北京应用物理与计算数学研究所
Institute of Applied Physics and Computational Mathematics

XVII Zababakhin Scientific Talks

公开★

Particle-Fluid Hybrid Simulation of Acceleration and Pinch Dynamics in Dense Plasma Focus

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目录

Background

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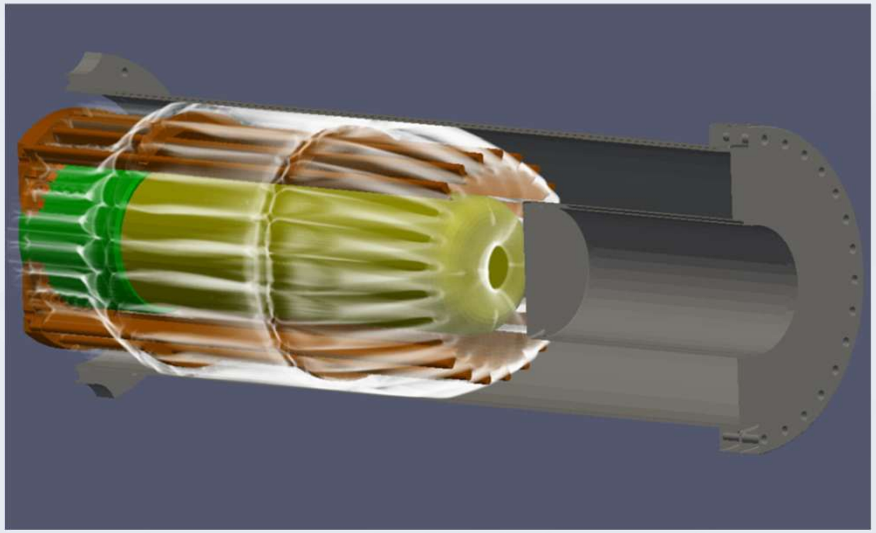
- **Dense plasma focus (DPF) is a compact and intense pulsed multi-radiation source which is driven by electromagnetic force to form high-temperature plasma.**

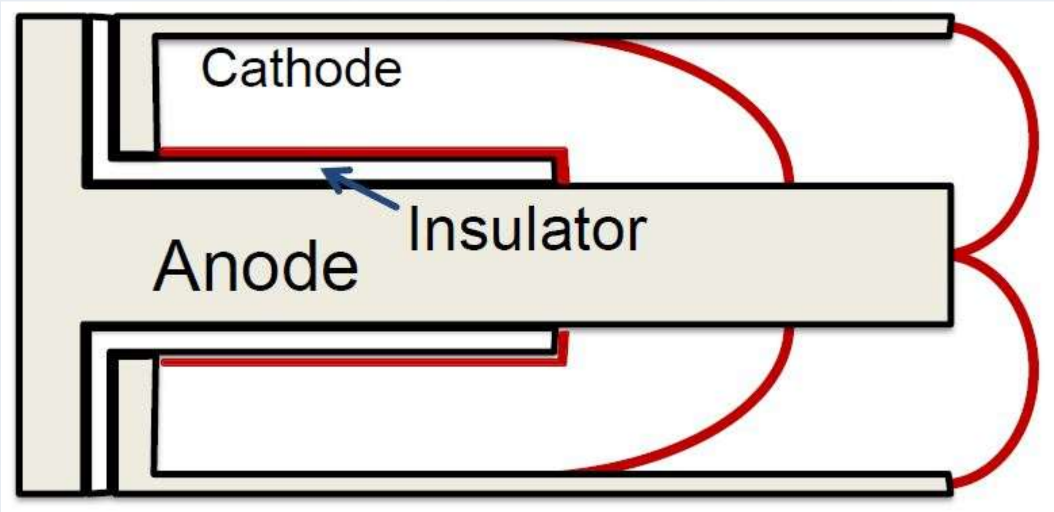
1 MJ MJOLNIR（America）



DPF Internal Simulation Image

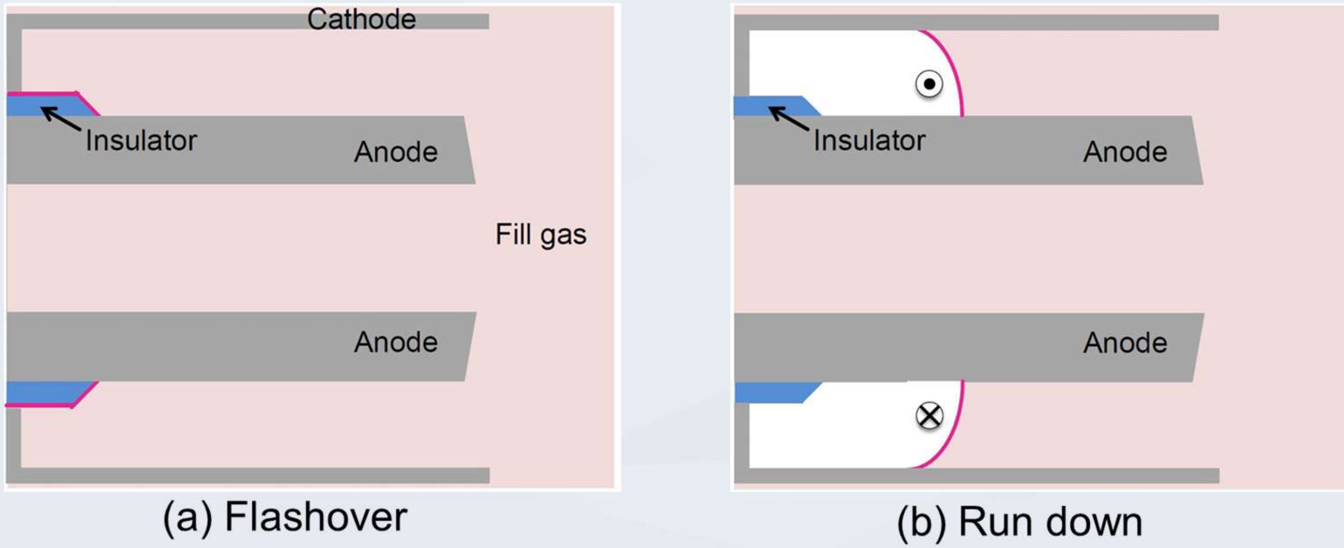
- **DPF was originally used to explore controlled nuclear fusion. With the further study of DPF, it is found that DPF is suitable for various pulsed radiation sources and has a wide range of application prospects.**

Mather Type DPF



A DPF Z pinch is a device consisting of two coaxially located electrodes with a high-voltage source at one end.

Background



Schematic of a dense plasma focus

(a) In the presence of a low-pressure gas, the high-voltage source induces a surface flashover and the formation of a current-conducting plasma sheath across an insulator at the upstream end of the DPF.

(b) During the “run-down” phase, the current sheath is accelerated down the length of the electrodes by magnetic pressure, ionizing and sweeping up neutral gas as it accelerates.

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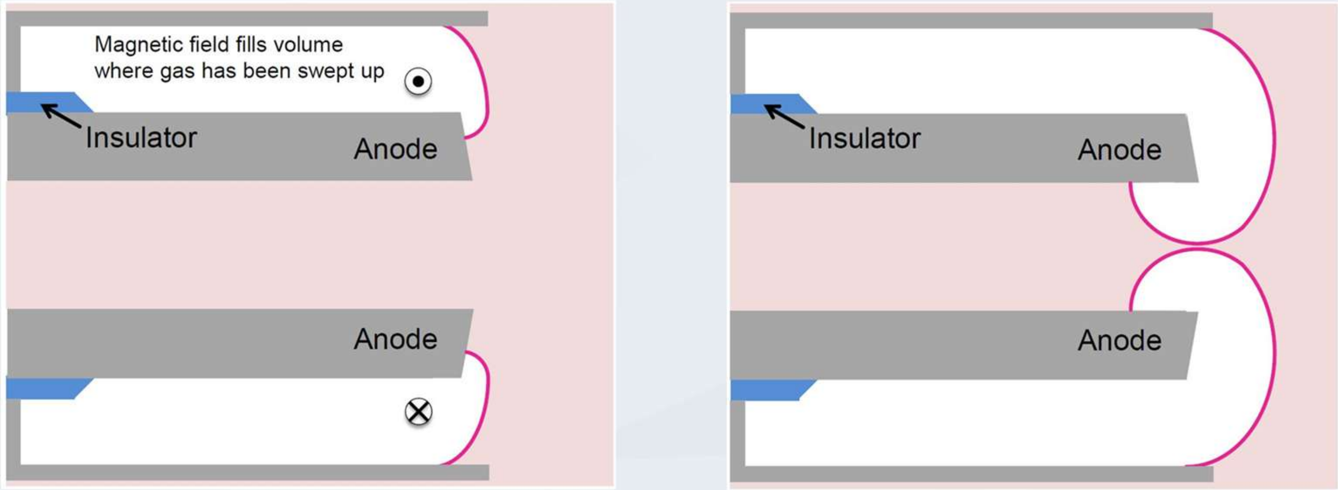


(c) Run in

(d) Pinch

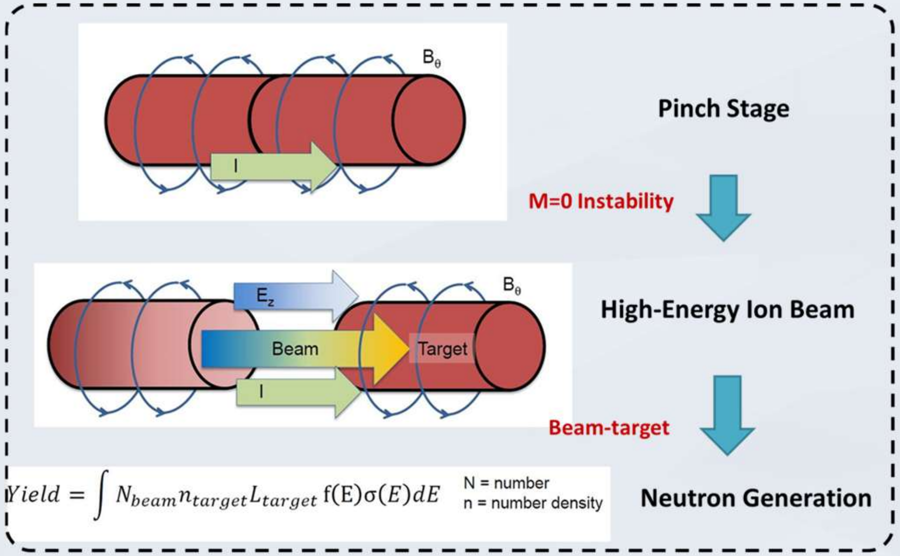
Schematic of a dense plasma focus

(c) When the plasma sheath reaches the end of the inner electrode, a portion is pushed radially inward during the "run-in" phase.

(d) When the leading edge of the current sheath reaches the axis, it "pinches" the plasma to create a hot, dense region that emits high-energy electron and ion beams, x rays, and (in the presence of D or D-T) neutrons.

Background

- In the pinch stage of a DPF device, instability usually occurs, causing the plasma to be partially crushed and the voltage to rise violently. Extremely high voltages will lead to the generation of high-energy ion beams, and eventually neutrons will be generated through beam-target interaction.



Neutron Generation Process

- The generation of high-energy particle beams is a typical particle dynamics behavior, and the traditional magnetohydrodynamics has been unable to describe the generation process of high-energy particles and the beam-target interaction mechanism.

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PIC METHODE

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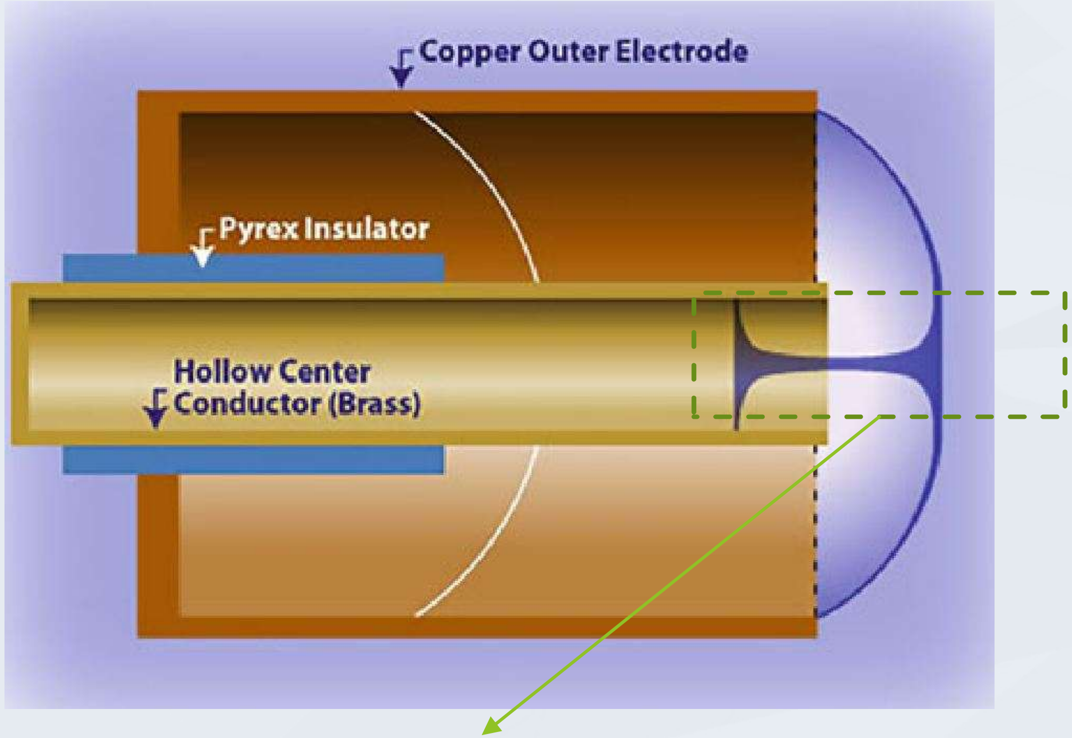
Pinch density: $\sim 10^{25}\ m^{-3}$ Temperature: $\sim 10^{3}$eV



Pinch scale: 2D far greater than $1mm \times 1mm$

Fully particle method (PIC) needs analysis:

Debye length: $\sqrt{\frac{\varepsilon_0 k_B T_e}{e^2 n_e}} < 10^{-7} m$

Reciprocal electron cyclotron frequency:

$$\sqrt{\frac{\varepsilon_0 m_e}{e^2 n_e}} < 5 \times 10^{-15} s$$

Number of meshes in space (two-dimensional): 10^8

Time steps: 10^8

The amount of calculation is unbearable!

Implicit PIC ——hybrid simulation

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- 1. Implicit scheme PIC does not need to analyze the Debye length (spatial calculation can save 10^4)
- 2. Fluid-PIC hybrid simulation does not need to analyze the Debye length and electron cyclotron frequency (spatial and time scale savings 10^6)

- **The main advantages of particle-fluid mixing simulation are::**
  - 1. Compared with pure particle PIC, for typical physical applications, ignoring the spatio-temporal scale of electron dynamics usually reduces the amount of calculation in each spatial and temporal dimension by the magnitude of $\sqrt{m_i/m_e}$ (for three-dimensional space, one-dimensional time$\sim 100^4$) .
  - 2. Ions are kinetic models that can deal with non-Maxwell distributed ions, which makes hybrid simulation methods intrinsically capable of capturing many kinetic effects that are difficult or impossible to simulate in fluids. (Such as DPF high-energy ion beam, neutron generation process by beam-target interaction)

Hybrid simulation

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- **Plasmas are coupled to electromagnetic fields in two main ways: the full Maxwell equation and the low-frequency magnetostatic (Darwin) model.**

Maxwell equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

$$\varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\mu_0} \nabla \times \boldsymbol{B} - \boldsymbol{j}$$

Magnetostatic (Darwin) model

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

$$0 = \frac{1}{\mu_0} \nabla \times \boldsymbol{B} - \boldsymbol{j}$$

Generalized Ohm's Law:

$$\boldsymbol{j} = \sigma \left(\boldsymbol{E} + \boldsymbol{v_e} \times \boldsymbol{B} + \frac{\nabla p_e}{e n_e} \right)$$

$$\boldsymbol{E} = -\boldsymbol{v_e} \times \boldsymbol{B} - \frac{\nabla p_e}{e n_e} + \frac{\boldsymbol{j}}{\sigma}$$

Advantages: The electromagnetic field is self-consistently treated, allowing electromagnetic waves in vacuum.

Fast calculation speed, low-frequency approximation, ignoring light waves, only need to deal with ion time scales

Disadvantage: In FDTD scheme, the time step size is strictly limited

The transmission of magnetic field in low density or vacuum region is greatly limited

Fully-electromagnetic model

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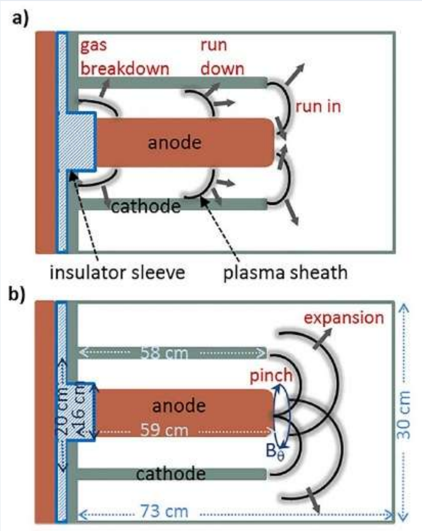
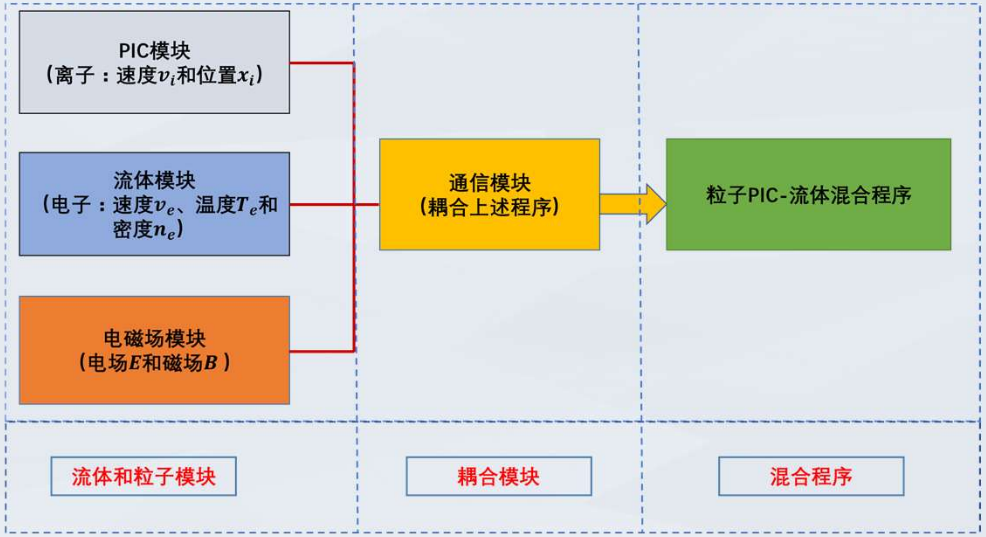
- Dense Plasma Focus (DPF) is a typical system with **high density plasma region and vacuum region**.
- Therefore, in order to **self-consistently describe the electromagnetic waves** in the vacuum region, we use the **fully electromagnetic model and the FDTD method** to solve the electromagnetic field equations.



Schematic diagram of DPF discharge



Hybrid simulation schematic diagram

混合模型方程

PIC Module
（Ion）

$$\frac{dx_i}{dt} = \boldsymbol{v_i}$$

$$m_i \frac{d\boldsymbol{v_i}}{dt} = q_i(\boldsymbol{E} + \boldsymbol{v_i} \times \boldsymbol{B}) - \boldsymbol{f}$$

Fluid Module
（Electron）

$$n_e = q_i n_i$$

$$n_e m_e \frac{d\boldsymbol{v_e}}{dt} = \boldsymbol{0} = -en_e(\boldsymbol{E} + v_e \times \boldsymbol{B}) - \nabla p_e + \frac{en_e\boldsymbol{j}}{\sigma}$$

$$\frac{\partial(\frac{3}{2}n_e k T_e)}{\partial t} + \frac{3}{2} n_e k \boldsymbol{u_e} \cdot \nabla T_e + p_e(\nabla \cdot \boldsymbol{u_e}) = \nabla \cdot (k_e \nabla T_e) + \frac{j^2}{\sigma} - Q_{ei}$$

FDTD module
（E and B ）

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

$$\varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\mu_0} \nabla \times \boldsymbol{B} - \boldsymbol{j}$$

PIC-Fluid Hybrid Model

Ion module（PIC）：

$$\frac{dx_i}{dt} = \boldsymbol{v_i} \quad (1)$$

$$m_i \frac{d\boldsymbol{v_i}}{dt} = q_i(\boldsymbol{E} + \boldsymbol{v_i} \times \boldsymbol{B}) - \frac{e\boldsymbol{j}}{\sigma} \quad (2)$$

Electron fluid module:

$$n_e = q_i n_i$$ （准中性近似）（3）

$$p_e = n_e k T_e$$ （理想气体）（4）

Electromagnetic field module（FDTD）：

$$\varepsilon_0 \frac{\partial E}{\partial t} = \frac{1}{\mu_0} \nabla \times \boldsymbol{B} - \boldsymbol{j} \quad (9) \qquad \frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} \quad (10)$$

$$\boldsymbol{j} = \sigma\left(\boldsymbol{E} + \boldsymbol{v_e} \times \boldsymbol{B} + \frac{\nabla p_e}{e n_e}\right) \quad (8)$$ Generalized Ohm's Law

$$\boldsymbol{V_e} = \boldsymbol{V_i} - \frac{\boldsymbol{j}}{e n_e} \quad (7)$$

Electron temperature（1） $T_e = T_i$ （5）

（Thermodynamic equilibrium assumption）

（2） $$\frac{\partial\left(\frac{3}{2} n_e k T_e\right)}{\partial t} + \frac{3}{2} n_e k \boldsymbol{u_e} \cdot \nabla T_e + p_e(\nabla \cdot \boldsymbol{u_e}) = \nabla \cdot (k_e \nabla T_e) + \frac{j^2}{\sigma} - Q_{ei} \quad (6)$$

（Solving the internal energy equation of electrons）

时间推进流程

The position of the initial particle $\boldsymbol{x}^n$ and the electric field $\boldsymbol{E}^n$at the integer time step，the velocity of the particle $\boldsymbol{v_p}^{n-1/2}$ the magnetic field $\boldsymbol{B}^{n+1/2}$at the half time step。

Ion PIC Propulsion:

1. **First, in the PIC equation of motion, the particle velocity advances from $\boldsymbol{v_p}^{n-1/2}\boldsymbol{v_p}^{n+1/2}$：**

$$\frac{\boldsymbol{v}^{n+1/2}-\boldsymbol{v}^{n-1/2}}{\Delta t}=\frac{q}{m}\left(\boldsymbol{E}^n+\frac{\boldsymbol{v}^{n+1/2}+\boldsymbol{v}^{n-1/2}}{2}\times\boldsymbol{B}^n\right)-\frac{\boldsymbol{f}^n}{m}$$

2. **The particle position is then pushed from $\boldsymbol{x}^n$ to $\boldsymbol{x}^{n+1}$：**

$$\frac{\boldsymbol{x}^{n+1}-\boldsymbol{x}^n}{\Delta t}=\boldsymbol{v}^{n+1/2}$$

During this process, the ion orientation velocity V_i（N+1/2 time step）, ion average temperature （N+1/2 time step）, and ion density （N+1 time step）， in the grid can be calculated, which are used by later fluid modules.

电子流体推进

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3. The electron density and temperature are obtained by quasi-neutral approximation and thermodynamic equilibrium $n_e = q_i n_i \qquad T_e = T_i$

4. Calculation of current density by generalized Ohm's law

$$J^{n+1/2} = \sigma^{n+1/2}\left(\boldsymbol{E}^{n+1/2} - \frac{J_e^{n+1/2}}{e n_e^{n+1/2}} \times \boldsymbol{B}^{n+1/2} + \frac{\nabla p_e^{n+1/2}}{e n_e^{n+1/2}}\right)$$

$$\boldsymbol{E}^{n+1/2} \approx (\boldsymbol{E}^{n+1} + \boldsymbol{E}^{n})/2$$

$$\frac{\boldsymbol{E}^{n+1} - \boldsymbol{E}^{n}}{\Delta t} = c^2 \nabla \times \boldsymbol{B}^{n+1/2} - \frac{1}{\epsilon_0} J^{n+1/2}$$

$$J_r^{n+1/2} = \frac{DA_r + CA_z}{D^2 + C^2}$$

$$J_z^{n+1/2} = \frac{DA_z - CA_r}{D^2 + C^2}$$

其中$D \equiv 1 + \sigma^{n+1/2}\Delta t/2\varepsilon_0$，$C \equiv \sigma^{n+1/2} B_\theta^{n+1/2}/e n_e^{n+1/2}$，矢量$\boldsymbol{A}$为：

$$\boldsymbol{A} \equiv \sigma^{n+1/2}\left(\boldsymbol{E}^{n} + \frac{c^2 \Delta t^2}{2} \nabla \times \boldsymbol{B}^{n+1/2} + \frac{1}{e n_e^{n+1/2}} \boldsymbol{J}_i^{n+1/2} \times \boldsymbol{B}^{n+1/2} + \frac{\nabla p_e^{n+1/2}}{e n_e^{n+1/2}}\right)$$

电磁场推进

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5. By ampere law and FDTD method, the electric field is solved

$$\frac{\boldsymbol{E}^{n+1}-\boldsymbol{E}^{n}}{\Delta t} = c^2\nabla\times\boldsymbol{B}^{n+1/2}-\frac{1}{\epsilon_0}\boldsymbol{J}^{n+1/2}$$

6. By Faraday's Law and FDTD method, the magnetic field is solved

$$\frac{\boldsymbol{B}^{n+3/2}-\boldsymbol{B}^{n+1/2}}{\Delta t} = \nabla\times\boldsymbol{E}^{n+1}$$

7. When a complete iteration of the time loop is completed, the current density J^{n+1} at the next moment must be updated。 （For particle propulsion in the next time step）

Prediction-Correction

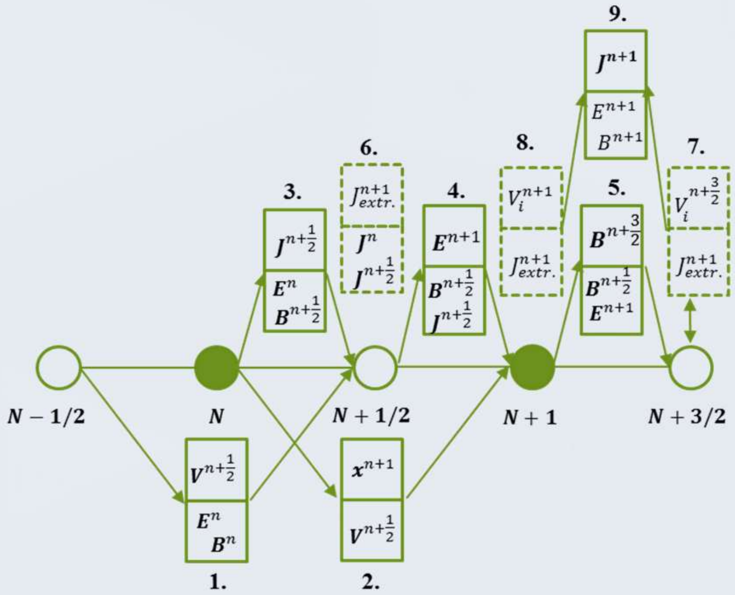
（i）预估n+1时刻的电流密度 $\boldsymbol{J}'^{n+1} = -\boldsymbol{J}^{n} + 2\boldsymbol{J}^{n+1/2}$

（ii）利用n+1时刻的电场和磁场推动离子，获得 $V_i'^{n+3/2}$、$T_i'^{n+3/2}$

（iii）对离子定向速度和温度平均，得到$V_i'^{n+1}$、$T_i'^{n+1}$

（iv）得到修正后的n+1时刻的电流密度 $\boldsymbol{J}^{n+1}$

$\boldsymbol{J}^{n+1/2}$



Schematic diagram of time advancement

离子PIC推进： $$\frac{v^{n+1/2}-v^{n-1/2}}{\Delta t} = \frac{q}{m}\left(E^n + \frac{v^{n+1/2}+v^{n-1/2}}{2}\times B^n\right) - \frac{f^n}{m} \quad (1)$$

$$\frac{x^{n+1}-x^n}{\Delta t} = v^{n+1/2} \quad (2)$$

电流密度推进： $$J^{n+1/2} = \sigma^{n+\frac{1}{2}}\left(\frac{E^{n+1}+E^n}{2} - \frac{J_e^{n+1/2}}{en_e^{n+1/2}}\times B^{n+1/2} + \frac{\nabla p_e^{n+1/2}}{en_e^{n+1/2}}\right) \quad (3)$$

电磁场推进（FDTD）：

$$\frac{E^{n+1}-E^n}{\Delta t} = c^2\nabla\times B^{n+1/2} - \frac{1}{\epsilon_0}J^{n+1/2} \quad (4)$$

$$\frac{B^{n+3/2}-B^{n+1/2}}{\Delta t} = \nabla\times E^{n+1} \quad (5)$$

预估-修正推进 J^{n+1}：

（i）预估n+1时刻的电流密度 $J'^{n+1} = -J^n + 2J^{n+1/2}$ （6）

（ii）利用n+1时刻的电场和磁场推动离子，获得 $V_i'^{n+3/2}$、$T_i'^{n+3/2}$ （7）

（iii）对离子定向速度和温度平均，得到 $V_i'^{n+1}$、$T_i'^{n+1}$ （8）

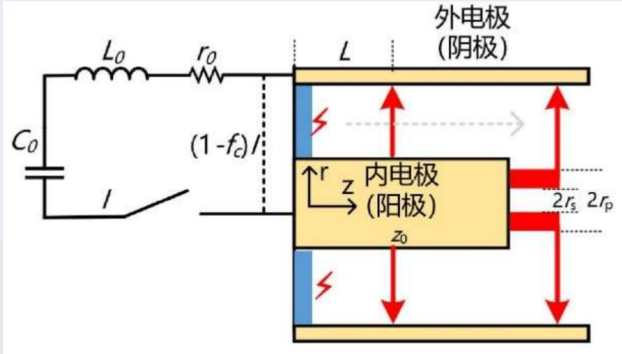
（iv）得到修正后的n+1时刻的电流密度 （9）

Time advancement step

- **The entire time advancement step is shown in the figure above, including the prediction-corrected flow from the current density value at time n+1 of the consistent advancement.**

External Circuit and Magnetic Field Boundary

The external circuit model of DPF is established and coupled with the boundary conditions of hybrid simulation.



The external circuit： $$\frac{d}{dt}[L_0 I] = V_0 - r_0 I - \int \frac{Idt}{C_0} - U_{PF}$$

U_{PF} **is the voltage of DPF device, it needs to be determined self-consistently by the mixed simulation results:**

$$U_{DPF} = \frac{d(\int Bds)}{dt}$$

The magnetic field in the entire DPF system is integrated in each time step to obtain the system magnetic flux, and then the time derivative is obtained to obtain the DPF system voltage.

The whole equation is an ordinary differential equation, which can be discrete into the following first-order explicit scheme:

$$\frac{I^{n+1}-I^n}{\Delta t} = \frac{1}{L_0}\left(V_0 - r_0 I^n - \frac{Q^n}{C_0} - {U_{DPF}}^n\right) \qquad \frac{Q^{n+1}-Q^n}{\Delta t} = I^n$$

Overall calculation flow

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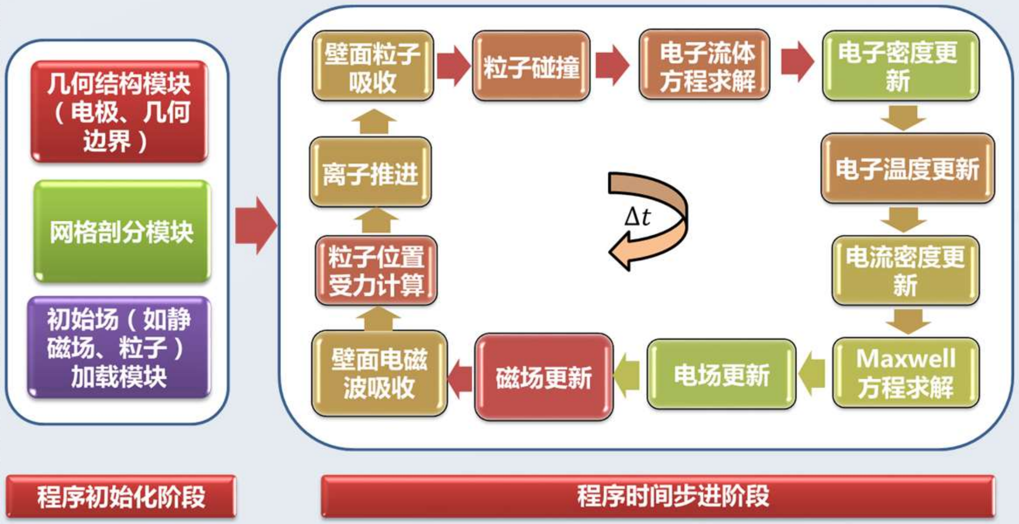
- **Finally, the following iterative algorithm flow is formed**



离子PIC推进：

$$\frac{\boldsymbol{v}^{n+1/2}-\boldsymbol{v}^{n-1/2}}{\Delta t}=\frac{q}{m}\left(\boldsymbol{E}^n+\frac{\boldsymbol{v}^{n+1/2}+\boldsymbol{v}^{n-1/2}}{2}\times\boldsymbol{B}^n\right)-\frac{f^n}{m}$$

$$\frac{\boldsymbol{x}^{n+1}-\boldsymbol{x}^n}{\Delta t}=\boldsymbol{v}^{n+1/2}$$

电流密度推进：

$$\boldsymbol{J}^{n+1/2}=\sigma^{n+\frac{1}{2}}\left(\frac{\boldsymbol{E}^{n+1}+\boldsymbol{E}^n}{2}-\frac{\boldsymbol{J}_e^{n+1/2}}{en_e^{n+1/2}}\times\boldsymbol{B}^{n+1/2}+\frac{\nabla p_e^{n+1/2}}{en_e^{n+1/2}}\right)$$

电磁场推进（FDTD）：

$$\frac{\boldsymbol{E}^{n+1}-\boldsymbol{E}^n}{\Delta t}=c^2\nabla\times\boldsymbol{B}^{n+1/2}-\frac{1}{\epsilon_0}\boldsymbol{J}^{n+1/2}$$

$$\frac{\boldsymbol{B}^{n+3/2}-\boldsymbol{B}^{n+1/2}}{\Delta t}=\nabla\times\boldsymbol{E}^{n+1}$$

预估-修正推进$\boldsymbol{J}^{n+1}$：

（i）预估n+1时刻的电流密度 $\boldsymbol{J}'^{n+1}=-\boldsymbol{J}^n+2\boldsymbol{J}^{n+1/2}$

（ii）利用n+1时刻的电场和磁场推动离子，获得 $V_i'^{n+3/2}$、$T_i'^{n+3/2}$

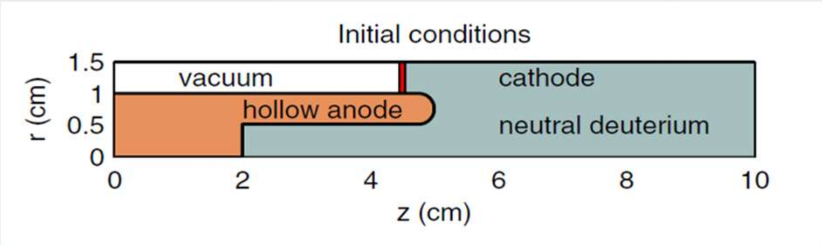
（iii）对离子定向速度和温度平均，得到$V_i'^{n+1}$、$T_i'^{n+1}$

（iv）得到修正后的n+1时刻的电流密度

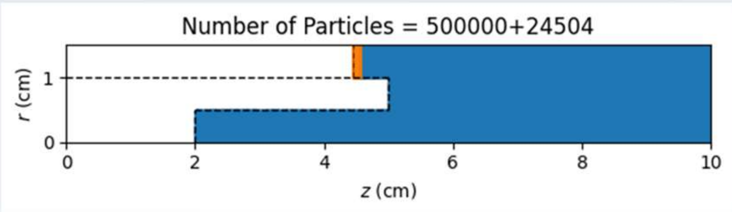
Validation 1: LLNL Device

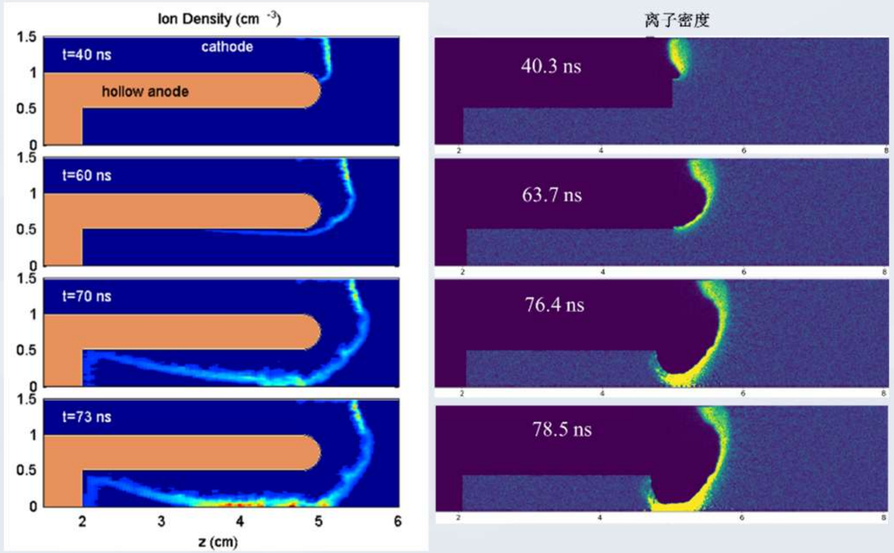
- The simulation results show that although there is a certain difference in plasma configuration (which may be caused by the round head and flat head structure), the overall evolution time of the plasma is very close, with a difference of less than 8%.

(a)



(b)





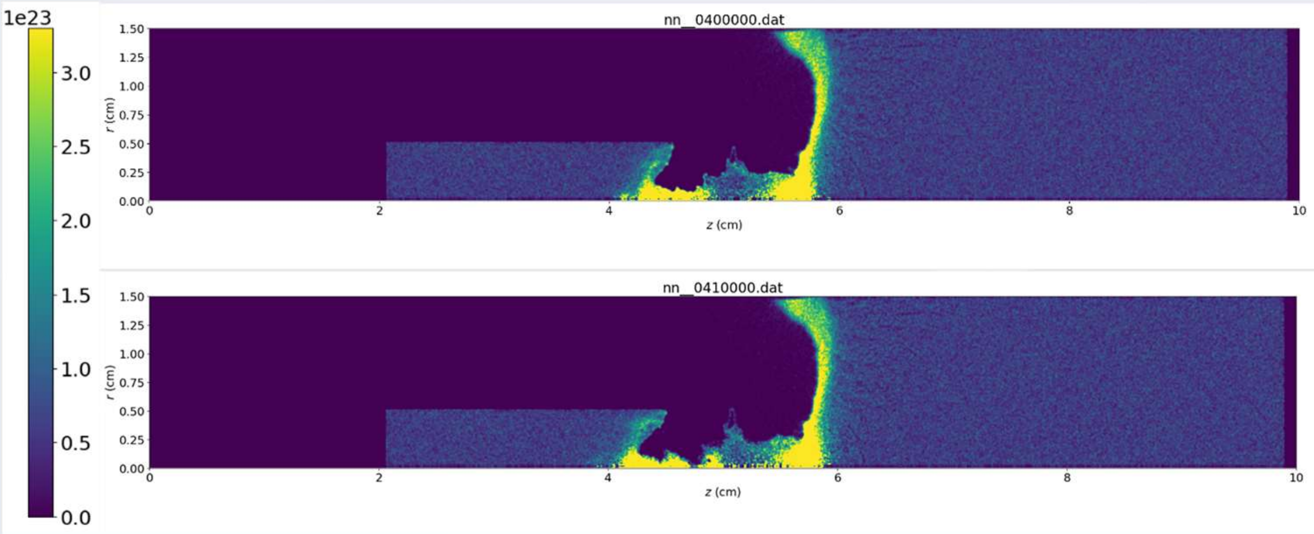
a) LLNL Lab DPF Device Structure (b) Hybrid Simulation Initial Setup

Comparison with LLNL simulation results in the United States (PRL article)

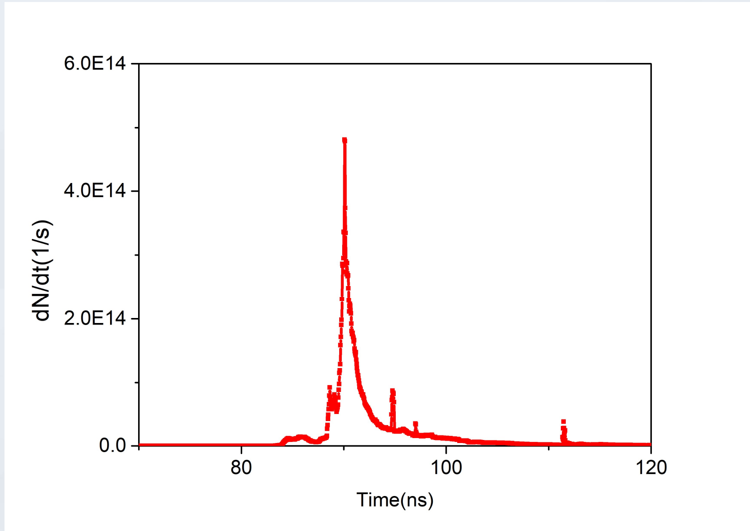
（(We use a flat head shape for the right end of the anode here, while the United States uses a round head shape; other conditions are consistent)）

The left is PIC simulation results obtained by the LLNL laboratory using LSP software, and the right is ourhybrid simulation results.

- **Neutron generation process: The plasma is compressed on the symmetry axis to generate a high electric field, and the beam-target mechanism generates neutrons.**



Physical image after pinch



Neutron production rate varies with time

Hybrid simulation neutron yield：2.64e6

American LSP neutron yield：8.6e6

Experiment neutron yield：2.0e7

Validation 2: UNU Device

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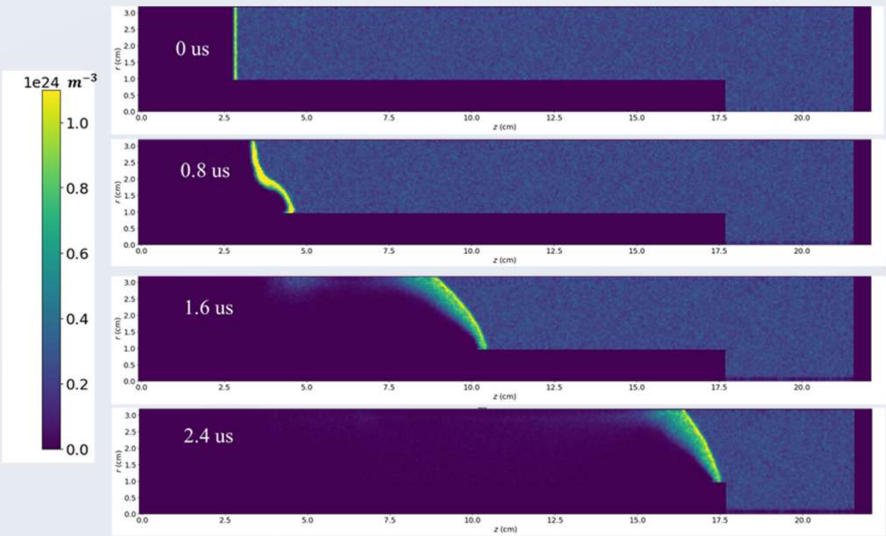
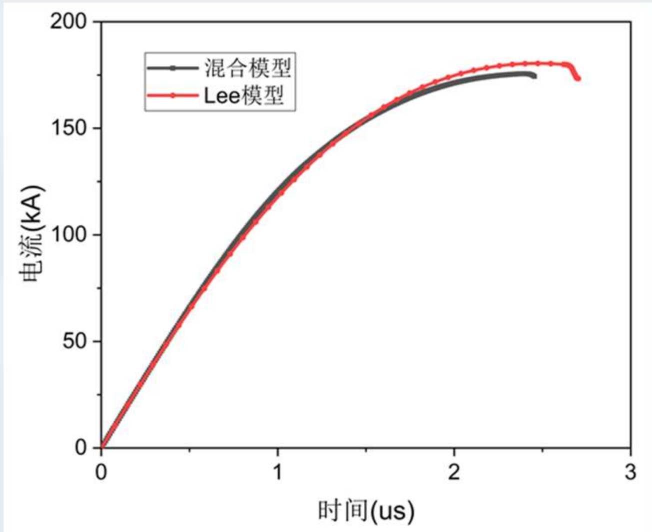
- **In the following, we further verify the accuracy of the hybrid simulation in the axial stage and the coupling matching with the external circuit through a small UNU device.**



Simulation Results of Axial Phase of UNU Device



Computational current comparison results

- **Comparing with the results of the UNU device, the accuracy of the hybrid model in the axial phase of plasma motion and external circuit simulation is basically verified。**

VALIDATION 3: MA DPF DEVICE

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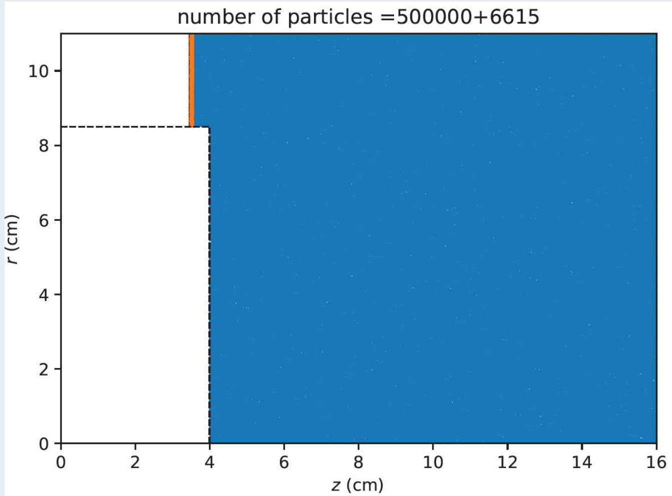
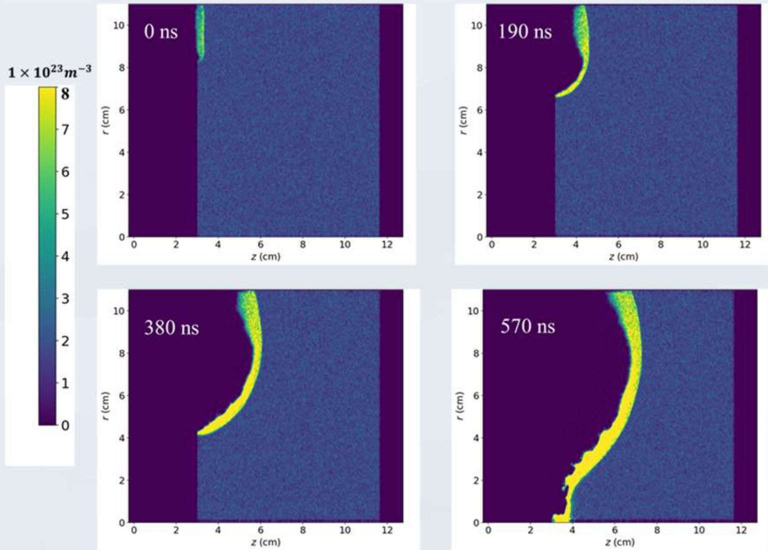
- Finally, the hybrid simulation is applied to MA-level DPF (1.5 MA) to verify the computational power and model accuracy of the program on MA-level DPF devices.



DPF Device Structure and Initial Macro Particle Setup (Radial Phase)



Simulation Results of Plasma Density Distribution at Different Time

- **The MA-level DPF device can complete the whole radial implosion stage in about 570ns, which is in good agreement with the experimental results and Lee model results.**

☐ Overall, the current hybrid program has the ability

to accurately simulate the axial, radial and pinch stages.

The whole Process DPF Simulation

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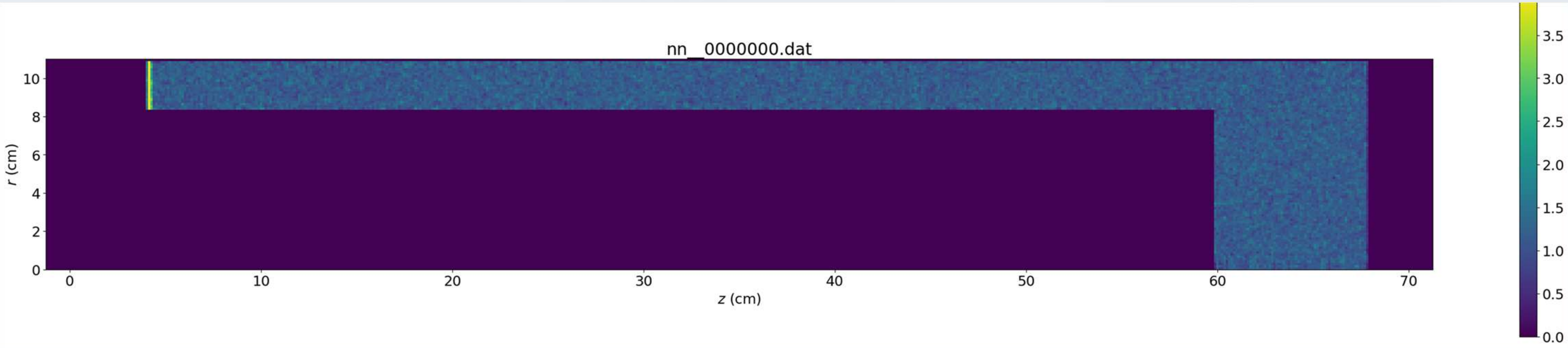
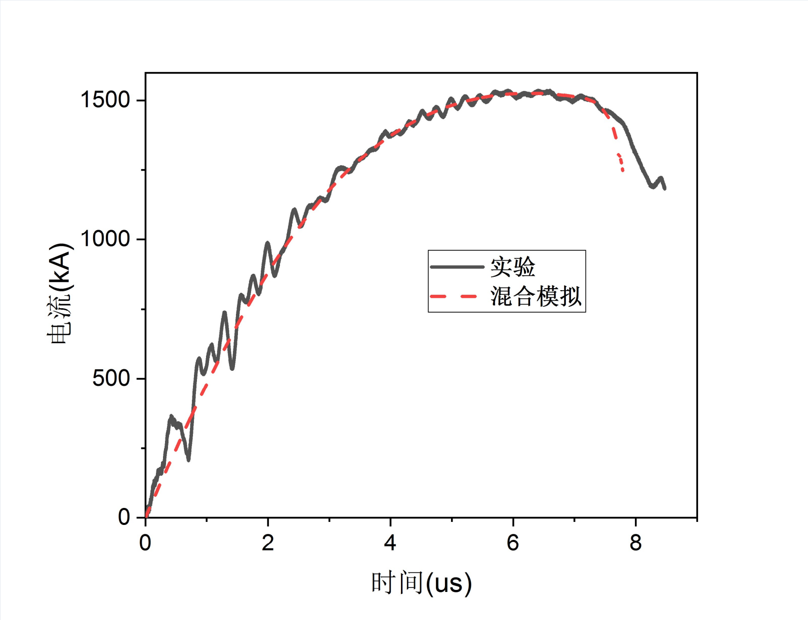
- **The DPF full physical process calculation is performed using hybrid simulation (the background gas needs to be multiplied by a certain coefficient according to the Lee model).**

The anode radius is 8.5 cm, the cathode radius is 11.0 cm, and the initial background D ion density is $0.8 \times 10^{23}\ cm^{-3}$



The current is calculated by the external circuit： $\frac{d}{dt}[L_0 I] = V_0 - r_0 I - \int \frac{I dt}{C_0} - U_{PF}$

The magnetic field is injected at the inlet: $B_\theta = \frac{\mu I}{2\pi r}$



Hybrid simulation and experimental comparison

- **The hybrid simulation can realize the whole process simulation, and the simulated current trajectory is in good agreement with the experiment (axial phase)**

目录

Conclusion

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In this paper, a particle-fluid hybrid model is established to study the dynamic process of dense plasma focus.

- **Aiming at the time step advancement of hybrid simulation, the self-consistent explicit iteration of hybrid simulation is realized by using the prediction-correction method.**
- **In the hybrid simulation, the particle method is used to simulate kinetic ions, the fluid method is used to simulate electrons, and the Maxwell equation and FDTD method are used to solve the electromagnetic field.**
- **We compare the simulation results with the experimental data. The simulation results are in good agreement with the experimental, which verifies the effectiveness and reliability of the hybrid simulation method.**