SINGLE-LAYER NEURAL NETWORKS IN PROBLEMS OF APPROXIMATION OF MULTIDIMENSIONAL FUNCTIONS AND SOLVING ELLIPTIC EQUATIONS

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The development of Kolmogorov's theorem [1] on the decomposition of multidimensional continuous functions into a sum of superpositions of one-dimensional functions led to Cybenko's theorem [2], which states that the set of functions representable by a single-layer neural network is dense in the space of continuous functions. These KAN-functions [3] are a sum of parameterized sigmoidal transformations of one-dimensional linear functions. With the exclusive level of software support to neural networks, it becomes possible to create highly efficient programs for approximating multidimensional functions and domain boundaries as a finite sum of simple, infinitely smooth functions, including potentials of equations of state and solutions to mathematical physics problems.

Highly efficient programs have been developed in Python for the torch tensor module [3] for solving such problems. This module includes a number of stochastic gradient descent [4] programs for finding dozens, hundreds, or even thousands of parameters of the minimum of the residual functional. The automatic differentiation of this functional with respect to search parameters and the capabilities of tensor algebra allow the minimization problem to be written in several short lines. Programs on request .cuda() are executed on an RTX3070 video card from Nvidia with CUDA practically within a few minutes.

The first Dirichlet problem for an elliptic equation is considered as an example where a mathematical physics problem is solved as a multidimensional KAN-function without the use of grid methods and grid differential. The problem is divided into a sum of solutions to the Poisson problem and the Laplace problem with the boundary condition in the form of the difference between the given boundary conditions and those remaining from the solution of the Poisson problem. The proposed method easily solves the elliptical problem with an arbitrarily shaped domain which is very difficult to solve with grid methods.

For a KAN-function approximated problem, the calculation of the normal derivatives of the intermediate KAN-surface solution of the Laplace equation at an arbitrary point on the domain boundary can easily be written in tensor form. This makes it easy to write a simple program for solving Neumann's problem for an arbitrarily shaped domain. The solution obtained in the form of an infinitely smooth KAN-function allows further processing and use in a convenient and effective manner.

So, the analytical solution of the Laplace problem in a square domain with the linear boundary conditions generated by a planar solution is described by infinite Fourier expansions, and any finite sums in these expansions have the Gibbs phenomenon in the corners of the domain, but the phenomenon is absent in the KAN-solution which describes the plane with very high accuracy.

References

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