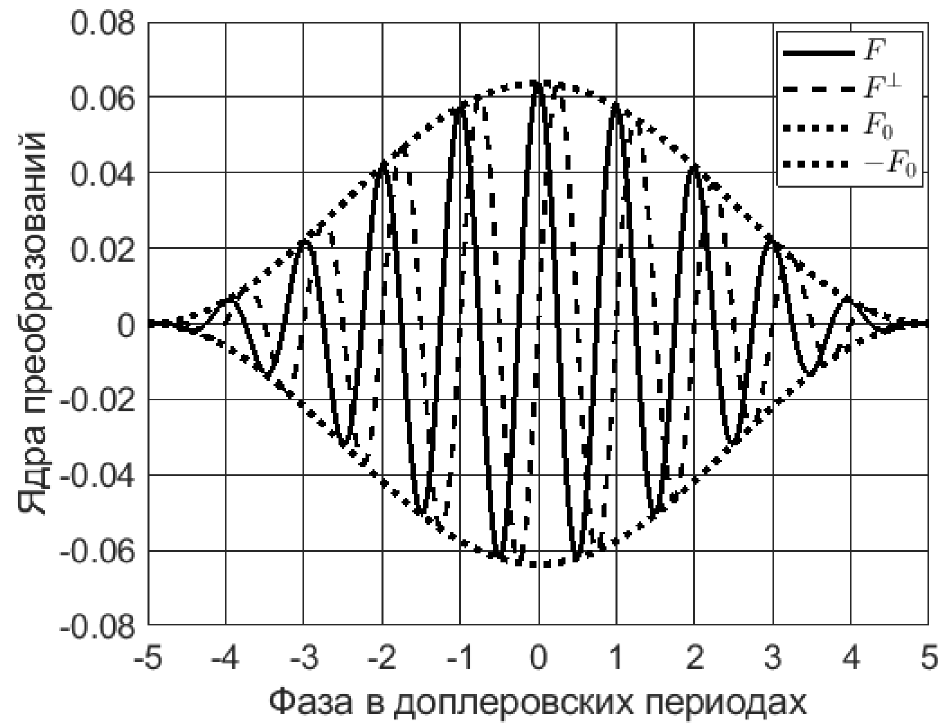
- *The obtained signal's deviation from the one, which is quadrature to the true first harmonic, will be a value of the second order of smallness from the values of* $\frac{\partial A}{\partial \varphi}$ *and* $\varepsilon = (\psi - \varphi)$.

Alternative approach. Kernel functions

n=2



n=10



Alternative approach. Doppler phase reconstruction algorithm

- *The approximate first Doppler harmonic and it's quadrature signal are convenient to present in the complex form:*

$$s_1(\varphi) = S_1(\varphi) + i\, S_1^T(\varphi)$$
$$s_1^*(\varphi) = S_1(\varphi) - i\, S_1^T(\varphi)$$

- *Square signal amplitude:*

$$A_1^2(\varphi) = s_1^* s_1(\varphi)$$

- *True Doppler phase:*

$$\frac{d\psi}{d\varphi} = \sqrt{\frac{s_1^{*\prime} s_1{}' - A_1'^2}{s_1^* s_1}}$$

Alternative approach. Doppler phase reconstruction algorithm

- *The squared uncertainty of the true phase's derivative with respect to the approximate one and the squared relative uncertainty of the velocity*

$$D[\psi'](\varphi) \approx \frac{\pi D_\omega}{2A_1^2(\varphi)} \int_{-\pi n}^{+\pi n} [F_0'(\theta) + \ln(A_1)'(\varphi) F_0(\theta)]^2 \omega_D(\varphi + \theta) d\theta$$

- *The uncertainty in approximation of the frequency constancy on the interval* [+πn,-πn] *:*

$$D[\psi'](\varphi) \approx \frac{D_\omega \omega_D(\varphi)}{2n^3 A_1^2(\varphi)} \{1 + 3n^2 [\ln(A_1)']^2(\varphi)\}$$

Alternative approach. Doppler phase reconstruction algorithm

➢ *The approximate solution taking into account the uncertainty. Control of the residual error ε′ = ψ′ −1 :*

$$\psi'_a(\varphi) \approx [\psi'(\varphi) - 1]\sqrt{1 - \frac{D[\psi'](\varphi)}{[\psi'(\varphi) - 1]^2}}$$

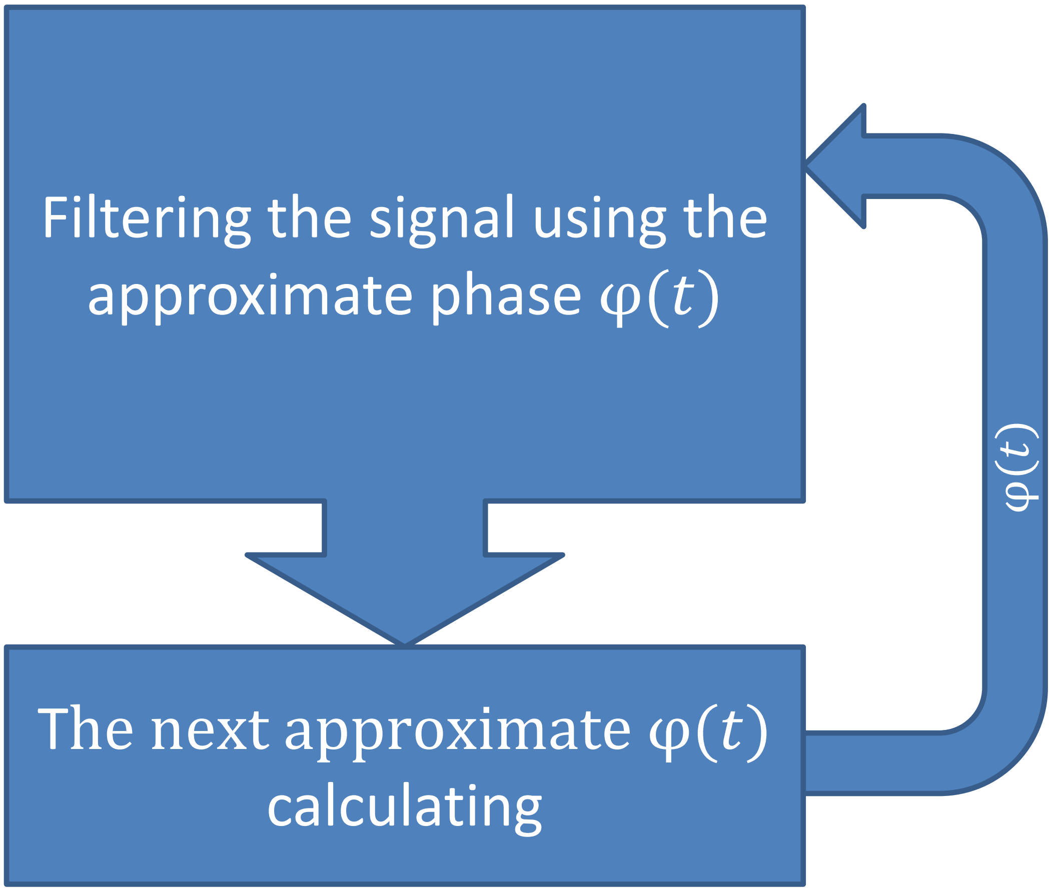
➢ *The regularized solution:*

$$\psi'_r(\varphi) \approx \begin{cases} \psi'_a(\varphi) & \text{если} \quad [\psi'(\varphi) - 1]^2 > rD[\psi'](\varphi) \\ 0 & \text{если} \quad [\psi'(\varphi) - 1]^2 \le rD[\psi'](\varphi) \end{cases} \qquad r = 2 \div 3$$

➢ *Doppler phase:*

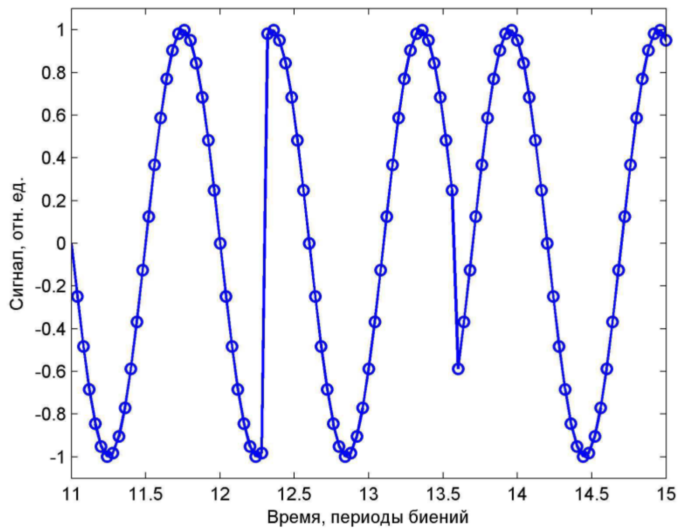
$$\psi(\varphi) \approx \varphi + \int_{\varphi_-}^{\varphi} \psi'_r(\tilde{\varphi})\, d\tilde{\varphi}$$

Alternative approach. Doppler phase reconstruction algorithm

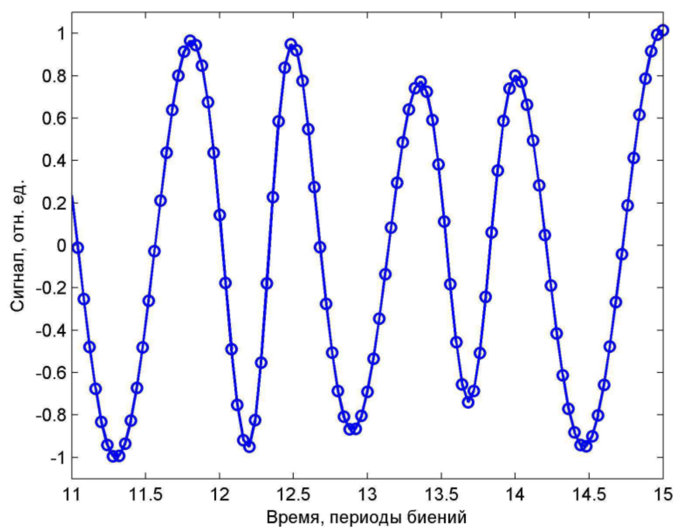


Resolving time. Synthetic signal

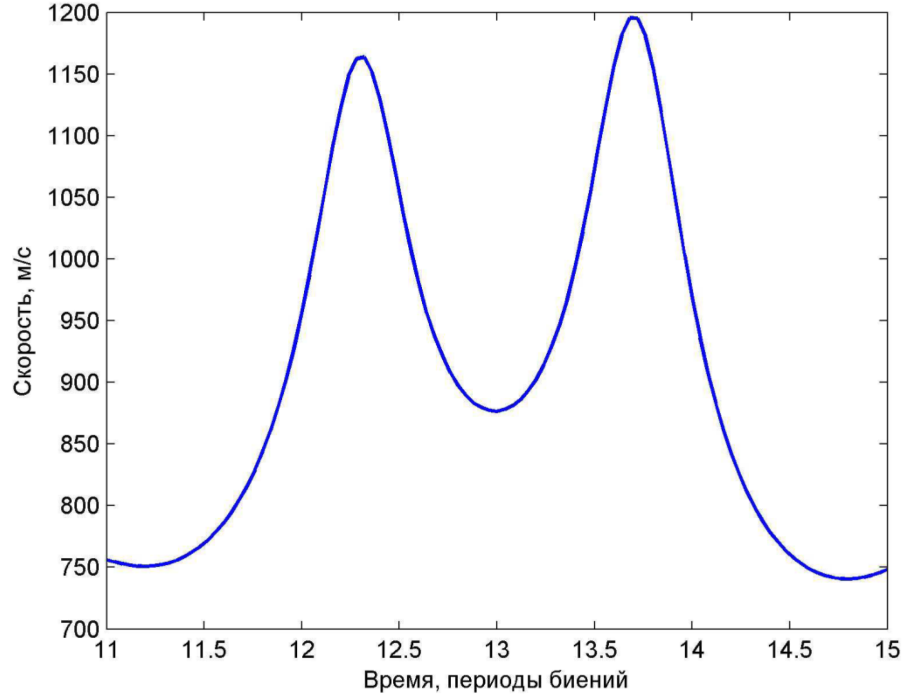
Original signal



Filtered signal



Reconstructed velocity profile

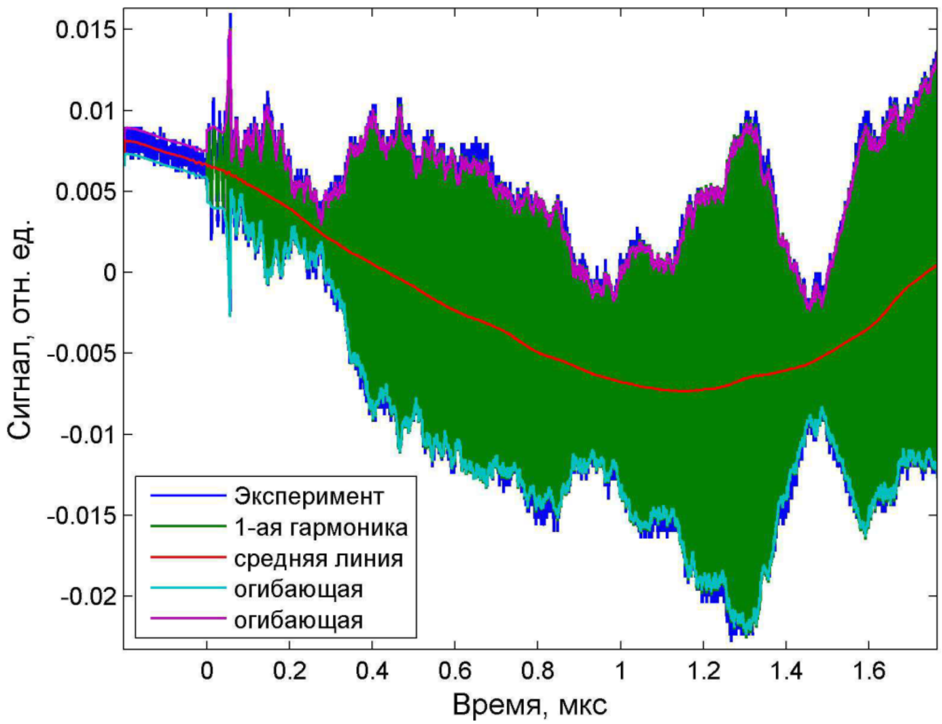


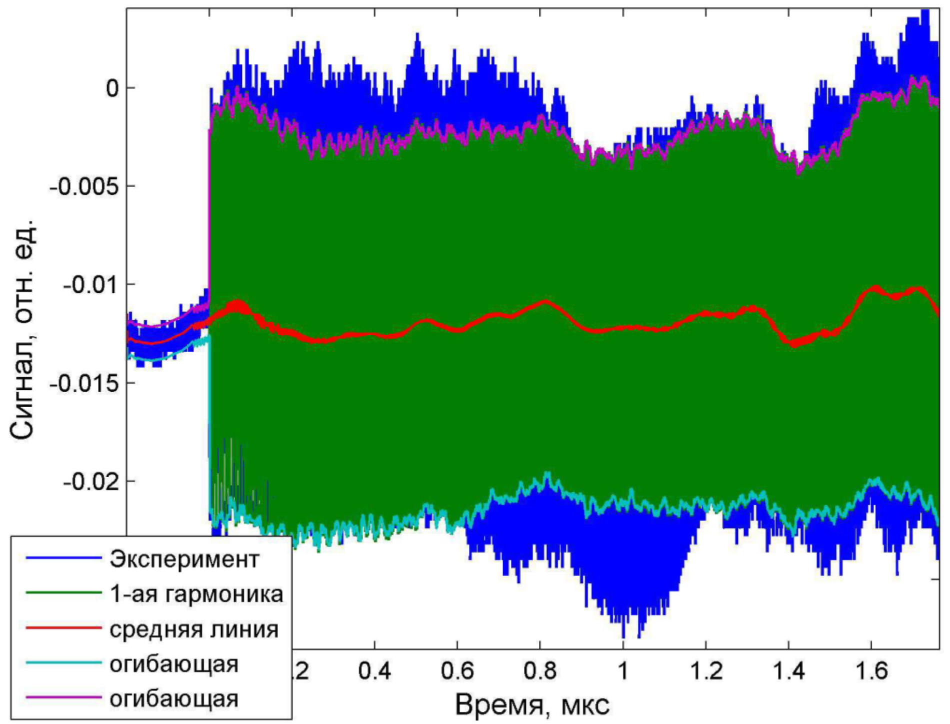
Resolving time depends on

integration interval

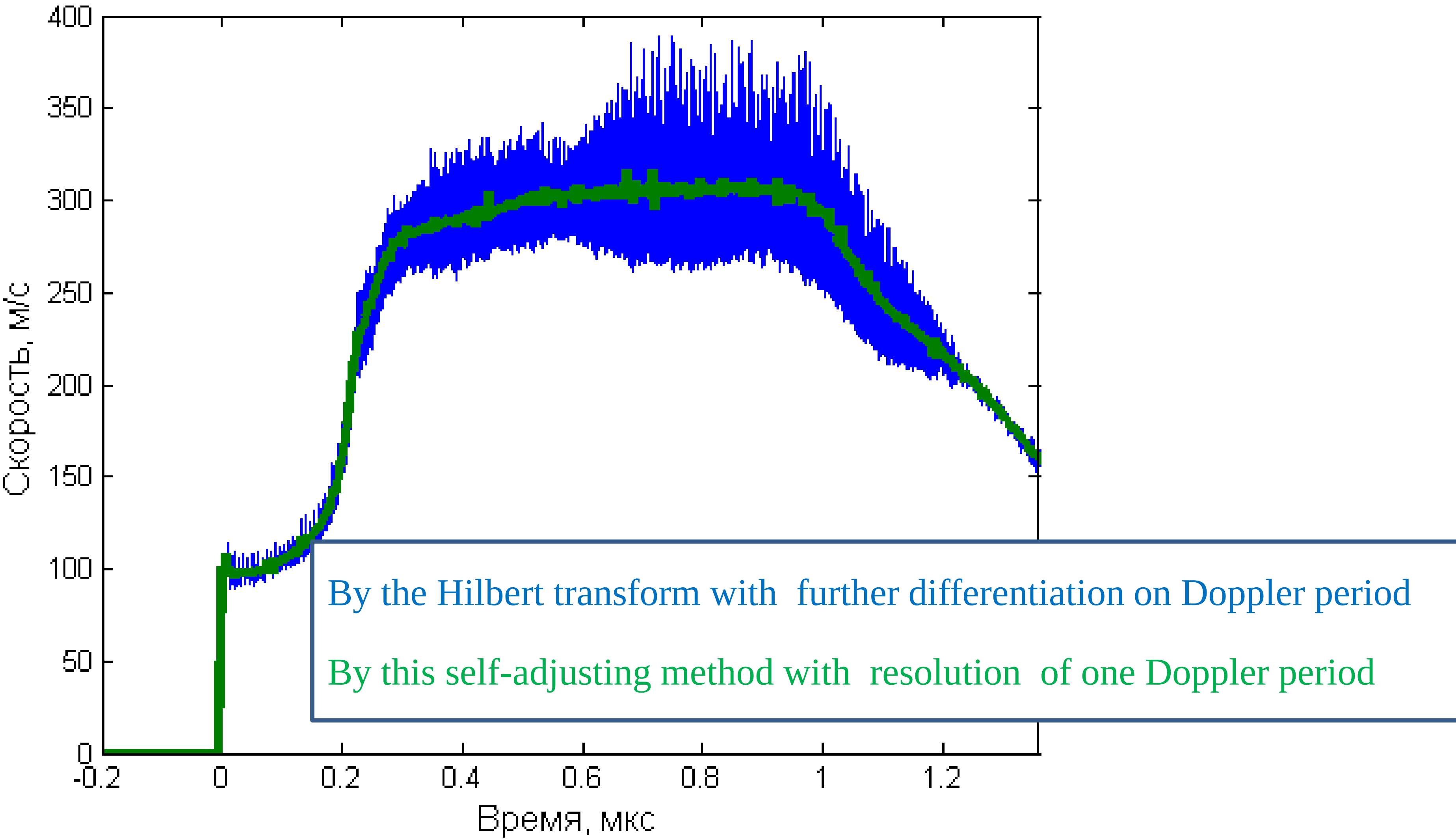
and is n / 2

Test profiles' reconstruction. Signals



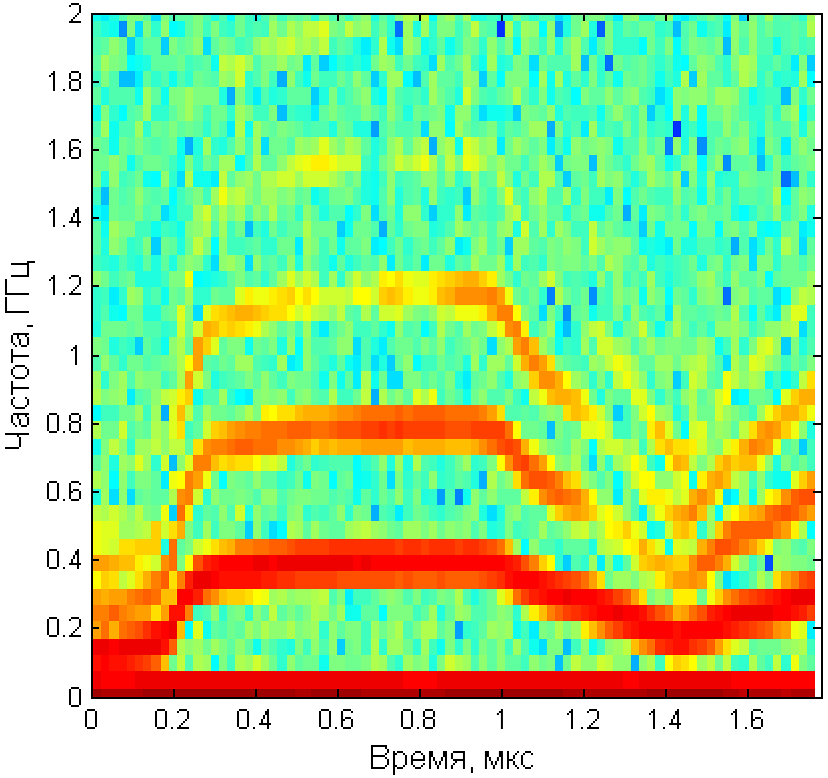


Test profiles' reconstruction. Comparing with the profile, obtained applying the Hilbert transform

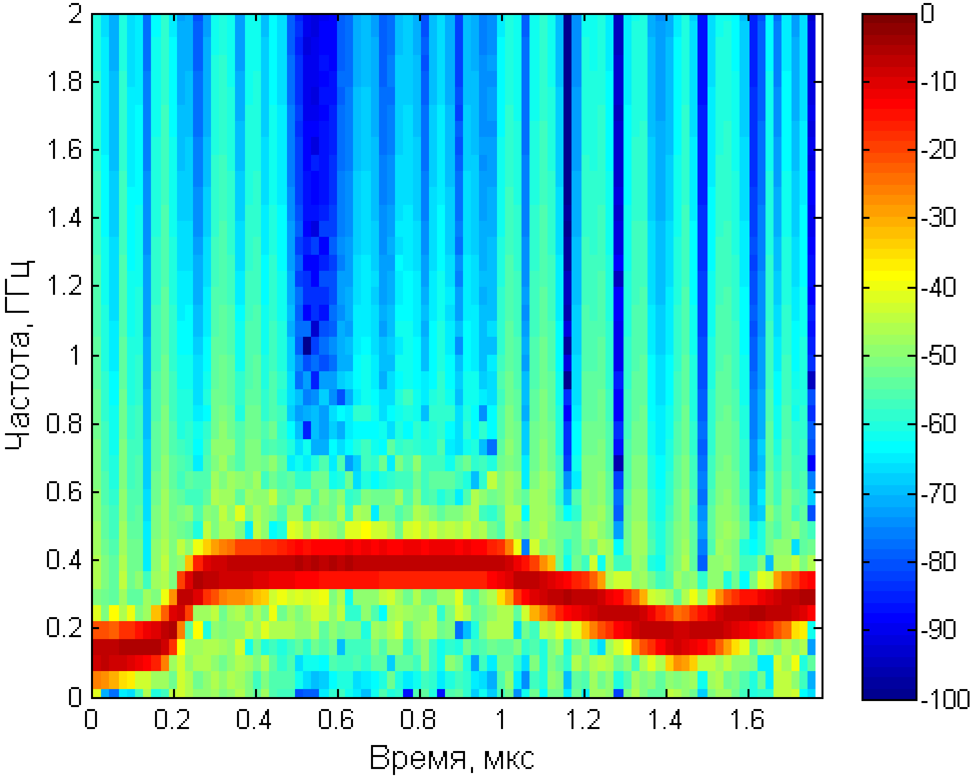


Test profiles' reconstruction. Spectrograms

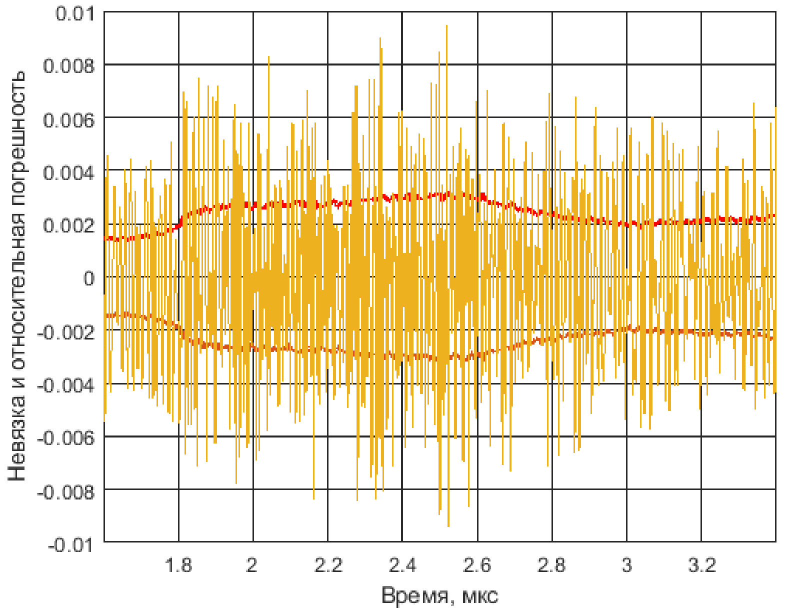
Original signal

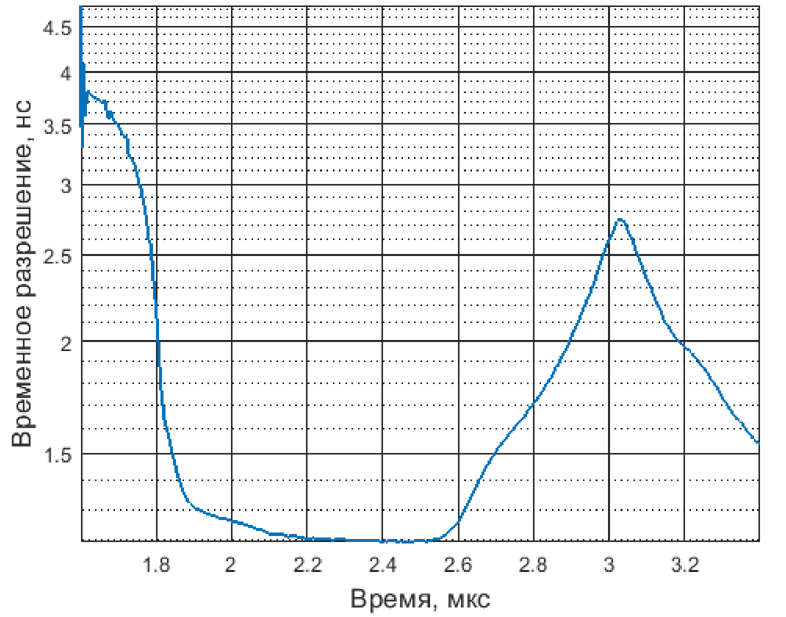


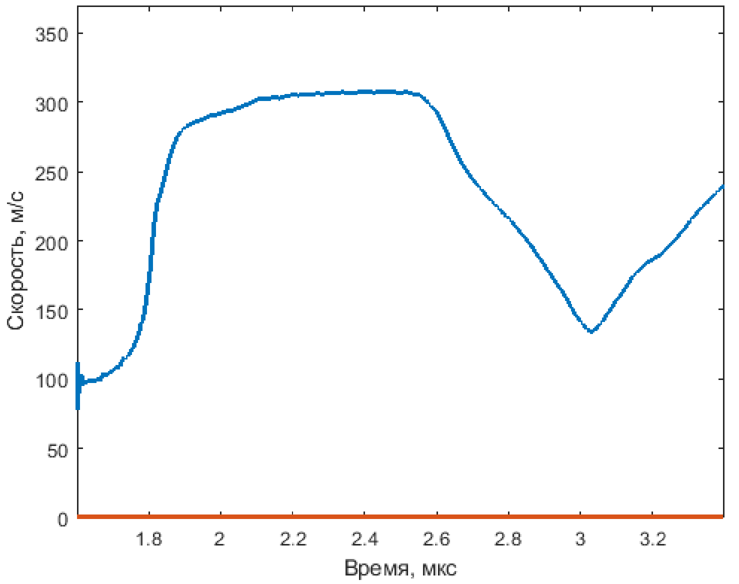
Filtered signal, 2 π



Test profiles' reconstruction. Velocity profile with resolution 2 π







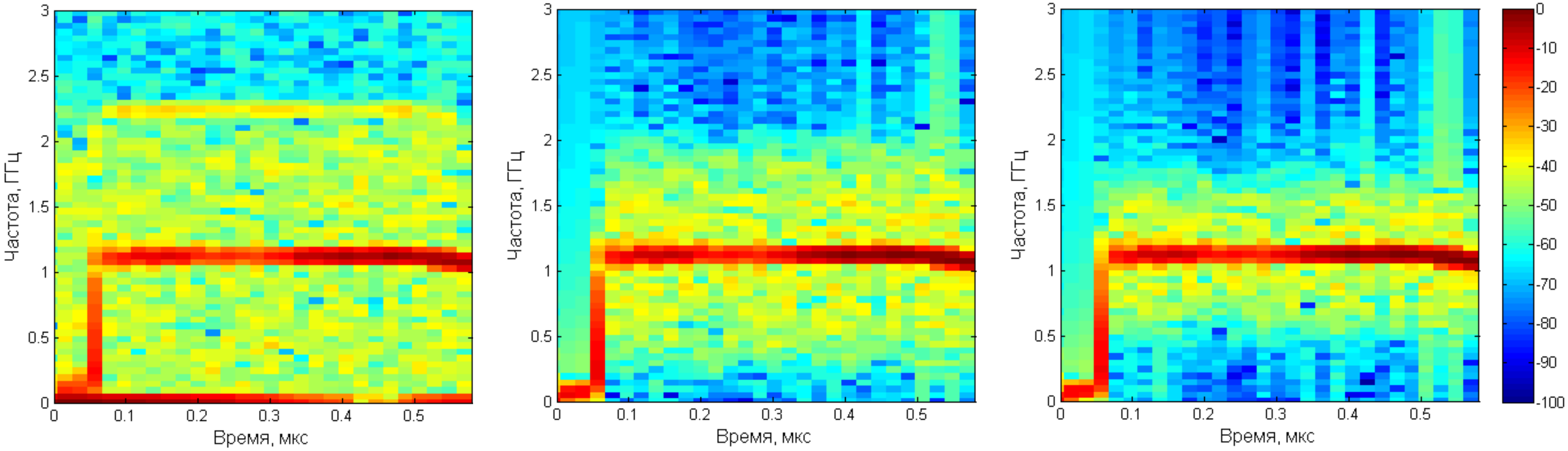
Test profiles' reconstruction. Spectrograms

Original signal

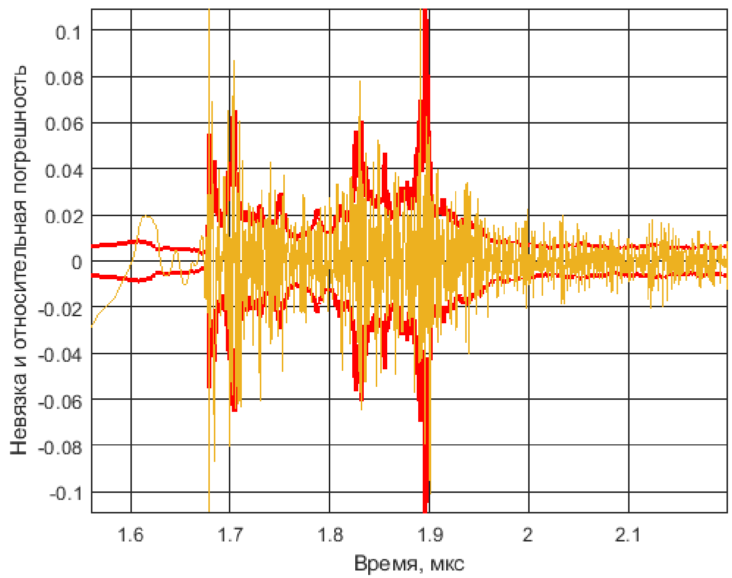
Filtered signal

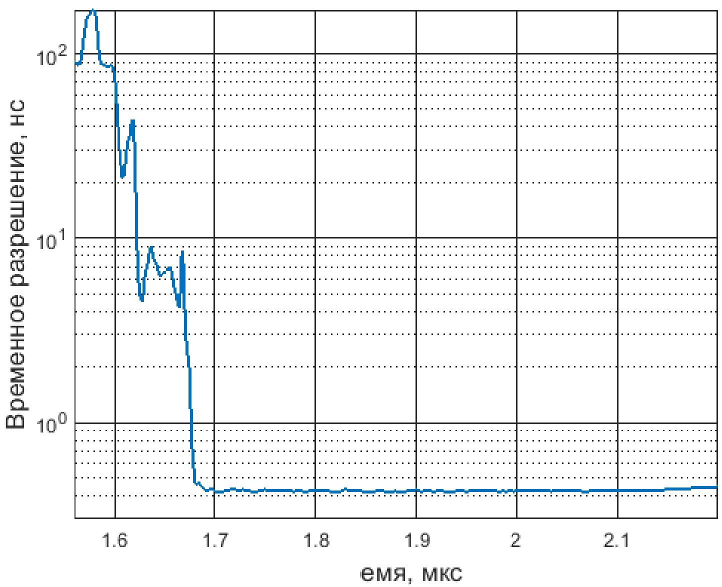
2 π

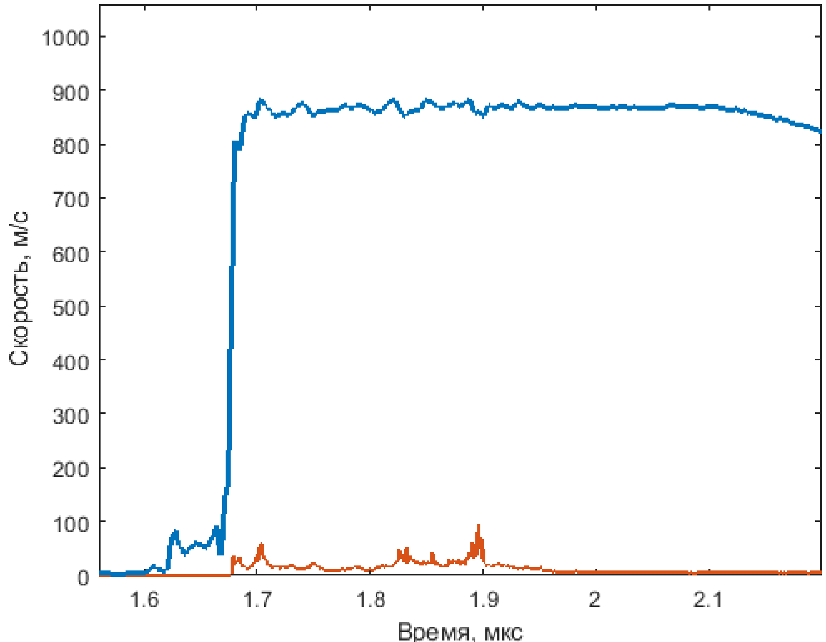
3 π



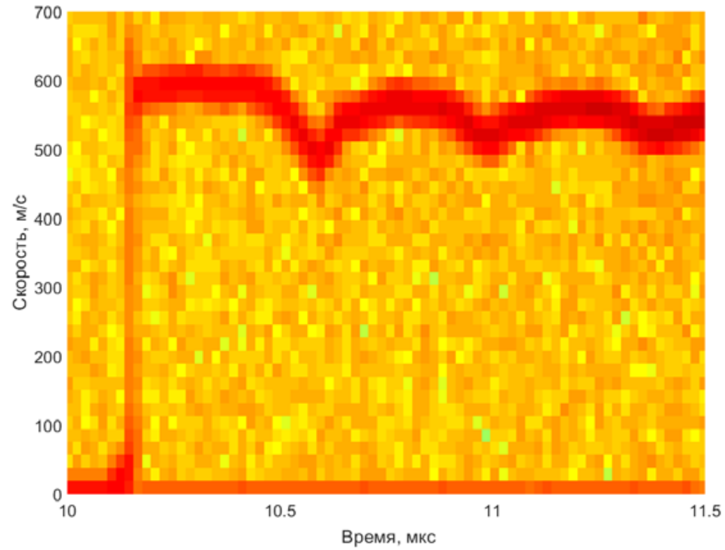
Test profiles' reconstruction. Velocity profile with resolution 2 π

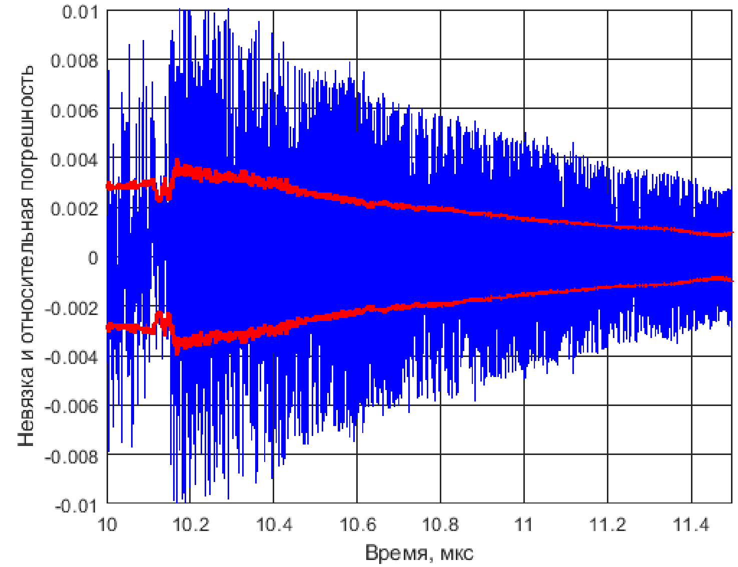


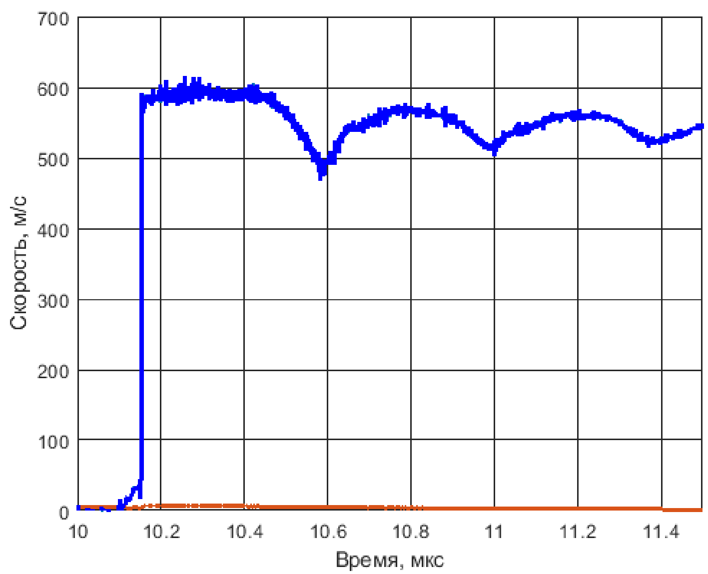


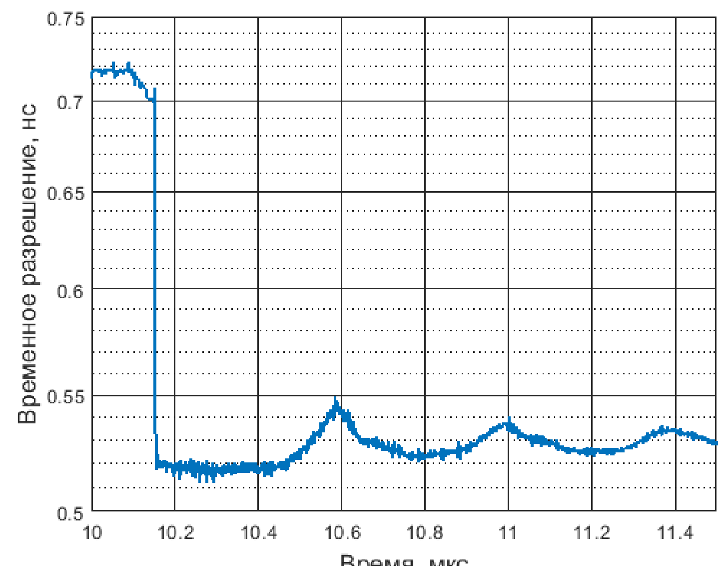


Test profiles' reconstruction with frequency shift (1990 MHz). Velocity profile with resolution 6 π

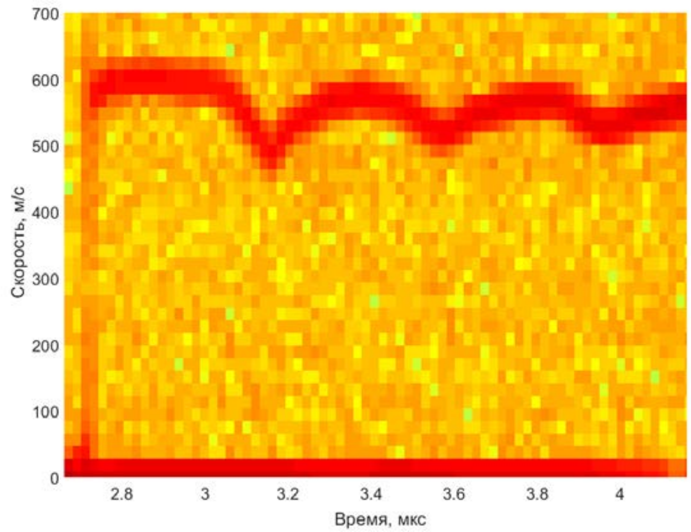


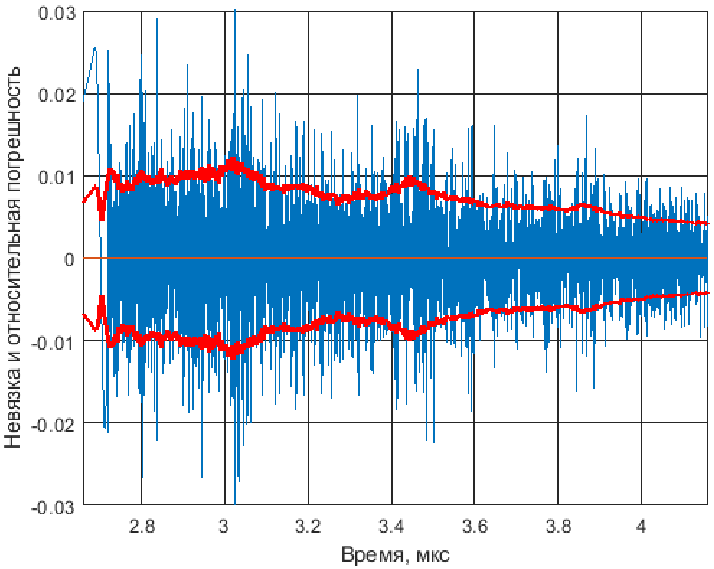


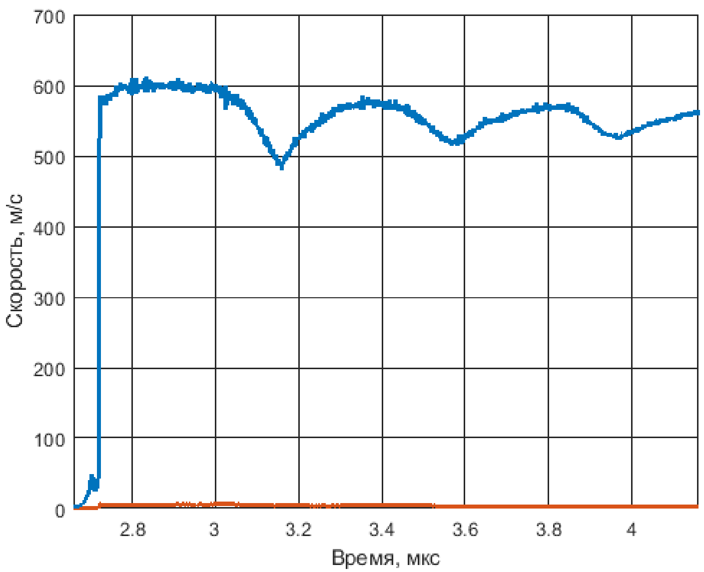


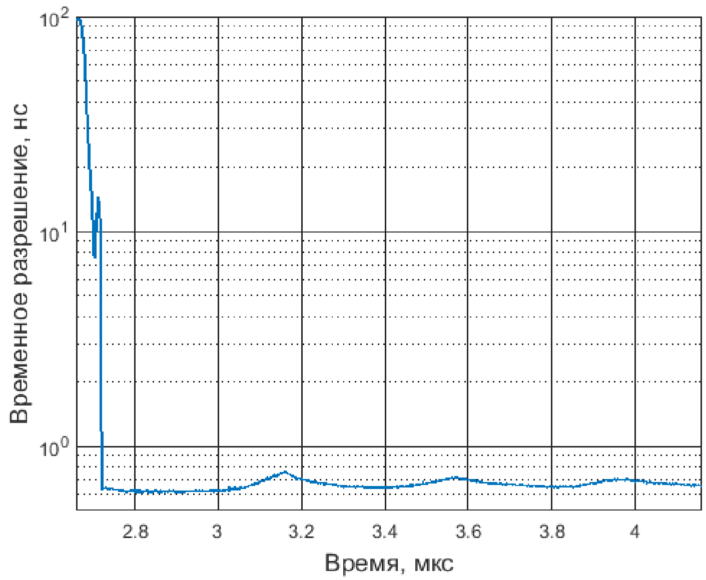


Test profiles' reconstruction. Velocity profile with resolution 2 π

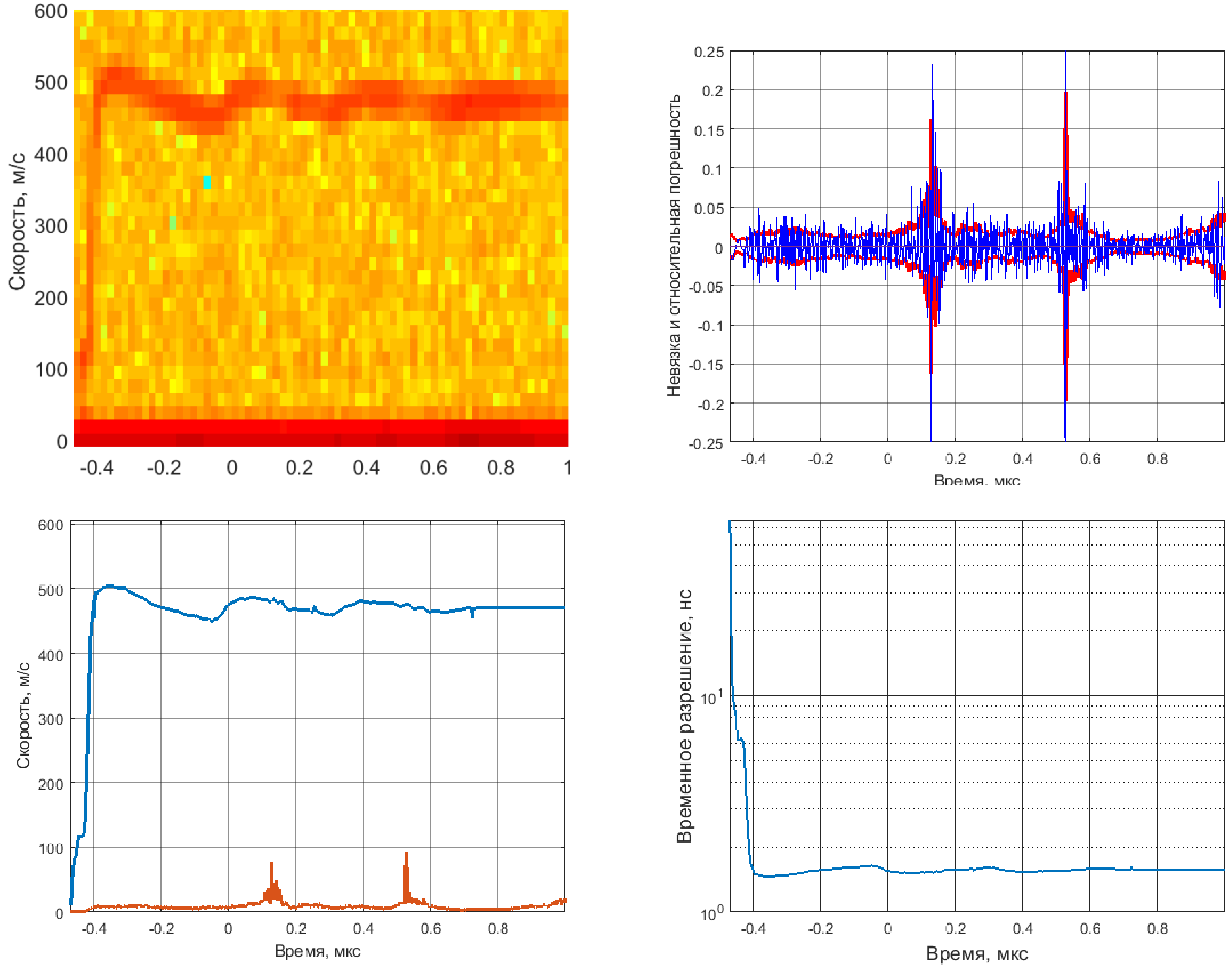








Test profiles' reconstruction. Velocity profile with resolution 4 π



Conclusion

- *An iterative method of a PDV-signal processing based on filtering the experimental signal by integral convolution has been proposed. This method is insensitive to the presence of higher Doppler harmonics and random noise in the original signal, changings of the amplitude of the Doppler harmonic and the presence of zero harmonic in the signal*

- *The method resolving time depends on the parameter n of the integral transform and in the limit is one period of the Doppler harmonic.*

- *An approach to the method's uncertainty estimation of the reconstructed velocity has been proposed*