INVESTIGATION OF THE BEHAVIOUR OF UNSTEADY TWO-DIMENSIONAL PERIODIC FLOWS DEFINED BY THE EQUATIONS OF MOTION

O. A. Karelina

MEPI National Nuclear Research University's Snezhinsk Physicotechnical Institute, Snezhinsk, Russia

FSUE «RFNC – VNIITF named after Academ. E. I. Zababakhin», Snezhinsk, Russia

As a mathematical model for approximate transfer of gas motions from the full system of Navier-Stokes equations, only the equations of motion are studied assuming constant values of thermodynamic parameters of density and temperature: $\rho = 1$, $T = 1$.

The velocity scale u_{00} is taken as $1/3 \cdot 10^3$ м/c, which is close to the speed of sound in air under normal conditions. The distance scale x_{00} is taken as the value corresponding to the geometric characteristics of the particular flow under study.

This paper considers the case of absence of z dependence and equality to zero of the third component of the gas velocity vector: $\partial/\partial z = 0$; $\upsilon_3 = 0$.

In this case, the system under consideration in a detailed form has the following form

$$\begin{cases} u_t = -uu_x - \upsilon u_y + \mu_0\left(u_{xx} + \dfrac{3}{4}u_{yy} + \dfrac{1}{4}u_{xy}\right), \\ \upsilon_t = -u\upsilon_x + \upsilon\upsilon_y + \mu_0\left(\upsilon_{yy} + \dfrac{3}{4}\upsilon_{xx} + \dfrac{1}{4}u_{xy}\right). \end{cases} \tag{1}$$

Further the system (1) will be referred to as a system of equations of motion. In this paper, the case of two spatial variables is considered and, taking into account the results from [1–5], the following representations of the sought functions are used u, υ.

$$\begin{aligned} u(t,x,y) &= \sum_{k=1}^{K}\left[u_{k,1}\sin(kx) + u_{k,2}\sin(ky)\right], \\ \upsilon(t,x,y) &= \sum_{k=1}^{K}\left[\upsilon_{k,1}\sin(kx) + \upsilon_{k,2}\sin(ky)\right]. \end{aligned} \tag{2}$$

With the help of constants $u_{k,1}^0$, $u_{k,2}^0$, $\upsilon_{k,1}^0$, $\upsilon_{k,2}^0$ at time $t = 1$ initial conditions are set for systems of ordinary differential equations in which the upper indices in the sums are taken equal to K, i. e., $M = K$. The variable ℓ in these systems takes integer values from one to K.

$$u'_{\ell,1} = -\frac{1}{2}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty} m u_{k,1}(t) u_{m,1}(t) b_{k,\ell,m} - \mu_0 \ell^2 u_{\ell,1}; \qquad \ell = 1, 2, 3, \ldots \tag{3}$$

$$\upsilon'_{\ell,1} = -\frac{1}{2}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty} m u_{k,1}(t) \upsilon_{m,1}(t) b_{k,\ell,m} - \frac{3}{4}\mu_0 \ell^2 \upsilon_{\ell,1}; \quad \ell = 1, 2, 3, \ldots \tag{4}$$

$$u'_{\ell,2} = -\frac{1}{2}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty} k \upsilon_{m,2}(t) u_{k,2}(t) b_{\ell,m,k} - \frac{3}{4}\mu_0 \ell^2 u_{\ell,2}; \quad \ell = 1, 2, 3, \ldots \tag{5}$$

$$\upsilon'_{\ell,2} = -\frac{1}{2}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty} k \upsilon_{k,2}(t) \upsilon_{m,2}(t) b_{\ell,m,k} - \mu_0 \ell^2 \upsilon_{\ell,2}; \qquad \ell = 1, 2, 3, \ldots \tag{6}$$

The obtained Cauchy problems for the systems are solved numerically at $0 \le t \le t_f$, where the finite moment t_f is chosen based on the meaning of the problem under consideration and its solutions.

References

1. **Bautin, S. P.** Nonstationary two-dimensional periodic solutions of the equations of motion [Text] / S. P. Bautin, O. A. Karelina, A. G. Obukhov. – Snezhinsk : SFTI NRNU MEPhI, 2023. – 57 c.
2. **Bautin, S. P.** Representation of the solutions of the system of equations of motion using trigonometric series [Text] / S. P. Bautin, O. A. Karelina, A. G. Obukhov // Bulletin of the National Research Nuclear University MEPhI. – 2023. – Vol. 12, No. 1. – P. 39–51.