NEURAL NETWORK APPROXIMATION OF A TABULATED EQUATION OF STATE FOR SOLVING GAS DYNAMICS PROBLEMS

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The equation of state relates the thermodynamic quantities (density, temperature, and pressure) and it is used for mathematical modeling of various physical processes. The multiphase equations of state are used to simulate such physical phenomena as indirect compression of targets in inertial fusion [1, 2] and directed X-ray radiation [3]. The development of these equations is the order of the day in science and technology [4–7]. A wide-range multiphase equation of state is often a piecewise function defined partly or completely by a table [8]. Interpolation with such tables introduces additional errors, and numerical differentiation of partial pressure derivatives especially at phase transition points is a separate problem which is difficult to solve. A solution to this problem is approximation in the form of a continuous differentiable function. The authors of Ref. [9] proposed a neural network based approach to the approximation of equations of state.

The paper proposes a method that uses a neural network to approximate tabulated equations of state as a pressure dependence on density and temperature in wide ranges. Tabulated values were approximated for various equations of state with specified analytical formulas. Tabulated data are generated on the basis of the analytical formulas and then fed to the neural network: density and temperature in the input layer and pressure in the output one. In the process of machine learning the network selects weights and biases so as to minimize the difference between the predicted and actual values. As a result we have obtained an explicit approximating function which can be used to calculate pressure at any density and temperature within their specified ranges.

In this work we approximated the pressure surfaces for the ideal gas equation of state, the binomial Mie-Grüneisen equation of state, and a model wide-range multiphase equation of state. The equation of state for ideal gas was tested on five gas dynamics problems [10] and the Mie-Grüneisen equation, on two ones [11, 12]. As for the wide-range multiphase equation, its pressure surface is smooth on the linear scale, while on the logarithmic one it shows noticeable phase transitions. The paper proposes a method for approximating a two-dimensional surface on the logarithmic scale for independent variables and the sought quantity as well. Approximation on the logarithmic scale helped describe the complicated structure of the wide-range multiphase equation of state more accurately.

The error of the proposed neural network approach is evaluated through comparison with results obtained with the analytical formulas specified for the equations of state. It's efficiency was verified through test calculations.

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