ON NEURAL NETWORK-BASED STRUCTURED GRID GENERATION FOR NUMERICAL PDE SOLVERS

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This work presents a novel approach to generating 2D body-fitted curvilinear coordinate systems (BFCs) using fully-connected neural networks (FNNs), interpreted as diffeomorphisms between computational and physical domains. This enables solving partial differential equations (PDE) on structured grids even in complex geometries.

The method employs a residual neural network architecture that treats each layer as a small perturbation of the identity map. This allows for exact computation of Jacobians and metric tensors at any point and enforces regularity constraints such as non-negative Jacobian determinants.

Both non-physics-informed neural networks (non-PINN, without mesh-loss) and physics-informed neural networks (PINN, with interior loss) formulations are investigated. The PINN approach includes the Winslow functional to encourage orthogonality and uniformity inside the domain.

Numerical experiments demonstrate the method's ability to fit complex, multi-connected domains with high fidelity. Grid refinement becomes efficient through a simple forward pass, unlike classical methods that require solving elliptic systems anew.

The proposed method holds potential in computational fluid dynamics (CFD), particle in cell (PIC) simulations, and inverse problems where adaptive yet structured meshing is beneficial. Limitations and directions for future work such as 3D mesh generation and time-varying domains are discussed.

References

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