WAVE TOMOGRAPHY IN MEDICINE: THEORY AND NUMERICAL METHODS

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Tomography has been developing intensively over the past two decades. The development of ultrasound early diagnosis methods for soft tissue malignancies is one of the key problems in medicine, and the absence of exposure to ionized radiation has become an urgent trend in medicine, which makes ultrasound tomography very promising.

Currently, ultrasound tomographs are being developed by various research groups around the world [1–5]. This work, which has been going on for more than 10 years, has reached the pilot production stage. However, there is still no commercially available ultrasound scanner.

One of the serious problems of ultrasound tomography is the development of effective and stable methods for solving nonlinear coefficient inverse problems.

The most adequate model is a three-dimensional inverse problem in which the speed of wave propagation, the density of the medium, and other acoustic parameters must be reconstructed from data recorded by detectors located at the boundary of the region.

Consider the following two-dimensional direct problem for a hyperbolic system of equations [6]

$$\frac{\partial u}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0,\ \ \frac{\partial v}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial y}=0,$$

$$\frac{\partial p}{\partial t}+\rho c^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\Theta_\Omega(x,y)I(t)$$

with zero initial data and non-reflective boundary conditions.

Here $u(x,y,t)$ is the velocity in the x direction, $v(x,y,t)$ is the velocity in the y direction, $p(x,y,t)$ is the pressure, $c(x,y)$ is the velocity of wave propagation in the medium, $\rho(x,y)$ is pressure, $\Omega=(x,y)\in[0:L]\times[0:L]$ is domain, $I(t)$ probing source, $\Theta_\Omega(x,y)$ is localization of sources.

Inverse problem is to find coefficients $c(x,y)$ and $\rho(x,y)$ by known the additional information [7]

$$f_i(t)=p(x_i,y_i,t),\ \ i=1\ldots N.$$

Here N is the number of receivers.

The solution of the inverse problem is reduced to the task of minimizing the target functional by the accelerated gradient method [4]. Formulas for the gradient of the functional are obtained using solutions to the corresponding conjugate problems. The results of numerical calculations and a comparative analysis of numerical algorithms, including neural network approaches, are presented [8].

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