Understanding radiation transport in stochastic mixtures

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Outline

- **Preliminary remarks**

  Basic definition, applications, recent progress, et al.

- **Theoretical method**

  Benchmark procedure, observables' definition, et al.

- **Results and analysis**

  Analytical model, benchmark simulations, et al.

- **Summary**

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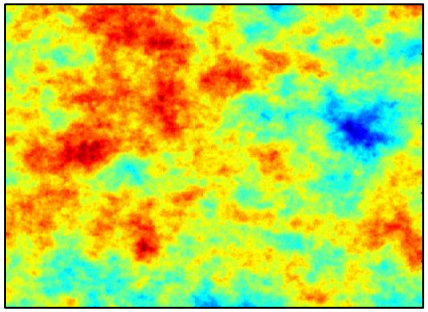
1. Preliminary remarks

Question 1: **what is stochastic mixture?**

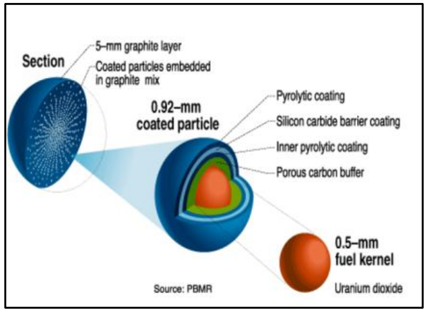
Stochastic mixture (SM) contains **two or more** **randomly mixed** **immiscible** materials, thus it is **only known statistically** by materials' occurrence probability.



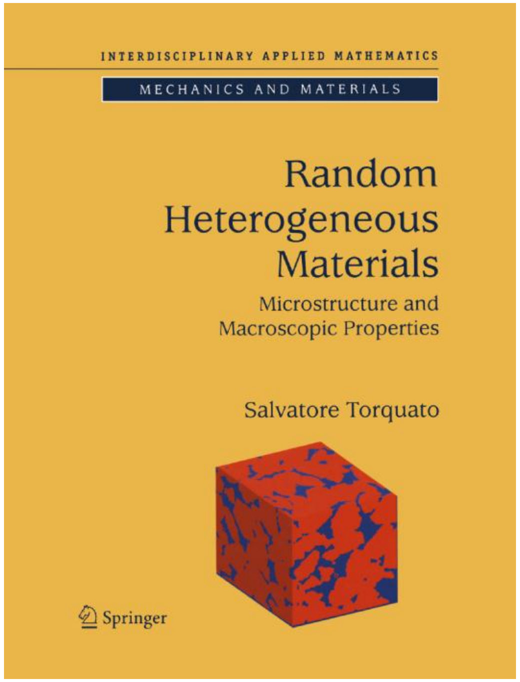
turbulence

interstellar clouds

terrestrial clouds



nuclear reactor

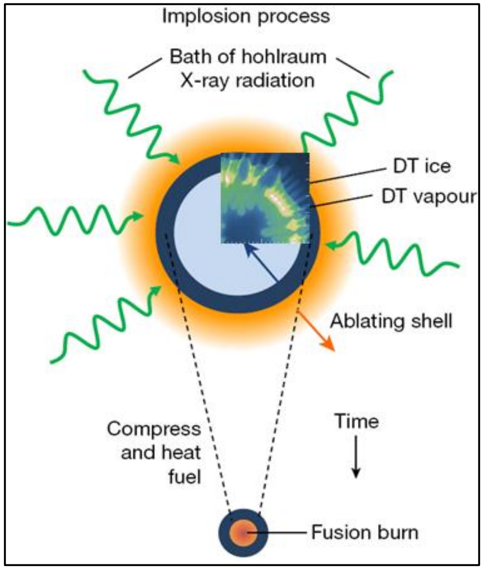


Springer Science & Business Media

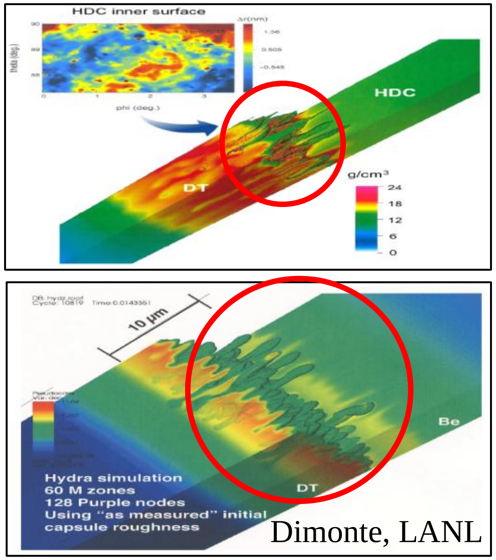
1. Preliminary remarks

Question 2: **where does SM involve in applications?**

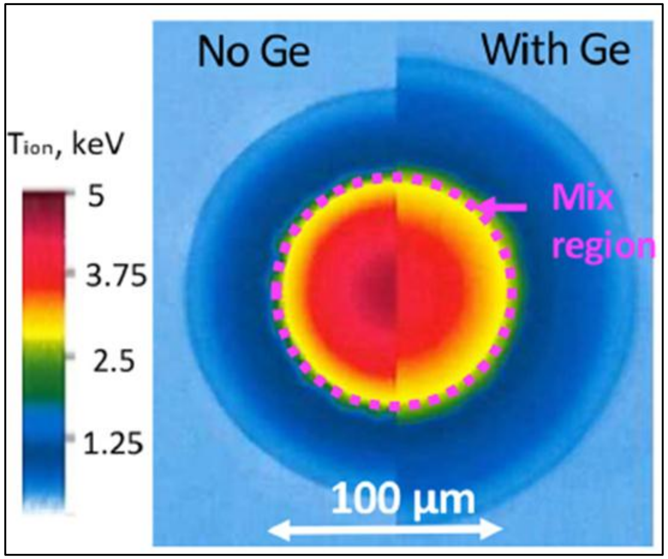
In inertial confinement fusion (ICF), the mixture of the fuel and shell initiated by the hydrodynamic instability appears, and **the radiation transported through the stochastic fuel-shell mixtures is believed to play a role in the performance of the fusion pellet**.



Schematic of the indirect-drive ICF
Nature 601, 542(2022)



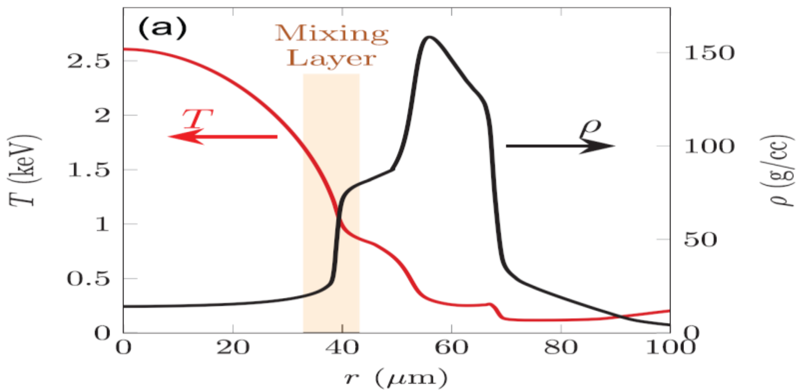
Interface mixing
PoP26, 050601 (2019)



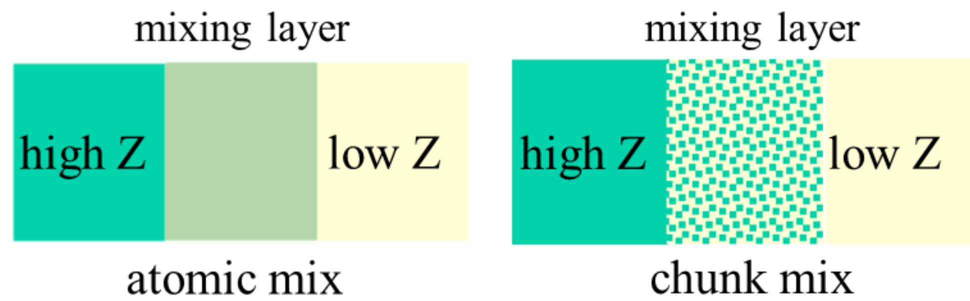
PoP26, 072705(2019)

1. Preliminary remarks

Question 3: **why does it affect the radiation transport?**



Typical plasma conditions
PoP27, 092704 (2020)



$$\kappa = f_H\,\kappa_H + f_L\,\kappa_L \qquad \frac{1}{\kappa} = \frac{f_H}{\kappa_H} + \frac{f_L}{\kappa_L}$$

(Density, Temperature)	$\sigma_{a,C}/\sigma_{a,DT}$	$\sigma_{a,Si}/\sigma_{a,DT}$
(20.3, 1.66)	43.06	495.86
(22.5, 1.48)	17.50	605.09
(27.9, 1.25)	25.58	1393.07
(71.0, 0.99)	48.72	3222.33
(78.1, 0.89)	58.42	4055.90

Ratio of species opacity **C&DT** **Si&DT**

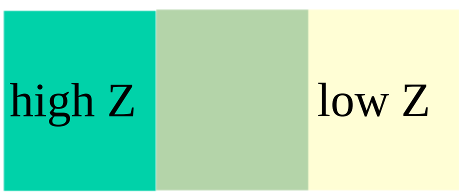
In the mixing layer of ICF, the opacity of carbon and silicon is, respectively, higher than that of DT fuels by **1 and 2–3 orders of magnitude**.

1. Preliminary remarks

Question 3: **why does it affect the radiation transport?**

The opacity of stochastic mixture strongly depends on the mixing, whose uncertainty greatly exceeds that of species opacity for (ρ, T).

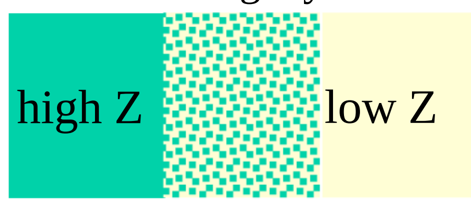
mixing layer



atomic mix

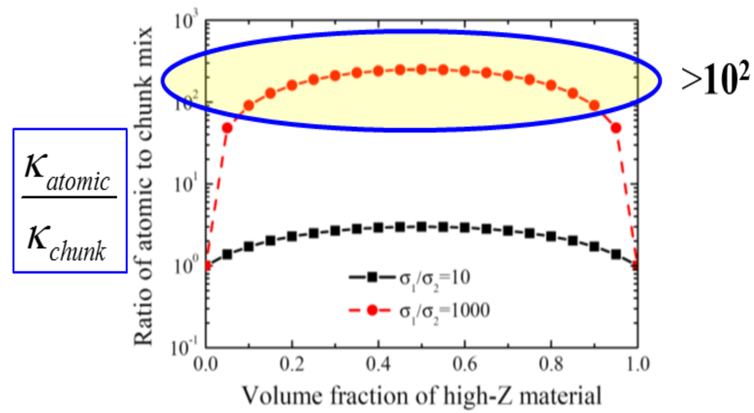
$$\kappa = f_H\,\kappa_H + f_L\,\kappa_L$$

mixing layer



chunk mix

$$\frac{1}{\kappa} = \frac{f_H}{\kappa_H} + \frac{f_L}{\kappa_L}$$



For stochastic mixture, **a direct solution of radiation transport equations is forbidden**, since the opacity is random for any positions.

$$\frac{1}{c}\frac{\partial}{\partial t}\psi(t,\mathbf{r},\mathbf{\Omega}) + \mathbf{\Omega}\cdot\nabla\psi(t,\mathbf{r},\mathbf{\Omega}) + \sigma_t(t,\mathbf{r})\psi(t,\mathbf{r},\mathbf{\Omega}) = \frac{c\sigma_a(t,\mathbf{r})}{4\pi}aT^4(t,\mathbf{r}) + \frac{\sigma_s(t,\mathbf{r})}{4\pi}\int\psi(t,\mathbf{r},\mathbf{\Omega}'\to\mathbf{\Omega})d\mathbf{\Omega}',$$

$$\rho(t,\mathbf{r})C_{\mathrm{v}}(t,\mathbf{r},T)\frac{\partial}{\partial t}T(t,\mathbf{r}) = -c\sigma_a(t,\mathbf{r})aT^4(t,\mathbf{r}) + [\sigma_t(t,\mathbf{r}) - \sigma_s(t,\mathbf{r})]\int\psi(t,\mathbf{r},\mathbf{\Omega})d\mathbf{\Omega}\,.$$

1. Preliminary remarks

Over the past decades, a substantial number of studies have been performed to understand the radiation transport in stochastic mixtures.

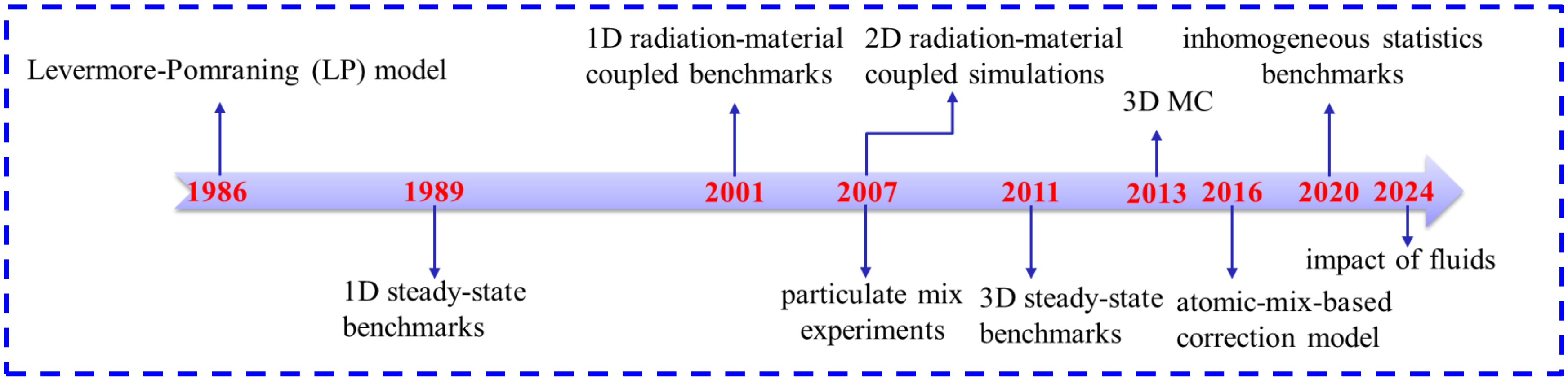


Table 10 Reflection and transmission results for planar geometry.

s		EXACT	MODEL
$\sigma_0 = 10/99$, $\sigma_{s0}/\sigma_0 = 0.00$, $\lambda_0 = 99/100$	$\sigma_1 = 100/11$, $\sigma_{s1}/\sigma_1 = 1.00$, $\lambda_1 = 11/100$		
0.1	$\bar{R}$	0.0491	0.0479
	Σ(R)	0.1182	
	$\bar{T}$	0.9331	0.9343
	Σ(T)	0.1132	
1.0	$\bar{R}$	0.2495	0.2187
	Σ(R)	0.2354	
	$\bar{T}$	0.5950	0.6254
	Σ(T)	0.2143	
10.0	$\bar{R}$	0.4342	0.3760
	Σ(R)	0.1616	
	$\bar{T}$	0.0146	0.0259
	Σ(T)	0.0152	

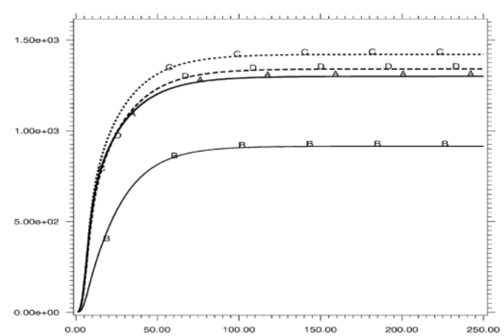
Table 11 Reflection and transmission results for planar geometry

s		EXACT	MODEL
$\sigma_0 = 10/99$, $\sigma_{s0}/\sigma_0 = 1.00$, $\lambda_0 = 99/100$	$\sigma_1 = 100/11$, $\sigma_{s1}/\sigma_1 = 0.00$, $\lambda_1 = 11/100$		
0.1	$\bar{R}$	0.0087	0.0086
	Σ(R)	0.0029	
	$\bar{T}$	0.9014	0.9004
	Σ(T)	0.2111	
1.0	$\bar{R}$	0.0548	0.0460
	Σ(R)	0.0307	
	$\bar{T}$	0.4841	0.4834
	Σ(T)	0.3632	
10.0	$\bar{R}$	0.0856	0.0591
	Σ(R)	0.0697	
	$\bar{T}$	0.0016	0.0015
	Σ(T)	0.0121	

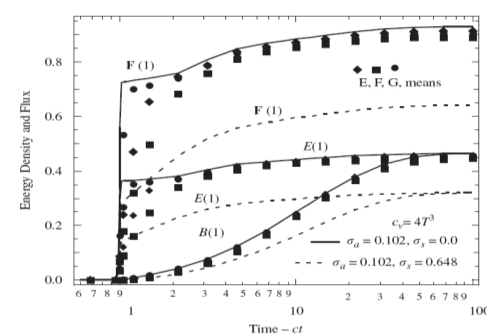
Table 12 Reflection and transmission results for planar geometry

s		EXACT	MODEL
$\sigma_0 = 10/99$, $\sigma_{s0}/\sigma_0 = 0.90$, $\lambda_0 = 99/100$	$\sigma_1 = 100/11$, $\sigma_{s1}/\sigma_1 = 0.90$, $\lambda_1 = 11/100$		
0.1	$\bar{R}$	0.0480	0.0473
	Σ(R)	0.0935	
	$\bar{T}$	0.9341	0.9344
	Σ(T)	0.1339	
1.0	$\bar{R}$	0.2563	0.2178
	Σ(R)	0.1546	
	$\bar{T}$	0.5985	0.6267
	Σ(T)	0.2832	
10.0	$\bar{R}$	0.4785	0.3707
	Σ(R)	0.0038	
	$\bar{T}$	0.0159	0.0237
	Σ(T)	0.0314	

M. L. Adams, et al., JQSRT42, 253(1989)



D. S. Miller, et al., JQSRT70, 115 (2001)

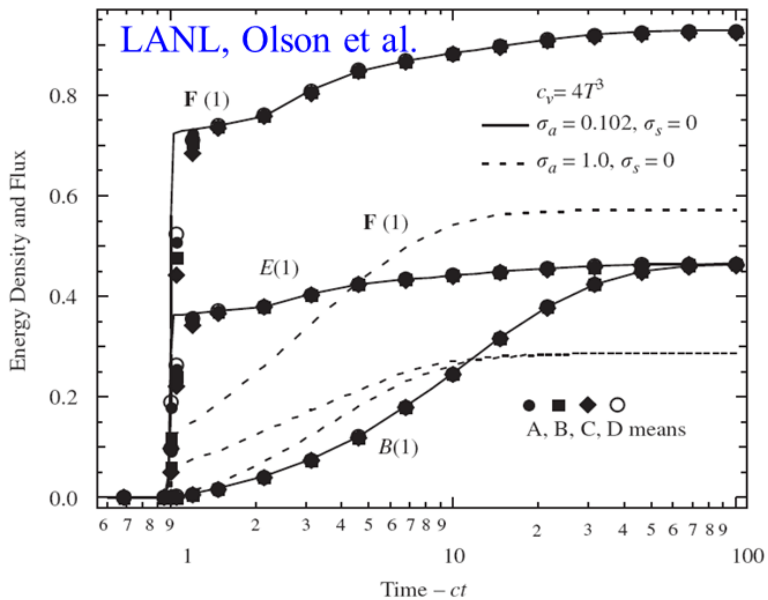


G. L. Olson, et al., JQSRT104, 86 (2007)

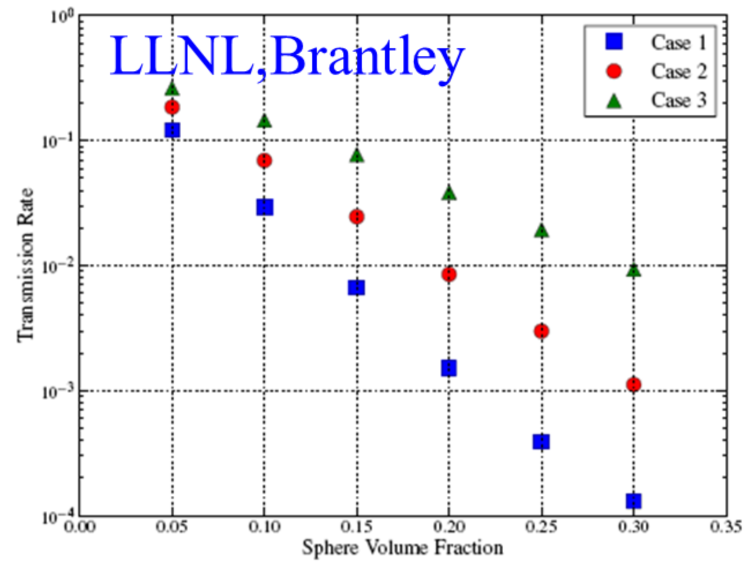
We have known manyyyy, but there are still some unclear ····

1. Preliminary remarks

A long-time debatable problem



JQSRT**104**, 86 (2007)



Tech. Rep. (LLNL, CA (United States), 2011)

Different mixing distributions and sizes **do not impact** the transmission flux and energy density.

Different mixing sizes **strongly affect** the transmission flux and energy density.

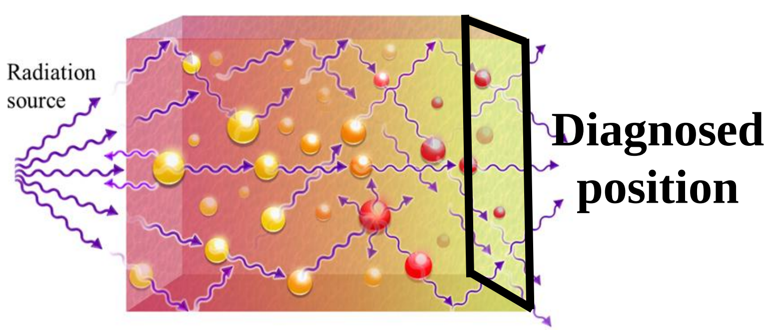
In this report, we shall provide the definite answer to this issue.

2. Theoretical method

We have **developed a massively-parallel code RAREBIT2D**("RAdiative tRansfEr in Binary stochastIc mixTures in Two Dimensions") from scratch.

RAREBIT2D

- Sampling different physical configurations
- Solving transport equations for each configuration
- Averaging the solutions over the ensemble



A physical realization of stochastic mixtures

Physical observables: ensemble-averaged radiation transmission flux, material energy density, and temperature

$$\langle J_{\text{trans}}(t,x,y)\rangle = \frac{1}{N}\sum_{n=1}^{N}\int_{\zeta>0}[\zeta\,\psi_n(t,x,y,\mathbf{\Omega})]d\mathbf{\Omega},$$

$$\langle E_{\text{mat}}(t,x,y)\rangle = \frac{1}{N}\sum_{n=1}^{N} aT_n^4(t,x,y),$$

$$\langle T_{\text{mat}}(t,x,y)\rangle = \frac{1}{N}\sum_{n=1}^{N} T_n(t,x,y).$$

$$\boxed{\langle q(t,x)\rangle = \frac{\int_0^{L_y}\langle q(t,x)\rangle dy}{L_y}}$$

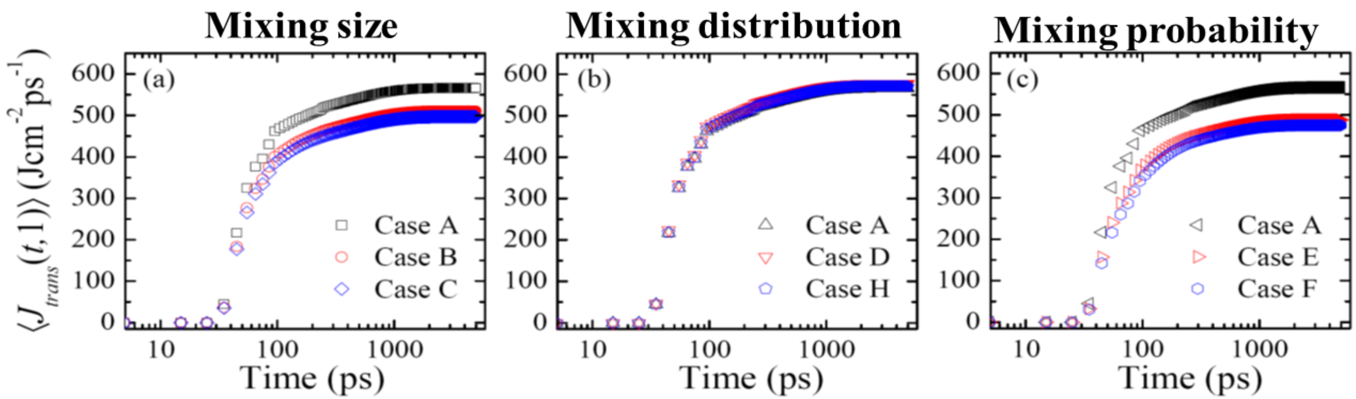
3. Results and Analysis

Parameter	Set 1 (Olson[20])	Set 2 (Brantley and Martos[25])
Domain size $L \times L$ (cm^2)	1.0×1.0	10.0×10.0
Grid representation $n_x \times n_y$	2000×2000	2000×2000
Material 1 density ρ_1 (g/cm^3)	2.7	2.7
Material 2 density ρ_2 (g/cm^3)	0.1	0.1
Mixing probability p_1	0.02–0.2	0.05–0.3
Average particle size $\langle r \rangle$ (cm)	0.001 91–0.019 40	0.087 54–0.700 28
Material 1 absorption opacity $\sigma_{a,1}$ (cm^{-1})	4.6–495.1	9.1
Material 2 absorption opacity $\sigma_{a,2}$ (cm^{-1})	0.1	0.1
Material scattering opacity σ_s	0	0
Initial material temperature T_0^m (keV)	0.003	0.003
Radiation source temperature T_0^r (keV)	0.3	0.3
Level of quadrature set s	16	16
Total simulation time τ (ps)	5000	20 000
Time step $d\tau$ (ps)	0.1	0.1
Convergence value of source iteration ϵ	10^{-6}	10^{-6}
Number of physical configurations N	10	10

Comparison of the parameters used by Olson et al. and Brantely et al.

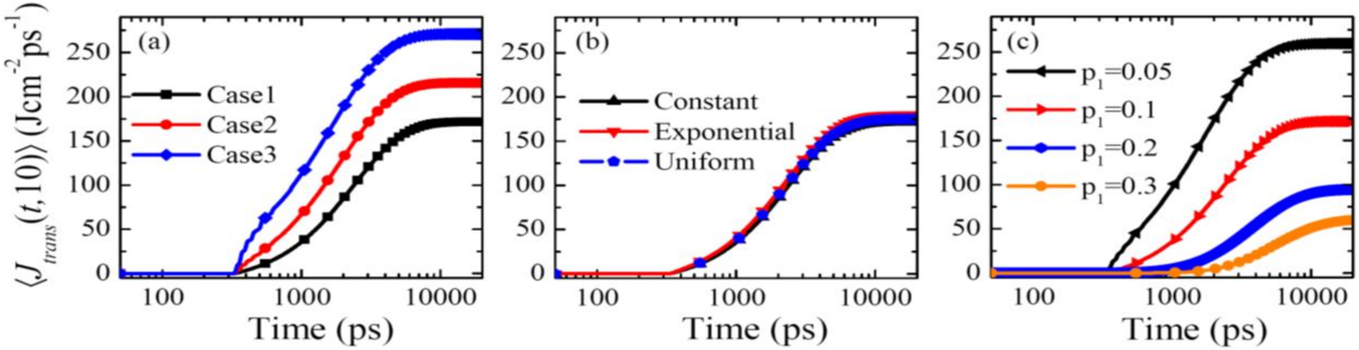
3. Results and Analysis

(a) Olson et al., JQSRT 104, 86 (2007)



10%-20%

(b) Brantely et al., Technical Report (Livermore, CA, 2011)



a factor of ~4.3

We have confirmed the inconsistency between Olson et al. and Brantely et al. using our RAREBIT2D code.

 C.-Z. Gao et al., Matter Radiat. Extremes 9, 067802 (2024)

3. Results and Analysis

Consider 1D, single-group, steady-state, and no scattering radiation transport,

$$\mu\frac{\partial}{\partial z}\psi(z,\mu)+\underline{\sigma_a(z)}\psi(z,\mu)=\underline{S(z)}$$

related to the material

We weaken the dependence of the photon's direction

$$\frac{\partial}{\partial z}\psi(z,\mu)+\sigma_a'(z)\psi(z,\mu)=S'(z)\ ,$$

$$\sigma_a'(z)=\frac{\sigma_a(z)}{\mu},\qquad S'(z)=\frac{S(z)}{\mu}=\sigma_a'(z)B(T)$$

The ensemble-averaged equations are

$$\frac{d\langle\psi\rangle}{dz}+\langle\sigma_a\rangle\langle\psi\rangle+\gamma\chi=\langle S\rangle$$

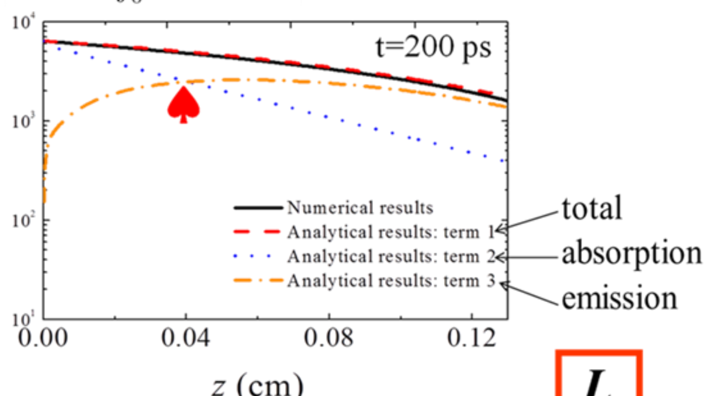
$$\frac{d\chi}{dz}+\gamma\langle\psi\rangle+\hat{\sigma}_a\chi=T$$

$$\chi=\sqrt{p_1p_2}(\psi_1-\psi_2)$$

The ensemble-averaged intensity can be analytically written as

$$\underbrace{\langle\psi_\mu(z)\rangle}_{\mathrm{I}}\approx\underbrace{\psi_0\left[we^{\varepsilon_+ z}+(1-w)e^{\varepsilon_- z}\right]}_{\mathrm{II}}+$$
$$\underbrace{\langle B(z)\rangle\frac{1+\langle\sigma_a(z)B(z)\rangle/\sigma_{a,1}\sigma_{a,2}\lambda_c\langle B(z)\rangle}{1+\langle\sigma_a(z)\rangle/\sigma_{a,1}\sigma_{a,2}\lambda_c}\left\{1-\left[we^{\varepsilon_+ z}+(1-w)\,e^{\varepsilon_- z}\right]\right\}}_{\mathrm{III}},$$

$$\langle J(z)\rangle=\int_0^1\mu\langle\psi_\mu(z)\rangle d\mu$$ transmission flux



A theoretical criterion $\boxed{\frac{L}{l_p}}$

3. Results and Analysis

Effective optical thickness

$$\tau_{eff} = \sigma_{eff} \cdot L = \frac{L}{l_p}$$

If the number of l_p through the domain is less than 1, the stochastic mixtures are optically thin, the probability of the photons interacting with the mixture is small, thus the influence of the binary stochastic mixture is limited.

Physical parameters from Olson et al., JQSRT 104, 86 (2007) result in optically thin stochastic mixtures!

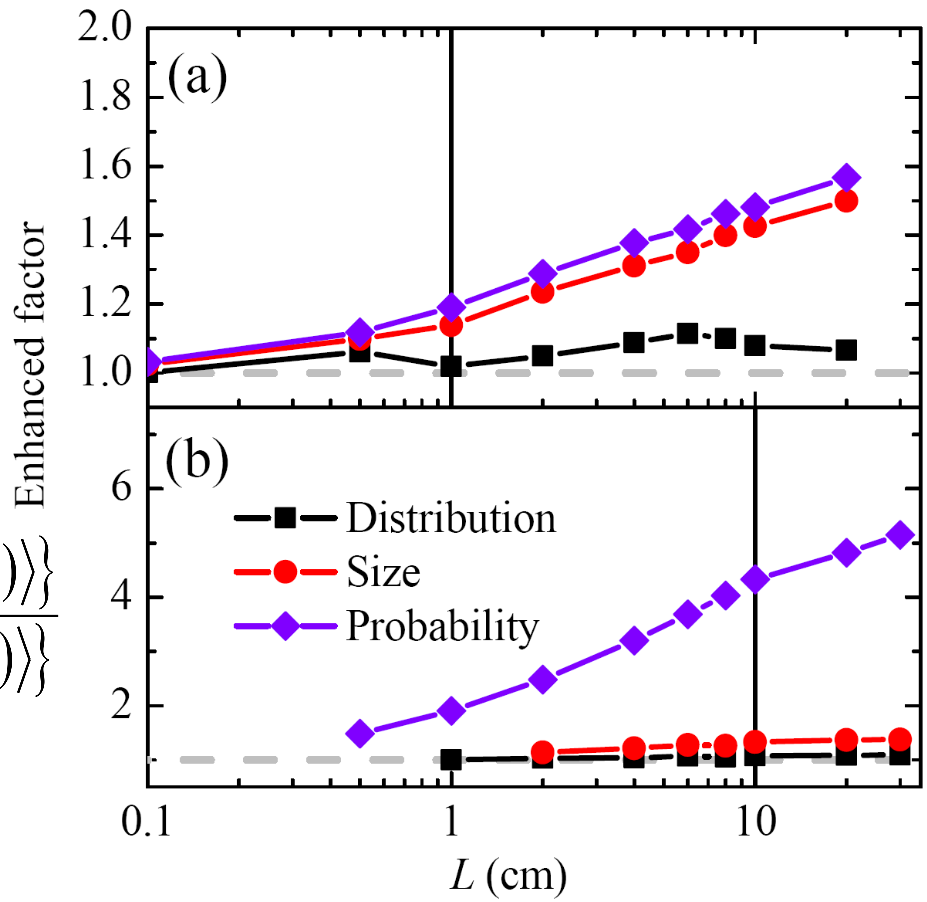
If the number of l_p through the domain exceeding 1, it is optically thick such that the photons most probably interacts with the mixture for multiple times before it finally traverses the domain, thereby the properties of the mixture do really matter.

Physical parameters from Brantely et al., Tech. Rep. (LLNL, 2011) result in optically thick stochastic mixtures!

The previous discrepancy between Olson et al. and Brantley et al. arises from different optical properties. In this sense, previous results are basically consistent.

C.-Z. Gao et al., Matter Radiat. Extremes 9, 067802 (2024)

3. Results and Analysis



$$\frac{\max\left\{\left\langle J_{trans}\left(t\rightarrow\infty,L\right)\right\rangle\right\}}{\min\left\{\left\langle J_{trans}\left(t\rightarrow\infty,L\right)\right\rangle\right\}}$$

The dependence of the radiation fluxes on the stochastic mixture can be (in)significant by varying the mixing width (L) and/or opacities (l_p).

 C.-Z. Gao et al., Matter Radiat. Extremes 9, 067802 (2024)

4. Summary

For the long-standing disputable problem of the impact of mixing on the radiation transport, **we have made efforts to understand the radiation transport in stochastic mixtures**:

- ✓ **Confirmed** the previous discrepancy between Olson et al. and Brantley et al.
- ✓ **Proposed** the effective optical thickness as a theoretical criterion.
- ✓ **Unveiled** the discrepancy arises from different optical properties, thus previous results are basically consistent.

More details can be found in
C.-Z. Gao, et al., **Benchmark simulations** of radiative transfer in participating binary stochastic mixtures in two dimensions. Matter and Radiation at Extremes 9, 067802 (2024).

Thanks for your attention!

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理解随机混合物的辐射流

在惯性约束聚变(ICF)中，聚变靶丸的结构中存在非常不同的材料使得在界面附近产生瑞利-泰勒不稳定性。这些流体力学不稳定性导致多物质混合区的形成，这被认为极大地改变了辐射输运，最终影响了内爆性能。由于随机混合物的材料组分仅在统计上可知，用于均匀介质的传统确定性和/或蒙特卡罗辐射输运方法可能不适用。因此，随机混合物的辐射输运数值模拟和物理建模是一个挑战。

首先我们针对文献中关于混合对辐射输运的影响这一长期存在争议的问题开展研究，通过发展一个大规模并行代码来直接模拟随机混合物中的辐射输运，我们首先证实了这种差异。基于稳态辐射输运方程导出的模型，我们提出有效光学厚度作为理论判据，并发现先前的差异是由不同的光学性质引起的，这意味着前人结果本质上是一致的。