COMPARATIVE ANALYSIS OF NUMERICAL METHODS FOR SOLVING THE INVERSE PROBLEM OF THE SOURCE OF WAVE PROPAGATION

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This report investigates the inverse problem of determining the source of waves in a two-dimensional case.

The direct problem for the acoustic equation in the $\Omega = \{(x,y): x \in (0,L_x), y \in (0,L_y)\}$ domain is considered:

$$\begin{aligned} u_{tt} &= div \Big(c^2 \nabla u \Big), \ \, \big(x,y \big) \in \Omega, \ \, t \in \big(0,1 \big), \\ u\big|_{t=0} &= q \big(x,y \big), \ \, u\big|_{t=0} = 0, \\ u\big|_{\partial \Omega} &= 0. \end{aligned}$$

Similar problems arise in many applications. For example, in problems of tsunami wave propagation, $c(x,y) = \sqrt{gh(x,y)}$ is the wave propagation speed, h(x,y) is the ocean depth, g = 9.81 m/s² is the acceleration of gravity [1].

The inverse problem consists of determining the function q(x, y) using additional information [2]:

$$u(x_n, y_n, t) = f_n(t), \quad n = \overline{1, N}.$$

Here (x_n, y_n) is the location of receivers, N is the number of receivers.

In operator form, the inverse problem is formulated as Aq = f.

A comparative analysis of numerical methods for solving this problem based on the matrix representation of the problem, neural networks, gradient methods was carried out. Measurements of the speed of the solution were also carried out when parallelizing algorithms and transferring them to the GPU.

References

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