THE DEPENDENCE OF VELOCITY AND PRESSURE DURING THE MOVEMENT OF BULK MATERIAL IN A VERTICAL TUBE OF SQUARE AND CIRCULAR CROSS-SECTION

E. A. Zhuravlev, R. N. Sadykov, M. A. Zakharov, K. E. Bartosh, N. R. Sadykov

Federal State Autonomous Educational Institution of Higher Professional Education «Snezhinsk Physical-Technology Institute (SPhTI) –National Research Nuclear University MEPhI», Snezhinsk, Russia

E-mail: n.r.sadykov@rambler.ru

According to both hydrodynamic movement of bulk material (BM for short) and dependence of viscosity on pressure, it is possible to derive the transverse velocity υ_z in a vertical tube of square and circular cross-section. In order to obtain the velocity, the mass flow rate of BM has been experimentally measured. It is shown that in order to determine the dependence of pressure p on the vertical coordinate *z*, it is necessary to consider the problem of stationary BM motion along an inclined plane. Results gained experimentally, confirm the same ones obtained by computational and theoretical calculations. The problem of the BM movement is rather relevant in modernity. For instance, patterns of such movement are manifested in the occurrence and movement of snow avalanches, in transporting solutions through flexible pipes, in loading fixed-mass building materials through conical bunkers [1]. In the report [2, 3] it was proposed that the viscosity of the BM depends on the hydrostatic pressure $\eta(z) = Kp(z)$ in BM, where $K = \text{const}$. In this case, an STF viscous stress tensor of the second rank of third dimension will have the following form:

$$\sigma_{ij} = \eta\left(\frac{\partial\upsilon_i}{\partial x_j} + \frac{\partial\upsilon_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial\upsilon_k}{\partial x_k}\right), \quad \sigma_{ii} = 0 \leftrightarrow \sum\nolimits_{i=1}^{3}\sigma_{ii} = 0, \tag{1}$$

where δ_{ij} is the Kronecker delta, e_{ijk} is the antisymmetric Levi-Civita's tensor of rank three.

The stress tensor does not contain symmetric $\sigma_{ij}^{(1)}$ and antisymmetric $\sigma_{ij}^{(2)}$ tensors of the second rank.

$$\sigma_{ij}^{(1)} = \eta_1\delta_{ij}\partial\upsilon_k/\partial x_k = \eta_1\delta_{ij}\,\mathrm{div}\,\upsilon = 0, \quad \sigma_{ij}^{(2)} = \eta_2\left(\partial\upsilon_i/\partial x_j - \upsilon_j/\partial x_i\right) = \eta_2 e_{ijk}\left(\mathrm{rot}\,\upsilon\right)_k = 0. \tag{2}$$

Taking into account the viscous force, the incompressibility of BM, where $\rho = \text{const}$ $\left(\mathrm{div}\,\upsilon = \partial\upsilon_k/\partial x_k = 0\right)$, gravity and pressure drop, it is possible to write the Navier-Stokes equation in the stationary case.

$$\rho\upsilon_k\frac{\partial\upsilon_i}{\partial x_k} = \frac{\partial}{\partial x_k}\left\{\eta\left(\frac{\partial\upsilon_i}{\partial x_k} + \frac{\partial\upsilon_k}{\partial x_i}\right)\right\} - \frac{\partial p}{\partial x_i} - \rho g\delta_{i3}, \tag{3}$$

where δ_{ik} is the Kronecker delta symbol, both *i* and *k* equal 1, 2, 3. Based on the method of separating variables, equations for velocity $\upsilon_z(x, y)$ and pressure $p(z)$ BM at $\eta = Kp$ in a long vertical tube of square and circular cross-section are derived from the equation (3).

$$\Delta_\perp\upsilon_z = \left(\rho g + \mathrm{d}p/\mathrm{d}z\right)/\left(Kp\right) = \alpha, \qquad p\left(z = H_0\right)\Big|_{t=0} = p_0, \tag{4}$$

z – vertical, and *x*, *y* (or *r*) are the transverse coordinates respectively. H_0 – BM column's height at *t* = 0; $\alpha = \text{const}$; $-h/2 \le x \le h/2$, $-h/2 \le y \le h/2$, $0 \le r \le R$; $\Delta_\perp = \partial$^2/∂x^2+∂^2/∂y^2; $p_0 = \text{const}$, the equation $\Delta_\perp\upsilon_z = \alpha$ is the Laplace equation; for vertical velocity, the first boundary condition is satisfied.

According to the equation (4), we are able to obtain the solution for p in the column of BM.

$$p(z) = \rho g/(K\alpha) + \left[p_0 - \rho g/(K\alpha)\right]\exp\left\{K\alpha\left[z - H_0\right]\right\}, \quad 0 \le z \le H. \tag{5}$$

The solution for the velocity in a vertical tube of square section in [5] is presented as the sum of two functions $\upsilon_z(x, y) = \upsilon_z^{(1)}(x, y) + \upsilon_z^{(2)}(x, y)$, where $\Delta_\perp\upsilon_z^{(1)} = \alpha$ (см. (4)), $\Delta_\perp\upsilon_z^{(2)} = 0$.

As a result, we get

$$\upsilon_z\left(x=\pm\frac{h}{2}, y=\pm\frac{h}{2}\right)=0,\ \upsilon_z^{(1)}=\frac{\alpha}{4}\left(x^2+y^2\right)+\Lambda;\ \upsilon_z(r)=\alpha\left(r^2-R^2\right)/4;\ \upsilon_z(r=R)=0,$$

$$\upsilon_z^{(2)}\left(x, y=\pm\frac{h}{2}\right)=-\upsilon_z^{(1)}\left(x, y=\pm\frac{h}{2}\right) \text{ or } \upsilon_z^{(2)}\left(x=\pm\frac{h}{2}, y\right)=-\upsilon_z^{(1)}\left(x=\pm\frac{h}{2}, y\right), \tag{6}$$

where $\Lambda=\text{const}$.

Solution for $\upsilon_z^{(2)}$ will be derived as a sum of products of the cosine function and the hyperbolic sine, which will satisfy boundary conditions from (4).

Calculated dependence of the longitudinal velocity component $\upsilon_z(x, y)$ of the bulk material in a square tube on x and y is shown on Fig. 2. In order to obtain two unidentified coefficients α и K in (4) two certain actions are necessary. The first one is to compare the experimental mass flow rate $\mathcal{M}_{\text{экс}}^{(1)}$ with the calculated $\mathcal{M}_{\text{рас}}^{(1)}$ and calculate α from (4).

$$\mathcal{M}_{\text{рас}}^{(1)}=\rho\iint\upsilon_z(x, y)\,\mathrm{d}x\,\mathrm{d}y. \tag{7}$$

Second one is derived via the infamous inclined plane that was mentioned earlier.

$$\upsilon_1(x_3)=(x/K)\tan\theta,\quad \mathcal{M}_{\text{рас}}^{(2)}=\rho\frac{H^2\tan\theta}{2K}. \tag{8}$$

Where θ is the inclination angle. Axes OX and OY are coplanar to BM's movement, and OZ is perpendicular to it. This, and the experimental calculation of the mass flow, allows us to define the value of K.

Knowing K and α, according to (5), it's possible to infer the distribution of pressure.

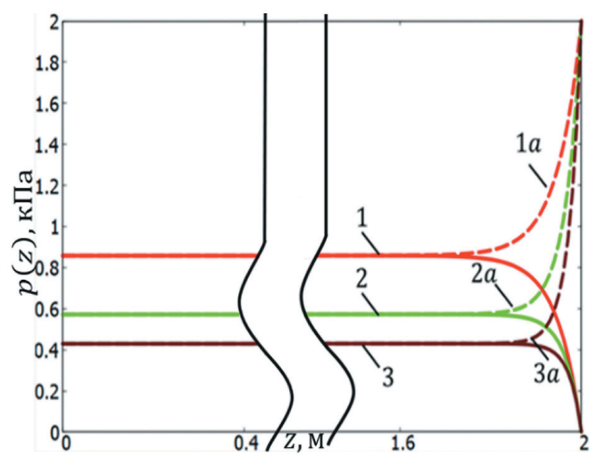

Fig. 1. Dependence of pressure on deepness of the layer z in BM, $H = 2$ m

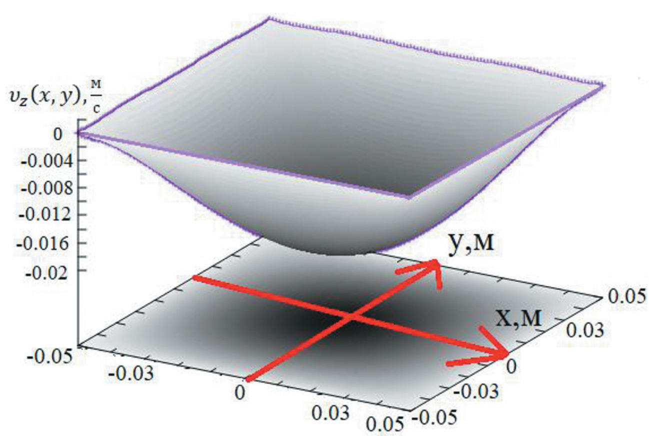

Fig. 2. Distribution of the transversal component of velocity in a tube of square cross-section

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