CLASSES OF EXACT SOLUTIONS WITH NONLINEAR DEPENDENCE ON SPATIAL COORDINATES FOR DESCRIBING THE MOTIONS OF COMPRESSIBLE MEDIA

E. Yu. Prosviryakov, N. V. Burmasheva

Ural Federal University, Yekaterinburg, Russia

Institute of Mechanical Engineering, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, Russia

The study of viscous fluid flows traditionally begins with the idea of one-dimensionality of the flow. Many classical solutions are based on this approach – the exact Couette solution, the Poiseuille gradient flow, the solution of the first and second Stokes problems, the Nusselt flow and other analytical results. This allowed us to obtain the first ideas about the structure of the flow at a constant temperature and gave impetus to the development of the hydrodynamics of exact solutions. These results were used to construct two-dimensional, shear and three-dimensional flows, as well as to describe flows taking into account the non-uniformity of the distribution of additional fields – the temperature field, the impurity concentration field, the magnetic field, etc. Later, attempts were made to generalize to the case of non-uniform flows, and not only in the class of functions linear in part of the coordinates, but with a significantly non-linear dependence on these same spatial coordinates.

Most of the research conducted to date is based on the hypotheses of Newtonian continuum mechanics, according to which a representative volume of a medium is considered as a material point with only three degrees of freedom, corresponding to the possibilities of translational motion in three directions. The possibility of interaction between two liquid particles along the boundary of their contact is also not taken into account. This idea does not correspond to reality, which is confirmed by numerous experiments. Work in this direction is complicated by the fact that when taking into account the moment (rotational) degrees of freedom, the order of the system of differential equations that form the basis of the model – the Navier–Stokes equation, supplemented by the incompressibility equation – sharply increases. Nevertheless, despite these difficulties, work in this direction is still being carried out, and to date a number of accurate results have been obtained.

It is worth noting that when describing flows of incompressible media (primarily liquids) by constructing exact solutions, the incompressibility equation plays a special role, since the choice of a class of exact solutions that identically satisfy this equation removes the problem of overdetermination of the reduced systems of equations that arise when substituting the selected class into the Navier–Stokes vector equation. This circumstance is largely decisive when dealing with the description of shear flows, since in this case the reduced system of equations of the model is overdetermined.

The approach based on the idea of incompressibility of the medium works to an acceptable degree for liquids, as well as for gases under isothermal flow conditions at speeds significantly lower than the speed of sound. But the problem of describing gas flows in a non-uniform temperature field and (or) at high flow speeds remains. The report discusses issues related to approaches to constructing classes of exact solutions for compressible media based on the speakers' accumulated research experience [1–7]. Variants of classes of exact solutions with a linear dependence of the velocity field components on spatial coordinates and with essentially nonlinear dependencies on the same parameters are considered. Issues of replicating classes of solutions are discussed.

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