NUMERICAL SOLUTION OF RADIATION TRANSPORT WITH A TEMPERATURE DISCONTINUITY

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When V. S. Vladimirov derived in 1952 his method of characteristics, he showed that in cases where discontinuities were present in the coefficients of the kinetic equation, its solution lost its smoothness on the characteristic tangent to the discontinuity line [1, 2]. Using the exact solution derived by E. S. Kuznetsov for a stationary transport equation in spherically symmetric geometry [3], B. N. Chetverushkin [4] constructed test problems to measure the loss of accuracy when the neutron transport equation is numerically integrated with no account for singularities on these characteristics. By analogy with Chetverushkin's tests, we constructed a radiative heat transfer problem to simulate the discontinuous temperature solution.

Kuznetsov's solution does not allow getting analytical expressions for radiation density and flux. In the limit of an optically dense medium in the center of the system the exact solution approaches an approximated solution for which these analytical expressions can be obtained but the solution is discontinuous. The approximated solution allows the analytical description of discontinuities in radiation density and flux and is then of interest for the construction of a test problem in this statement. The solution can also be used to evaluate the accuracy of the quadrature formulas used for radiation intensity approximation with the loss of smoothness on the characteristic tangent to the temperature discontinuity line.

The discrete-ordinates method (DS_n method) [5] which is used for solving radiative transfer offers the linear St and DD schemes that found wide use and became classical. The step-by-step St scheme have the first order of approximation and it is monotone and positive. The diamond DD scheme has the second order of approximation but it is non-monotone and non-positive. If in case of a negative solution in the DD scheme we change to the St scheme, we obtain the DD/St scheme. But oscillations in the positive region of the DD/St solution are still present, the accuracy of the solution reduces to order one, and changing from one scheme to the other may make the iteration converge slowly or diverge. That is why the classical St, DD, and DD/St schemes which are used for neutron transport simulation are almost never used for solving radiative heat transfer. These shortcomings of the DS_n schemes are removed by using the mechanisms of nonlinear corrections to the solution. So, in the DDAD (Diamond Difference with Artificial Dissipation) schemes [6], anti-dissipation in the DD scheme is removed by its subtraction from the residual term. In the ATVDR (Additive Total Variation Diminishing Reconstruction) schemes [7], the higher order of approximation is ensured by the use of TVD reconstruction and numerical non-monotonicity is reduced through the introduction of special limiter functions. In this case the ATVDR schemes, unlike the classical TVD ones, preserve their properties of the DS_n method, which allows the approximation to be done within one cell as well as the use of recurrent computing. In these schemes it is possible to keep the second order of approximation (except some points with extrema) and the positiveness of the solution.

The paper investigates radiation temperatures calculated on the boundary with vacuum for monotonicity and accuracy, and the presence of the Gibbs effect from sharply disturbed monotonicity. The Gibbs effect is a negative effect that occurs when discontinuities are interpolated as characteristic oscillations [8]. Calculations by the schemes of different approximation orders (St, DDAD, and ATVDR) with different Carlson quadratures ES_n [9] are provided.

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