APPLICATION OF TRIGONOMETRIC SERIES TO OBTAIN A SOLUTION TO THE NONLINEAR HEAT EQUATION IN THE ONE-DIMENSIONAL CASE

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Infinite trigonometric series have already been used to construct solutions to various nonlinear equations and systems of partial differential equations [1–4]. In this paper, for the nonlinear heat equation on a finite interval $[0, \pi]$, the Cauchy problem is posed with continuous initial conditions that are evenly continued to the interval $[-\pi, 0]$ and then with a period of 2π to the entire numerical axis. The solution to the resulting Cauchy problem is sought in the form of a trigonometric series in cosines of a spatial variable. The coefficients of the series are the desired functions that depend on time. Obtaining a solution to the equation under consideration in the form of a trigonometric series only in sines is impossible due to the specific type of nonlinearity of the original equation. For the desired coefficients of the series, an infinite system of ordinary differential equations is written out and local convergence of the series in time and the corresponding theorem on multiple frequencies are proved. To obtain approximate solutions, finite sums are considered and finite systems of ordinary differential equations for the coefficients of these sums are solved numerically. In the constructed solution, the temperature equalizes over time. At the same time, at all moments of time, the area of the figure under the corresponding curve is constant, which is a consequence of the thermal insulation conditions at the ends of the segment $[-\pi, \pi]$.

References

1. **Bautin, S. P.** Mathematical modeling of one-dimensional flows of viscous heat-conducting gas by trigonometric series [Text] / S. P. Bautin, V. E. Zamyslov, P. P. Skachkov. – Novosibirsk : Nauka ; Ekaterinburg : UrGUPS, 2014.

2. **Bautin, S. P.** Representation of solutions of the Burgers equation by trigonometric series [Text] / S. P. Bautin, V. E. Zamyslov // Bulletin of the National Research Nuclear University MEPhI. – 2022. – Vol. 11. – No. 4. – P. 305–318.

3. **Bautin, S. P.** Representation of solutions of a system of equations of motion using trigonometric series [Text] / S. P. Bautin, O. A. Karelina, A. G. Obukhov // Bulletin of the National Research Nuclear University MEPhI. – 2023. – Vol. 12. – No. 1. – P. 39–51.

4. **Bautin, S. P.** Solution of nonlinear partial differential equations by trigonometric series [Text] / S. P. Bautin, V. E. Zamyslov, O. A. Karelina, A. G. Obukhov. – Snezhinsk : SFTI NRNU MEPhI, 2024.