APPLICATION OF THE EXTENDED KALMAN FILTER TO IDENTIFY PARAMETERS OF EXTERNAL HEAT EXCHANGE OF EQUIPMENT UNDER AERODYNAMIC HEATING

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The analysis of temperature regime of aviation equipment components under direct aerodynamic heating requires an adequate mathematical model of heat exchange (MMHE) for the relevant physical system.

The highest uncertainty in such MMHE is caused by aerodynamic interference, i. e. aerodynamic interaction between the equipment components and surrounding structures.

To obtain information on aerodynamic interference effects, temperature of the equipment casing and approach flow parameters are measured during aerodynamic heating tests. Test results are processed in order to identify external heat exchange parameters for subsequent MMHE refinement.

In this work, the hybrid extended Kalman filter is used to identify external heat exchange parameters for the MMHE of the equipment [1]. This method has been chosen due to its inherent advantages in terms of convergence of solutions and lower computing cost [2, 3].

The lumped-parameter MMHE for the equipment is described by a system of N nonlinear ordinary differential equations (ODE). M temperature sensors are installed on the equipment casing. Measurements are carried out at specific time intervals with an increment, $\Delta\tau$. Permanent aerodynamic features are assumed in each sensor area due to geometry of the equipment and surrounding structures, while measured temperature values are independent of each other. Temperature measurement errors are assumed to be Gaussian white noises.

The problem under consideration can be represented by a nonlinear continuous-time model for system state evolution (1) and a linear discrete measurement model (2) to refine the system state:

$$\dot{\mathbf{x}}(\tau) = f(\mathbf{x}(\tau)); \tag{1}$$

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k, \; k = 1, 2, \ldots, S, \tag{2}$$

where $\mathbf{x}(\tau)$ is the system state vector, $\mathbf{x}_k = \mathbf{x}(\tau_k)$; $\tau_k = k\Delta\tau$ is the time interval corresponding to the k^{th} measurement; $\boldsymbol{f}(\mathbf{x}(\tau))$ is the nonlinear vector function; S is the number of measurements; $\mathbf{z}_k$ is the measurement vector of length $(M \times 1)$; $\mathbf{H}_k$ is the measurement matrix; $\mathbf{v}_k$ is the white Gaussian measurement noise with zero mathematical expectation, $\mathbf{E}[\mathbf{v}_k] = 0$, and covariance matrix, $\mathbf{R}_k$, of size $(M \times M)$.

We assume that components of random measurement error vector, $\mathbf{v}_k$, are not correlated with each other and have an a priori variance, σ^2.

To identify external heat exchange parameters, the extended system state vector, $\mathbf{x}(\tau)$, of length $((N+2M)\times 1)$ is used, which includes, in addition to temperatures of all model components, $2M$ unknown parameters of external heat exchange (heat exchange coefficients and the Mach number correction factors).

At the first step of filter operation, the system state vector and measurement error covariance matrix, $\mathbf{P}(\tau)$, of size $((N+2M)\times(N+2M))$ are predicted. Before the first iteration, it is necessary to define initial values of the state vector, $\hat{\mathbf{x}}_{0|0} = \mathbf{E}(\mathbf{x}(\tau_0))$, and covariance matrix, $\mathbf{P}_{0|0}$, which contains estimation error variances at the initial time.

To predict the state vector, $\hat{\mathbf{x}}_{k|k-1}$, a common numerical method for solving ODE systems can be used, in particular, the Runge-Kutta method.

The covariance matrix in the Kalman filter algorithm for nonlinear dynamic model of the system state is predicted as follows [1, 3]:

$$\mathbf{P}_{k|k-1} = \mathbf{\Phi}_{k,k-1} \mathbf{P}_{k-1} \mathbf{\Phi}_{k,k-1}^T, \tag{3}$$

where $\mathbf{\Phi}_{k,k-1}$ is the state transition matrix calculated by the formula:

$$\mathbf{\Phi}_{k,k-1} = \mathbf{I} + \mathbf{F}_k \Delta\tau + \frac{1}{2!}\mathbf{F}_k^2 \Delta\tau^2 + \ldots + \frac{1}{p!}\mathbf{F}_k^p \Delta\tau^p + \ldots, \tag{4}$$

where $\mathbf{F}_k$ is the Jacobian matrix of the vector-function, $\boldsymbol{f}(\hat{\mathbf{x}}_{k|k-1})$.

At the second step of the Kalman filter operation, the gain matrix, $\mathbf{K}_k$, is calculated based on the covariance matrix for the extrapolated state vector:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T \left(\mathbf{H}_k \mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{R}_k\right)^{-1}. \tag{5}$$

Using the gain matrix one can obtain refined estimates of the system state vector and covariance matrix of state vector estimation:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \left(\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}\right); \tag{6}$$

$$\mathbf{P}_{k|k} = \left(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k\right)\mathbf{P}_{k|k-1}. \tag{7}$$

The estimated system state vector and its covariance matrix can be further used in regression analysis to determine criterion relations for external heat exchange parameters of the equipment.

The algorithm has been implemented in numerical software application for the lumped-parameter MMHE. The developed algorithm has been validated against the experimental data. Numerical analysis shows that heat exchange coefficients and the Mach number correction factors commonly converge to stationary values in less than 60 steps of the Kalman filter.

References

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