Coefficient inverse problems for parabolic equations with nonlocal data


ИВМиМГ

Maxim Shishlenin
Institute of Computational Mathematics
and Mathematical Geophysics, Novosibirsk
e-mail: ***mshishlenin@ngs.ru***

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15 марта 2019 года состоялось заседание Ученого совета Института, посвященное 85-летнему юбилею **члена-корреспондента РАН Геннадия Алексеевича Михайлова**.

1956 -1965 – ВНИИТФ.

Outline

- Financial Mathematics. Coefficient Inverse problem for Black-Scholes equation

- Medicine. Coefficient inverse problems of pharmacodynamics

Coefficient inverse problems for parabolic equations

$$q(t)u_t = L\big(r(s), \sigma(s)\big)u, \qquad s \in \Omega, t \in (0, T)$$

$$u(s, 0) = u_0(s),$$

and boundary conditions. L is elliptic operator.

Inverse problem: it is required to recover functions $q(t), r(s)$ and $\sigma(s)$ by known additional information

- Nonlocal data: $\int_\Omega \boldsymbol{\chi}(s)u(s,t)dt = \boldsymbol{f_1}(t),$ $\quad \int_0^T \boldsymbol{\mu}(t)u(x,t)dt = \boldsymbol{f_2}(x)$

- Data on the given curve: $u(\boldsymbol{\varphi}(t), t) = \boldsymbol{f_3}(t).$

Coefficient inverse problems for financial mathematics.

Collaboration work with
Victor Isakov, Sergey Kabanikhin, Alexander Shananin and Shuhua Zhang

Financial mathematics is a relatively young discipline that has developed rapidly in recent decades.

This development was due to the formation of powerful financial markets with a variety of instruments for redistributing risks between market participants.

The changes that occurred in the oil market in connection with the creation of a cartel of OPEC oil-producing countries required the developed oil-exporting countries to adapt to the new conditions in the world economy.

The stock market can be studied separately from the real sector of the economy.

The activity of market participants is the management of risks associated with the uncertainty of the dynamics of financial instruments.

Introduction

Secondary financial instruments are used to manage risks.

The theory of pricing in the market of secondary financial instruments was investigated in (F. Black and M. Scholes, 1973; R. Merton, 1973; J. C. Cox, S. A. Ross, and M. Rubinstein, 1979).

The modern theory of arbitration, which is based on the modeling of pricing
in the market of secondary financial instruments, can be found in (G. Felmer and A. Shid, 2010, T. Bjork, 2010).

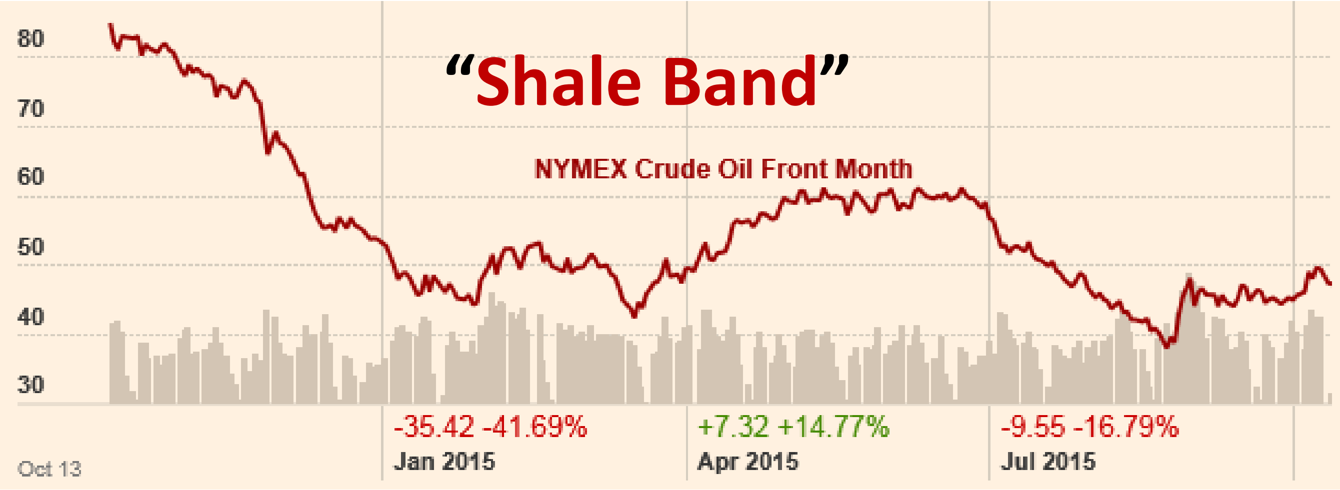
Important indicators of the stock market are aggregated indices,
such as the S & P 500 or SSE Composite.

Traditionally, the dynamics of the price of the stock market index is modeled as a geometric Brownian motion and is described by stochastic differential equation.

The form of this stochastic differential equation depends on the real sector of the economy.

Since the characteristic times of changes in the real sector of the economy are much greater than the characteristic times of changes in the conjuncture in the stock market, these equations can be considered quasi-stationary.

In this talk, we study the inverse problem of recovering functions which determine the form of a stochastic differential equation that simulates the dynamics of the stock market index from data of a European sales option.



Petromatrix, a consulting firm, has coined the phrase **“shale band”** for the price range **between $45 and $65**: below that range, American production falls sharply; above it, it surges. If so, there should be a tendency for prices to stay within that range. … Unless some large-scale conflict erupts that takes out some of the world’s biggest oilfields, the oil industry may be heading for a new normal in which the price of crude oscillates in the mid-double digits.

After OPEC: American shale firms are now the oil market’s swing producers // *The Economist* May 16th 2015

Price corridor ~ 40-60 $/bar.?

“We likely won’t see $100 a barrel again. The industry has been recalibrated to a lower price level.

A great struggle is unfolding in the world oil market. On one side are forces pushing to rebalance supply and demand; on the other, those pulling to recalibrate the business so that it operates at lower cost. That tension explains why the price keeps jumping toward $60 a barrel and then falling back near $40.”

Daniel Yergin, “The Struggle Behind Oil’s Ups and Downs”

***The Wall Street Journal* May 16, 2017 7:20 p.m. ET**

https://www.wsj.com/articles/the-struggle-behind-oils-ups-and-downs-1494976842

$$\frac{\partial u(S,t)}{\partial t} = rS\frac{\partial u(S,t)}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 u(S,t)}{\partial S^2} - ru(S,t),$$

$$u(S,\tau) = f(S).$$

u - the pay-off function

S - the stock price

r - the risk-free interest rate

σ - the volatility

t - the time, $t \in (\tau, T)$

T – the time of maturity

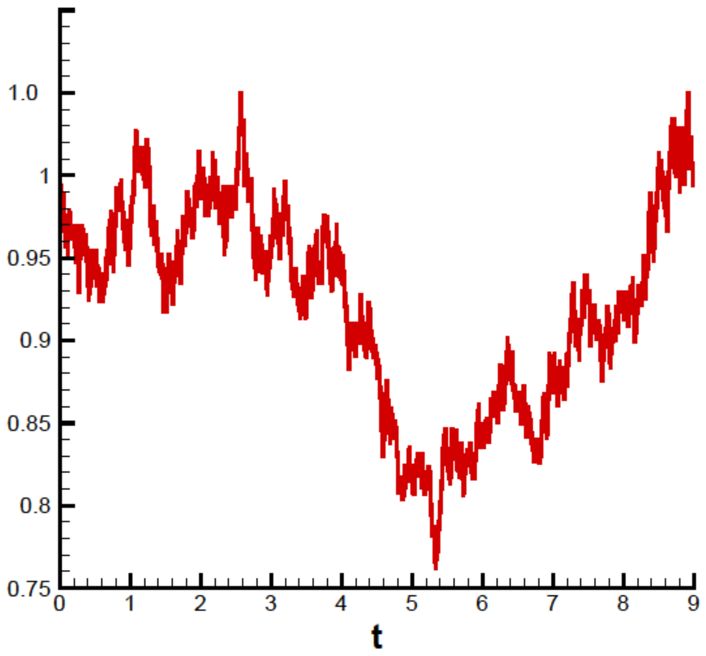
European call $\quad f(S) = \max(S - K, 0)$

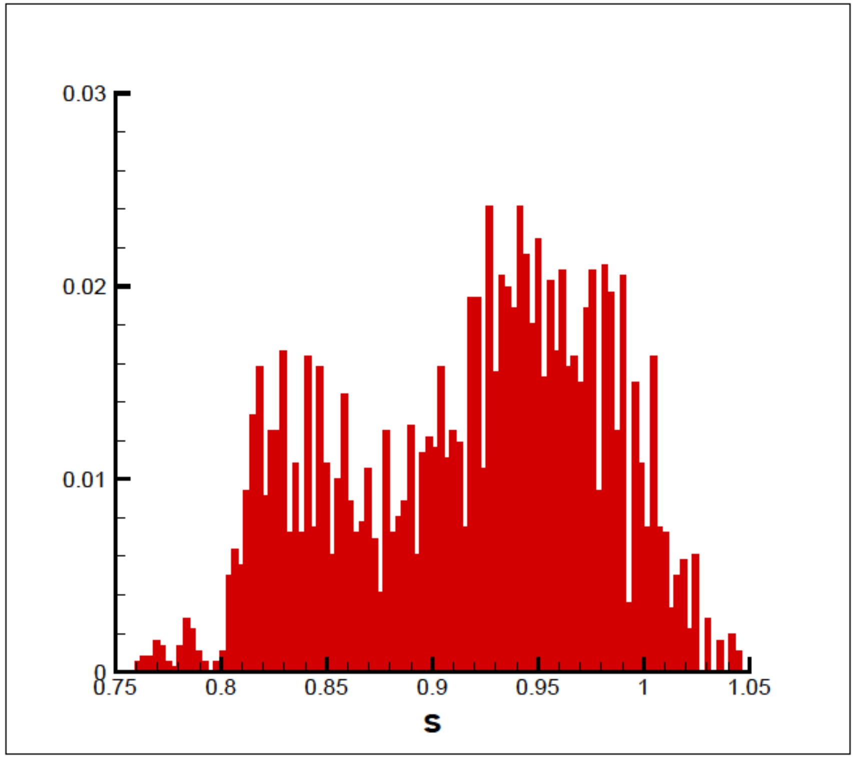
European put $\quad f(S) = \max(K - S, 0)$

Inverse problem data

$$u(\varphi(t), \boldsymbol{T-t}) = g(t)$$

Here $s = \varphi(t)$ is some curve.





Introduce $$\Phi(d) = \int_{-\infty}^{d} exp\left\{-\frac{y^2}{2}\right\} dy$$

$$d_1 = \frac{\ln\frac{S}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \qquad d_2 = \frac{\ln\frac{S}{K} + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

European call $$u_C(S,t) = S\Phi(d_1) - K\, exp\{-r(T-t)\}\Phi(d_2)$$

European put $$u_P(S,t) = K\, exp\{-r(T-t)\}\Phi(-d_2) - S\Phi(-d_1)$$

Black F and Scholes M. The pricing of options and corporate liabilities. J Pol Econ, 1973; 81: 637–659.
Wilmott, P, Howison, S, Dewynne, J., The mathematics of financial derivatives, Cambridge, 1997.
P. Wilmott, The mathematics of financial derivatives: a student introduction, Cambridge University Press, 1995.
E. G. Haug, The complete guide to option pricing formulas, vol. 2, McGraw-Hill New York, 1998.
Y.-K. Kwok, Mathematical models of financial derivatives, Springer, 2008.

S.R. Dunbar. Department of Mathematics. University of Nebraska-Lincoln. Stochastic Processes and Advanced Mathematical Finance: Solution of the Black-Scholes Equation (*http://www.math.unl.edu/~sdunbar1/MathematicalFinance/Lessons/BlackScholes/Solution/solution.pdf*)

- Authors used the following formula

$$u_C(S,t) = S\Phi(d_1) - K\ exp\{-r(T-t)\}\Phi(d_2)$$

$$u_P(S,t) = K\ exp\{-r(T-t)\}\Phi(-d_2) - S\Phi(-d_1)$$

- Which was established in

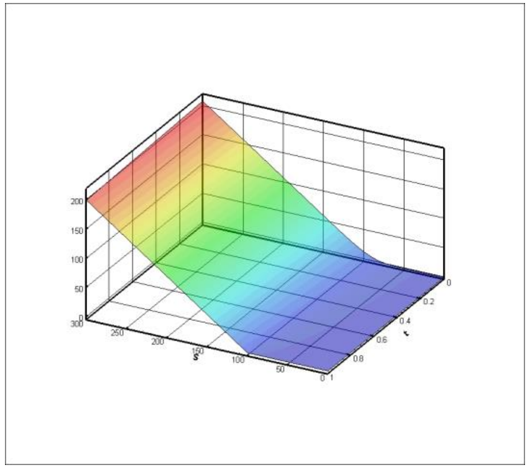
P. Wilmott, The mathematics of financial derivatives: a student introduction, Cambridge University Press, 1995.

E. G. Haug, The complete guide to option pricing formulas, vol. 2, McGraw-Hill New York, 1998.

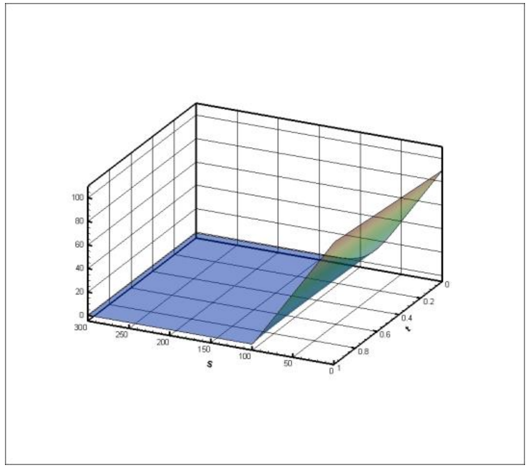
Y.-K. Kwok, Mathematical models of financial derivatives, Springer, 2008.

European call

European put



u(S,t)



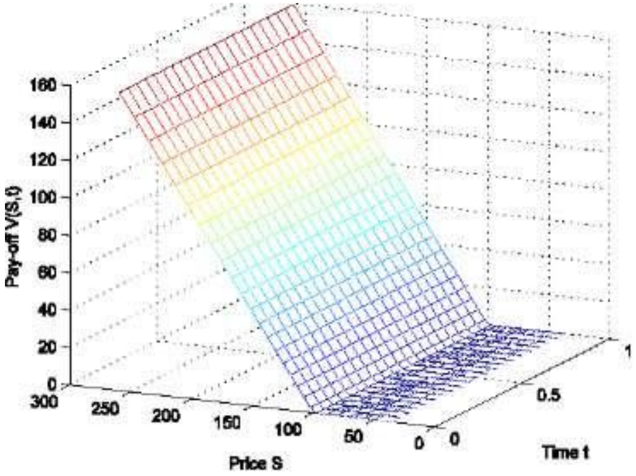
T = 1 year,

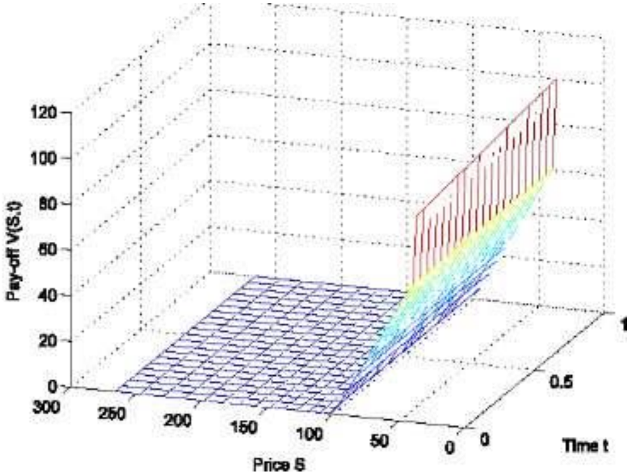
$S \in (0,300)$,

K = 100,

r = 0.1,

$\sigma = 0.2$





K.S. Uddin, N.-A-A. Siddiki, A. Hossain. Numerical Solution of a Linear Black-Scholes Model: A Comparative Overview Journal of of Statistics and Mathematical Sciences, Vol. 1(1) (2015), 1-7.

Inverse Problem for Black-Scholes equation

$u_t = sr(s)\, u_s + \frac{1}{2} s^2\sigma^2(s)\, u_{ss} - r(s)u, t \in (0,T), s \in (0,L),$
$u(s,0) = f(s) = \max(K - s, 0)\,,$

$u(0,t) = K,\ \ u(L,t)$ =0.

Here L=*10K*.

Inverse problem data

$u(\varphi(t), \boldsymbol{T} - \boldsymbol{t}) = g(t).$

Here $s = \varphi(t)$ is curve, where we know additional information.

Let us reduce inverse problem to the optimization problem $\boldsymbol{q} = (r, \sigma)$

$$J(\boldsymbol{q}) = \int_0^T (u(\varphi(t), \boldsymbol{T} - \boldsymbol{t}) - g(t))^2\ dt \rightarrow \min_{\boldsymbol{q}}$$

Gradient optimization method $\boldsymbol{q}^{\boldsymbol{n}+\mathbf{1}} = \boldsymbol{q}^{\boldsymbol{n}} - \boldsymbol{\alpha J}'(\boldsymbol{q}^{\boldsymbol{n}})$

Direct problem $t \in (0,T)$, $s \in (0,L)$, $L = 10K$:

$$u_t = sr(s)\,u_s + \frac{1}{2}s^2\sigma^2(s)\,u_{ss} - r(s)u,$$
$$u(s,0) = f(s) = \max(K-s,0)\,,$$
$$u(0,t) = K,\quad u(L,t) = 0.$$

Adjoint problem $t \in (0,T)$, $s \in (0,L)$:

$$\psi_t = (sr(s)\psi)_s - \frac{1}{2}(s^2\sigma^2(s)\psi)_{ss} + r(s)\psi + 2\delta(s-\varphi(t))[u(s,\boldsymbol{T}-\boldsymbol{t}) - g(t)],$$
$$\psi(s,T) = 0,$$
$$\psi(0,t) = \psi(L,t) = 0.$$

Gradient of the functional $J'(\boldsymbol{q}) = \left(\frac{s^2}{2}\int_0^T u_{ss}(s,t)\psi(s,t)dt\,,\ \int_0^T (su_s(s,t) - u(s,t))\psi(s,t)dt\right)$

Curve where data is measured

Curve $s = \varphi(t)$ where we collect the data is given by the following formulas. Let us define the probability

$$p(s_k) = \frac{e^{\sigma(s_k)\sqrt{h_t}} - 1 - r(s_k)h_t}{e^{\sigma(s_k)\sqrt{h_t}} - e^{-\sigma(s_k)\sqrt{h_t}}}$$

and the quantities

$$d(s_k) = e^{-\sigma(s_k)\sqrt{h_t}}, \qquad u(s_k) = e^{\sigma(s_k)\sqrt{h_t}}.$$

The algorithm is following:

(i) $s_0 = 1$;

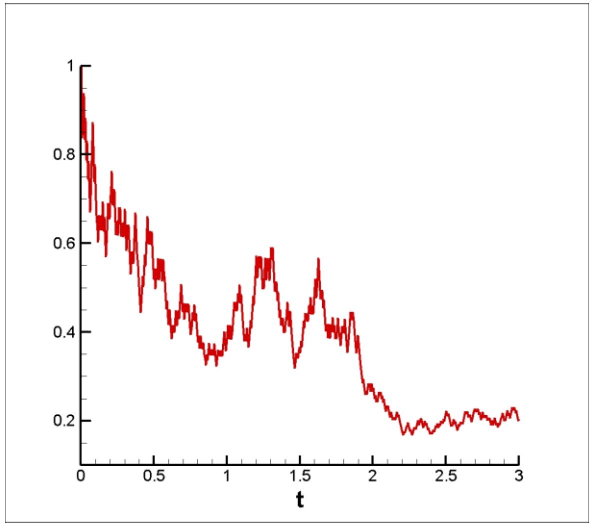
(ii)

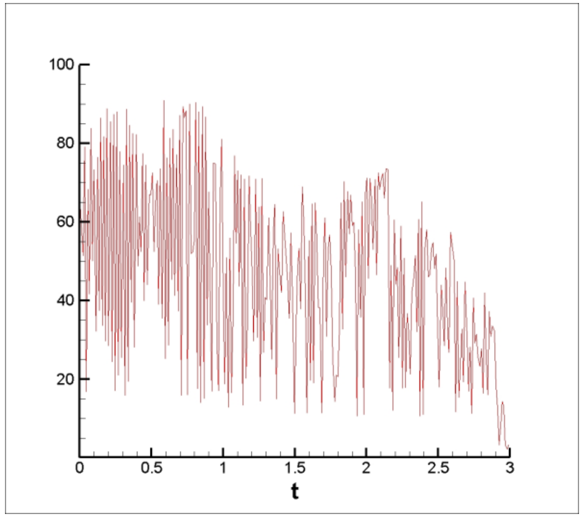
$$s_{k+1} = \begin{cases} d(s_k)s_k, & \text{with probability } p(s_k) \\ u(s_k)s_k, & \text{with probability } (1 - p(s_k)) \end{cases}, \qquad k = 1, 2, \ldots$$

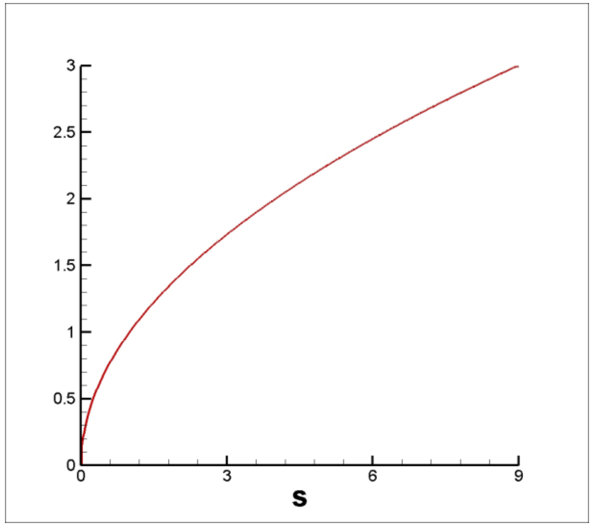
Inverse Problem data

T = 3 monthes, K = 0.9, L=90

r(s) ≡ const = 0.02







Curve $\varphi(t)$

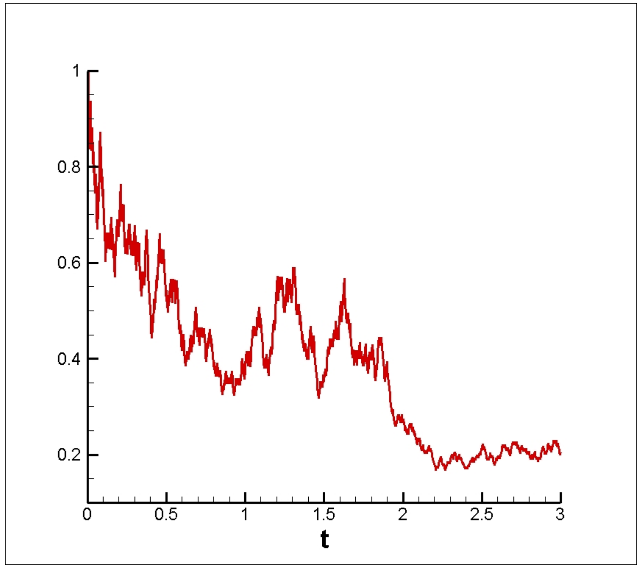
Inverse Problem data
$g(t) = u(\varphi(t), \boldsymbol{T - t})$

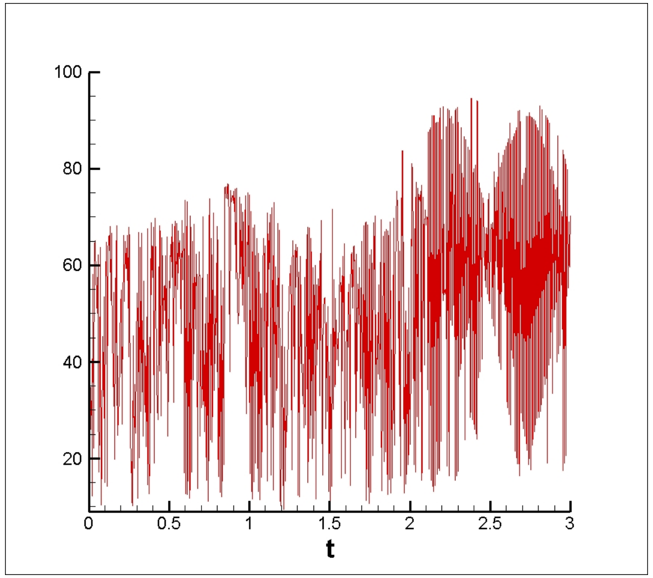
$\sigma(s) = \sqrt{s}$

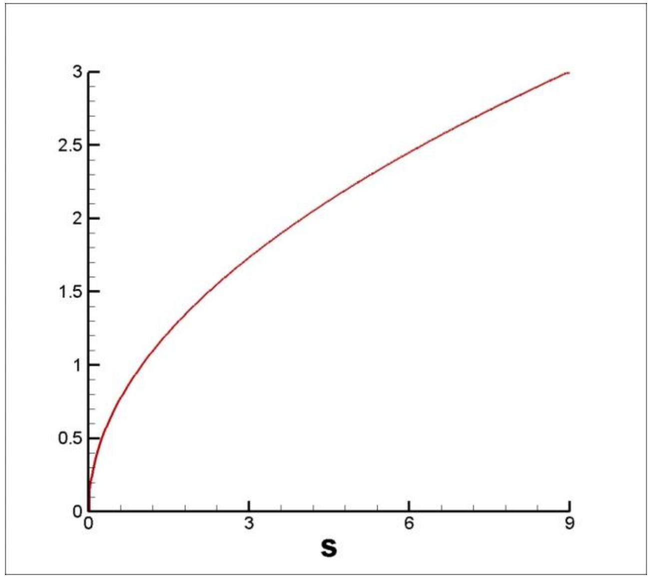
Inverse Problem data

T = 3 months, K = 0.9, L=90

r(s) ≡ const = 0.02







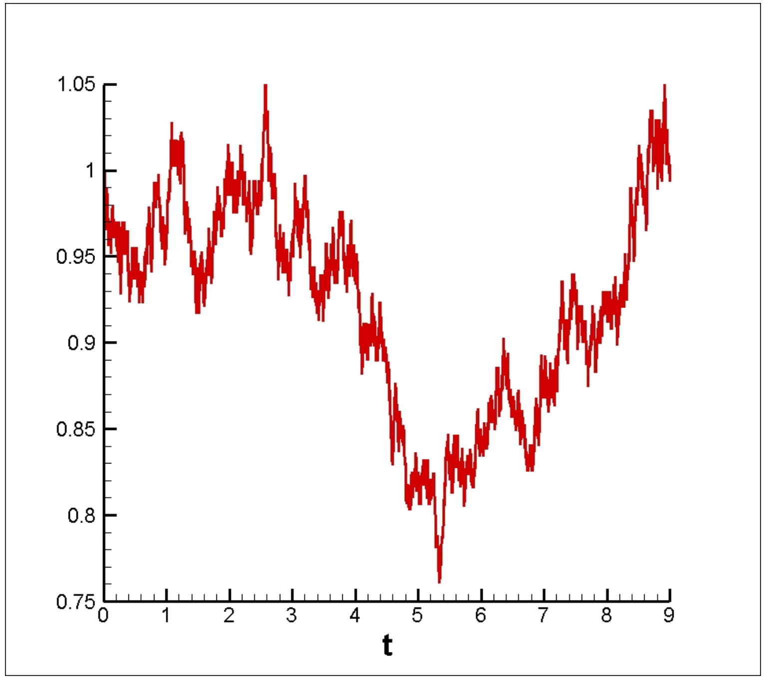
Curve $\varphi(t)$

Inverse Problem data
$g(t) = u(\varphi(t), \boldsymbol{T - t})$

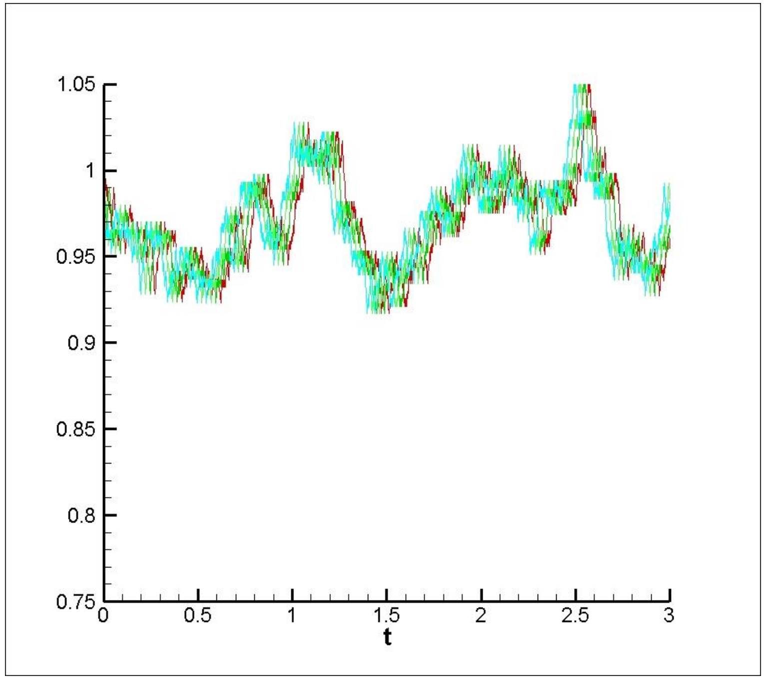
$\sigma(s) = \sqrt{s}$

Inverse Problem data

T, monthes	K	L	$r(s)$	$\sigma(s)$	τ, days	N_t	h_t	N_s	h_s	Iterations
3	0.9	9	0.05	$0.1\sqrt{s}$	0, 10, 20, 30	1200	0.025	900	0.01	5000



Curve $\varphi(t)$

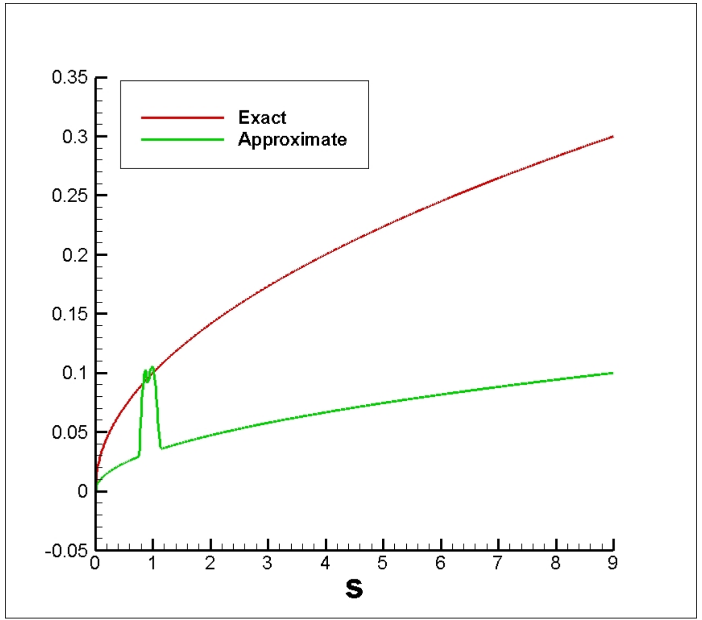


Curves $\varphi_0(t)$, $\varphi_{10}(t)$, $\varphi_{20}(t)$, $\varphi_{30}(t)$

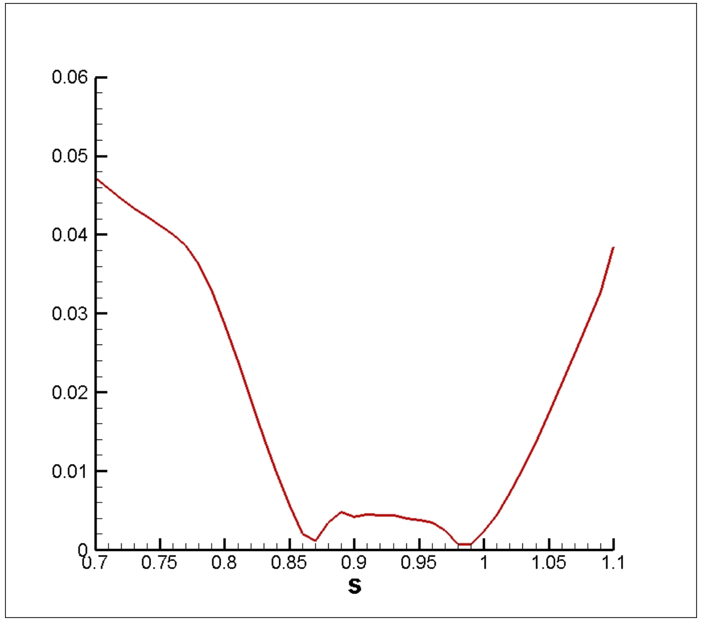
$$\varphi_0(t) = \varphi(t), t \in (0, T)$$
$$\varphi_\tau(t) = \varphi\left(t + \frac{\tau}{30}\right), t \in (0, T)$$

Inverse Problem solution. Recovering $\sigma(s)$ by $g_0(t)$.
Method of steepest descent

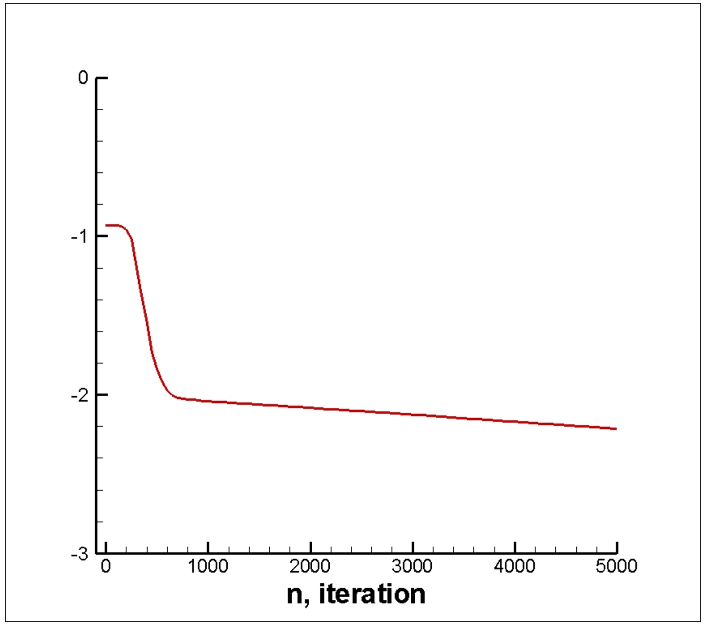
T, months	K	L	$r(s)$	$\sigma(s)$	N_t	h_t	N_s	h_s	Iterations
3	0.9	9	0.05	$0.1\sqrt{s}$	1200	0.025	900	0.01	5000



Inverse Problem solution



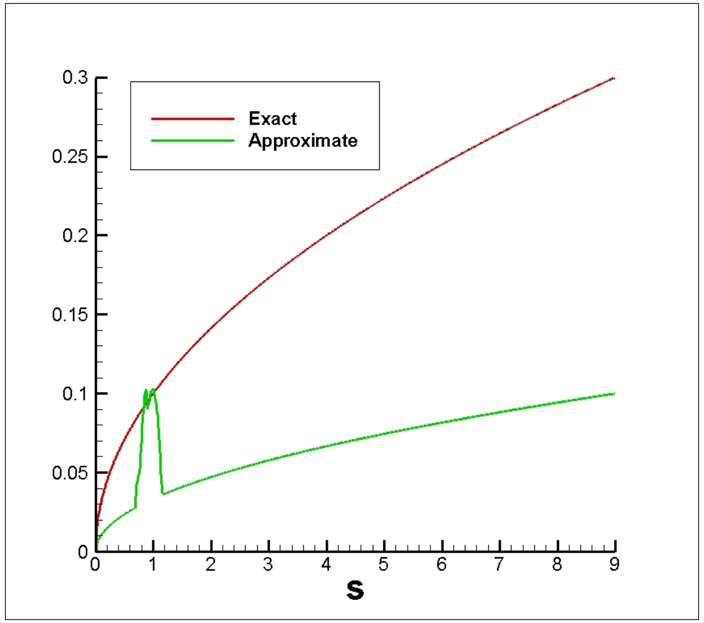
Difference



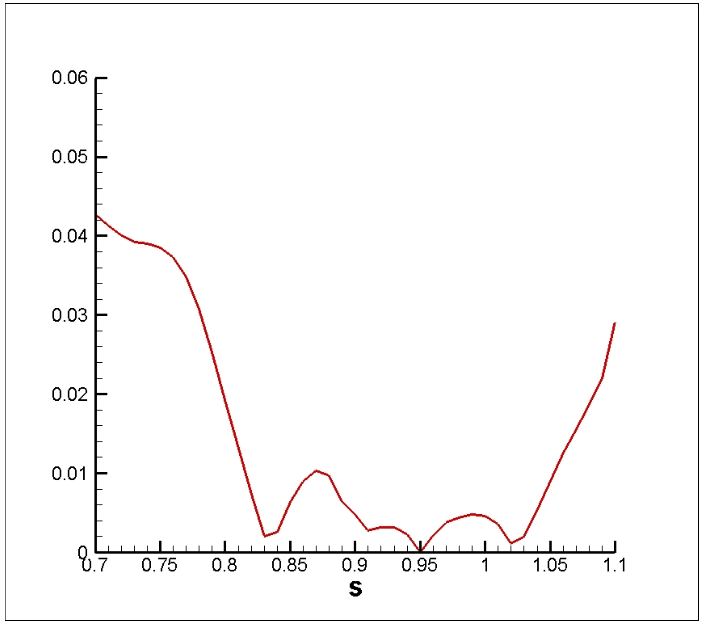
$$\log\left(u^{(n)}(\varphi_0(t), T-t) - g_0(t)\right)^2$$

Inverse Problem solution. Recovering $\sigma(s)$ by $g_0(t)$, $g_{10}(t)$, $g_{20}(t)$, $g_{30}(t)$. Method of steepest descent

T, months	K	L	$r(s)$	$\sigma(s)$	τ, days	N_t	h_t	N_s	h_s	Iterations
3	0.9	9	0.05	$0.1\sqrt{s}$	0,10,20,30	1200	0.025	900	0.01	5000



Inverse Problem solution



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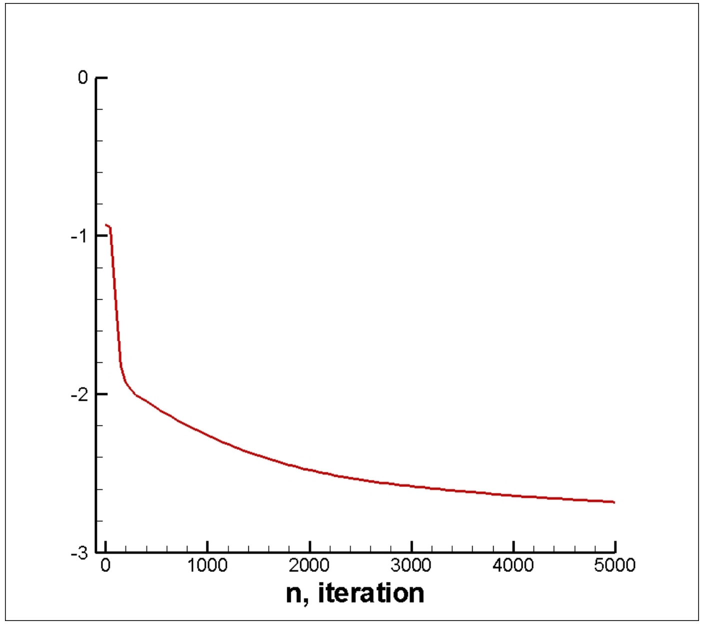
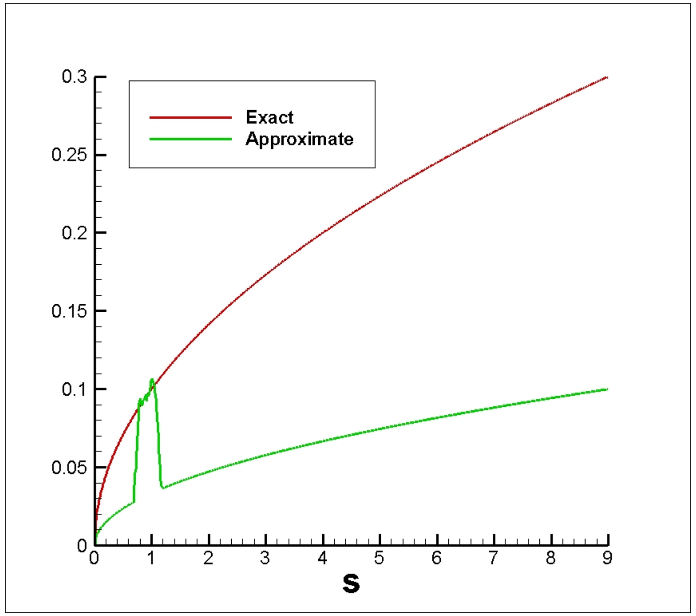


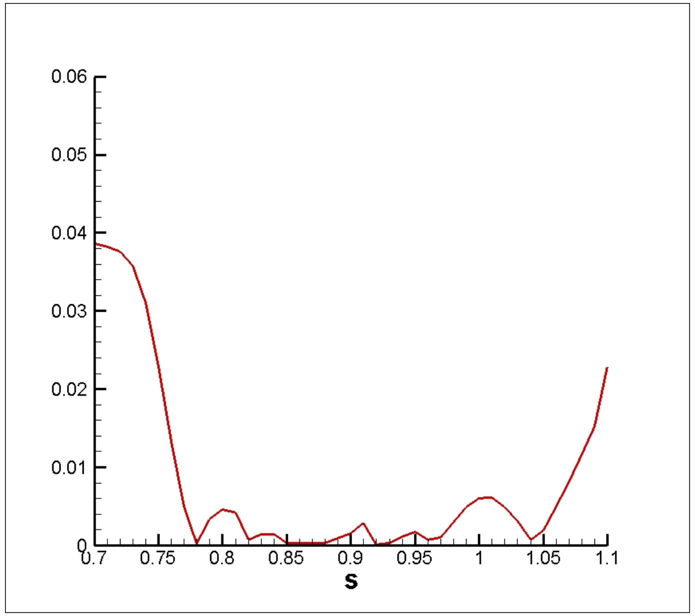
$$\log\Big(\big(u^{(n)}(\varphi_0(t),T-t)-g_0(t)\big)^2+\big(u^{(n)}(\varphi_{10}(t),T-t)-g_{10}(t)\big)^2 + \big(u^{(n)}(\varphi_{20}(t),T-t)-g_{20}(t)\big)^2+\big(u^{(n)}(\varphi_{30}(t),T-t)-g_{30}(t)\big)^2\Big)$$

Inverse Problem solution. Recovering $\sigma(s)$ by $g_0(t)$, $g_{10}(t)$, $g_{20}(t)$,…, $g_{110}(t)$. Method of steepest descent

T, months	K	L	$r(s)$	$\sigma(s)$	τ, days	N_t	h_t	N_s	h_s	Iterations
3	0.9	9	0.05	$0.1\sqrt{s}$	0,10,20,…,110	1200	0.025	900	0.01	5000



Inverse Problem solution



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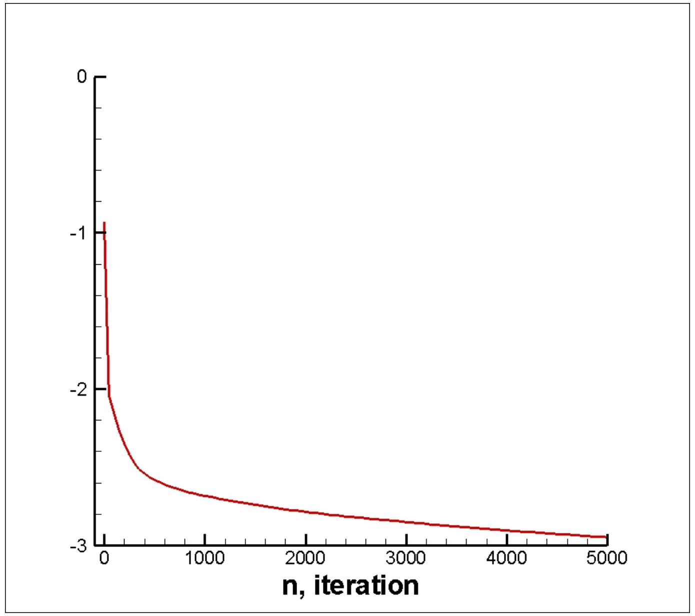
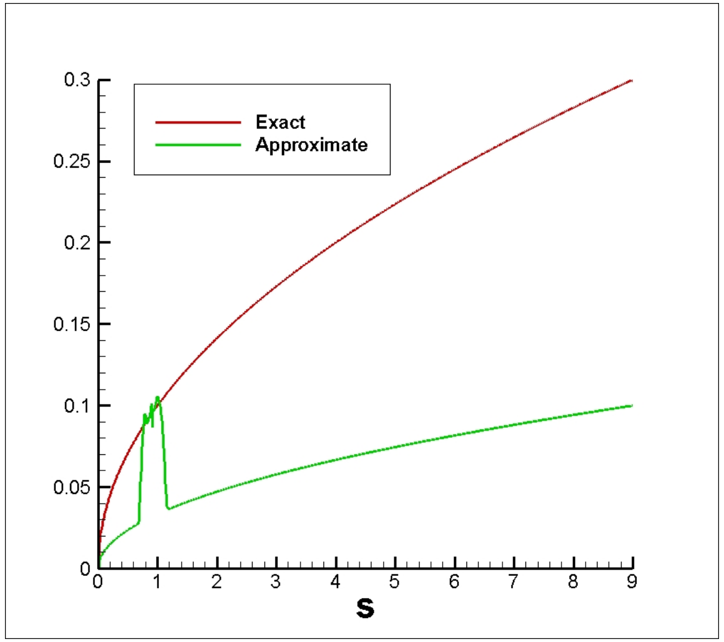


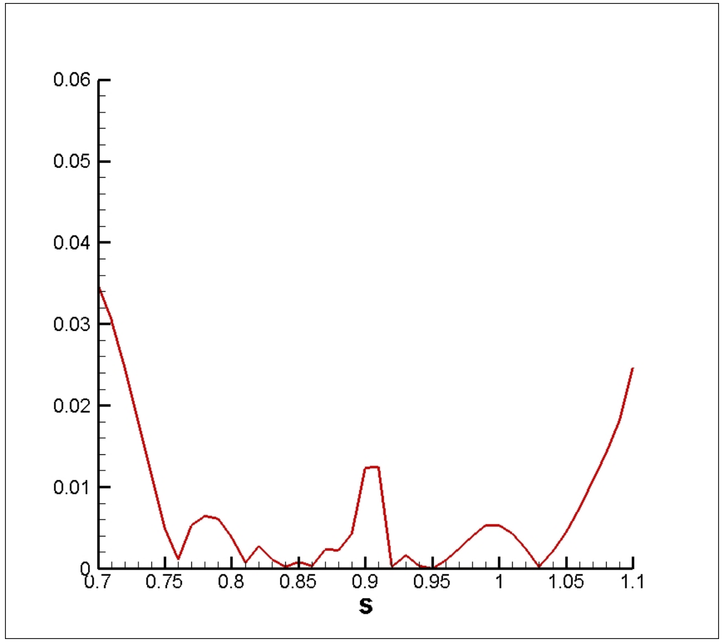
$$\log\Big(\big(u^{(n)}(\varphi_0(t),T-t)-g_0(t)\big)^2+\big(u^{(n)}(\varphi_{10}(t),T-t)-g_{10}(t)\big)^2 + \cdots + \big(u^{(n)}(\varphi_{110}(t),T-t)-g_{110}(t)\big)^2\Big)$$

Inverse Problem solution. Recovering $\sigma(s)$ by $g_0(t)$, $g_{20}(t)$, $g_{40}(t)$,…, $g_{160}(t)$. Method of steepest descent

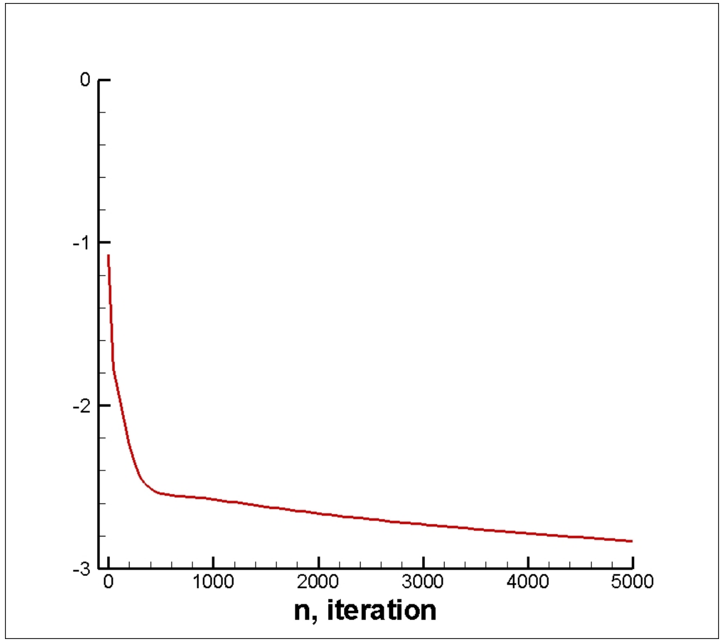
T, months	K	L	$r(s)$	$\sigma(s)$	τ, days	N_t	h_t	N_s	h_s	Iterations
3	0.9	9	0.05	$0.1\sqrt{s}$	0,20,…,160	1200	0.025	900	0.01	5000



Inverse Problem solution



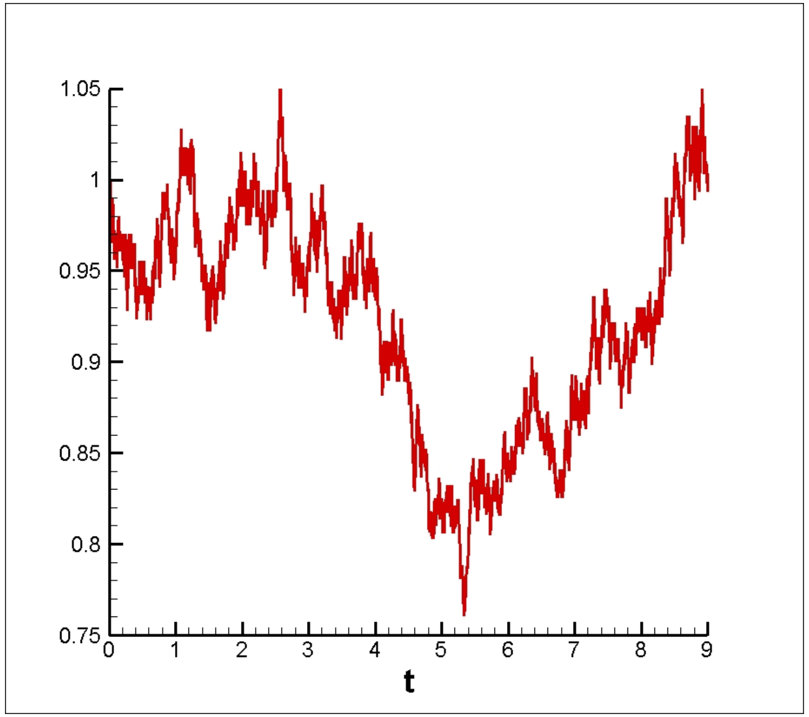
Difference



$$\log\Big(\big(u^{(n)}(\varphi_0(t),T-t)-g_0(t)\big)^2+\big(u^{(n)}(\varphi_{20}(t),T-t)-g_{20}(t)\big)^2 + \cdots + \big(u^{(n)}(\varphi_{140}(t),T-t)-g_{160}(t)\big)^2\Big)$$

Inverse Problem data

T, monthes	K	L	$r(s)$	$\sigma(s)$	τ, days	N_t	h_t	N_s	h_s	Iterations
3	0.9	9	0.05	0.02	0, 10, 20, 30	1200	0.025	900	0.01	5000



Curve $\varphi(t)$

Inverse Problem solution. Recovering $\sigma(s)$ by $g_0(t)$. Method of steepest descent

T, months	K	L	$r(s)$	$\sigma(s)$	N_t	h_t	N_s	h_s	Iterations
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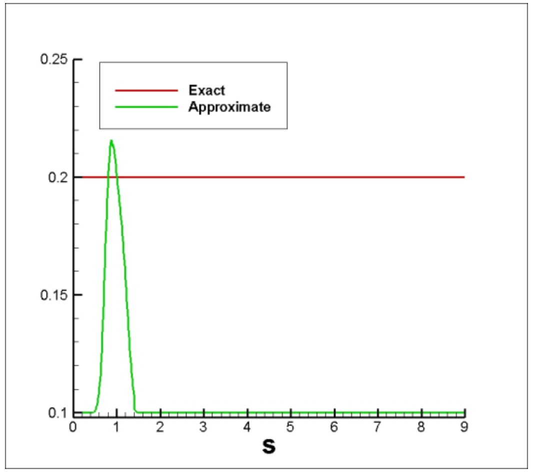
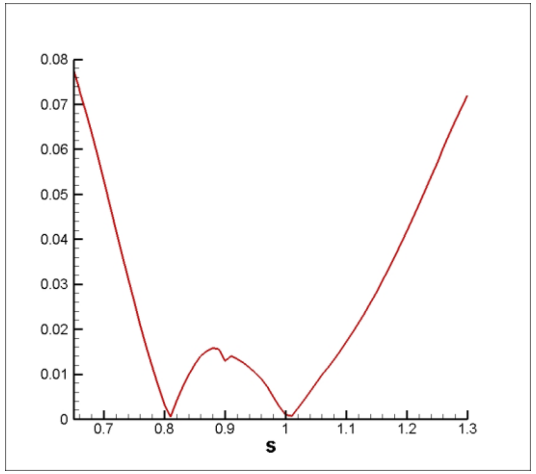
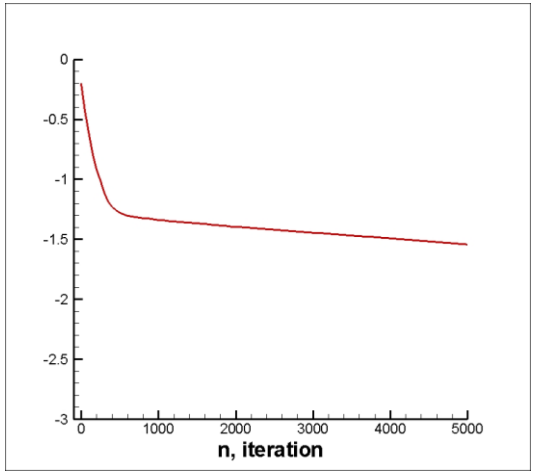


IP solution



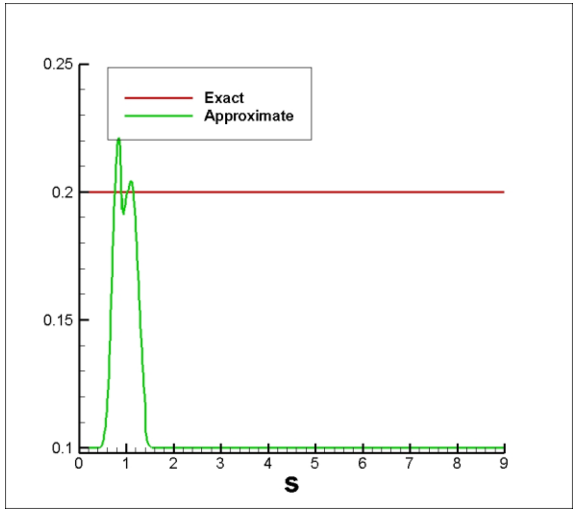
Difference



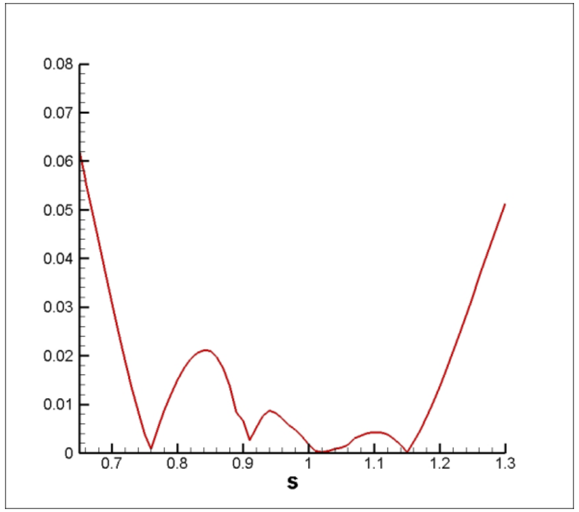
$$\log\left(u^{(n)}(\varphi_0(t), T-t) - g_0(t)\right)^2$$

Inverse Problem solution. Recovering $\sigma(s)$ by $g_0(t)$, $g_{10}(t)$, $g_{20}(t)$, $g_{30}(t)$. Method of steepest descent

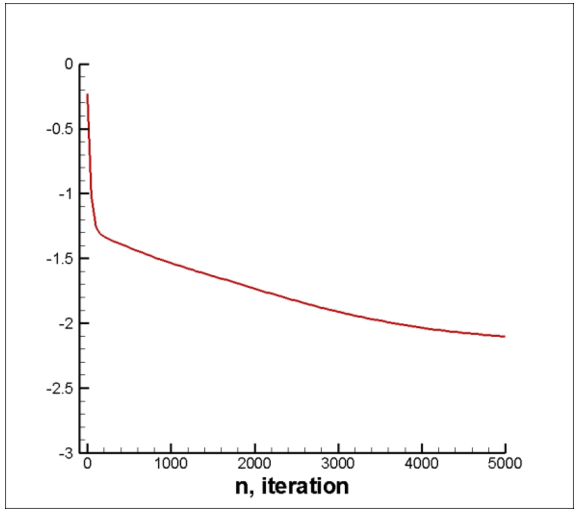
T, months	K	L	$r(s)$	$\sigma(s)$	τ, days	N_t	h_t	N_s	h_s	Iterations
3	0.9	9	0.05	0.02	0,10,20,30	1200	0.025	900	0.01	5000



IP solution



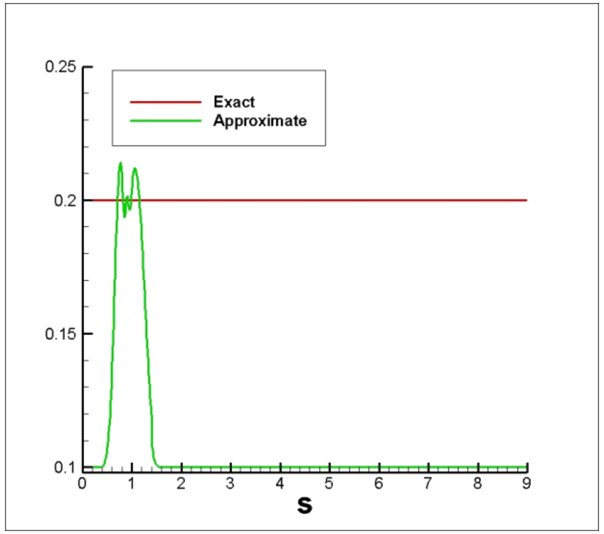
Difference



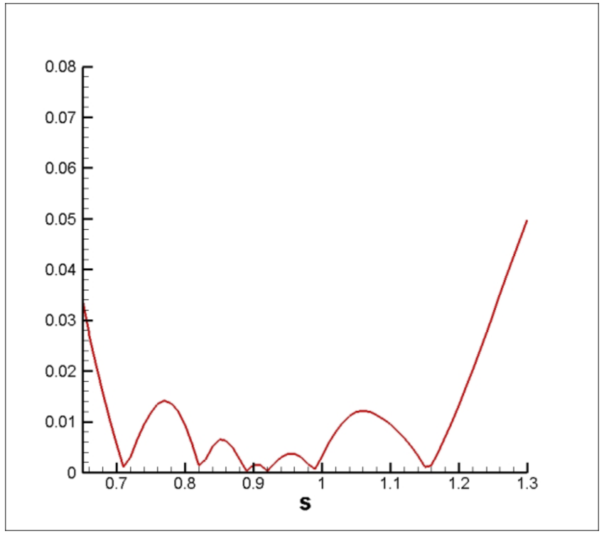
$$\log\Big(\big(u^{(n)}(\varphi_0(t),T-t)-g_0(t)\big)^2+\big(u^{(n)}(\varphi_{10}(t),T-t)-g_{10}(t)\big)^2 + \big(u^{(n)}(\varphi_{20}(t),T-t)-g_{20}(t)\big)^2+\big(u^{(n)}(\varphi_{30}(t),T-t)-g_{30}(t)\big)^2\Big)$$

Inverse Problem solution. Recovering $\sigma(s)$ by $g_0(t)$, $g_{10}(t)$, $g_{20}(t)$,…, $g_{160}(t)$. Method of steepest descent

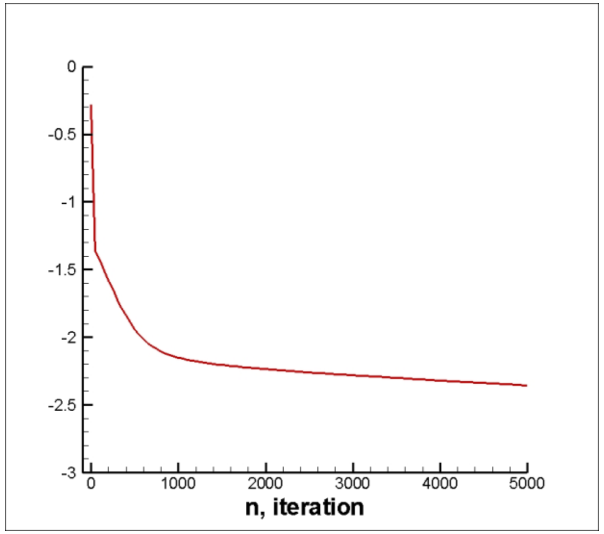
T, months	K	L	$r(s)$	$\sigma(s)$	τ, days	N_t	h_t	N_s	h_s	Iterations
3	0.9	9	0.05	0.02	0,10,20,…, 160	1200	0.025	900	0.01	5000



IP solution

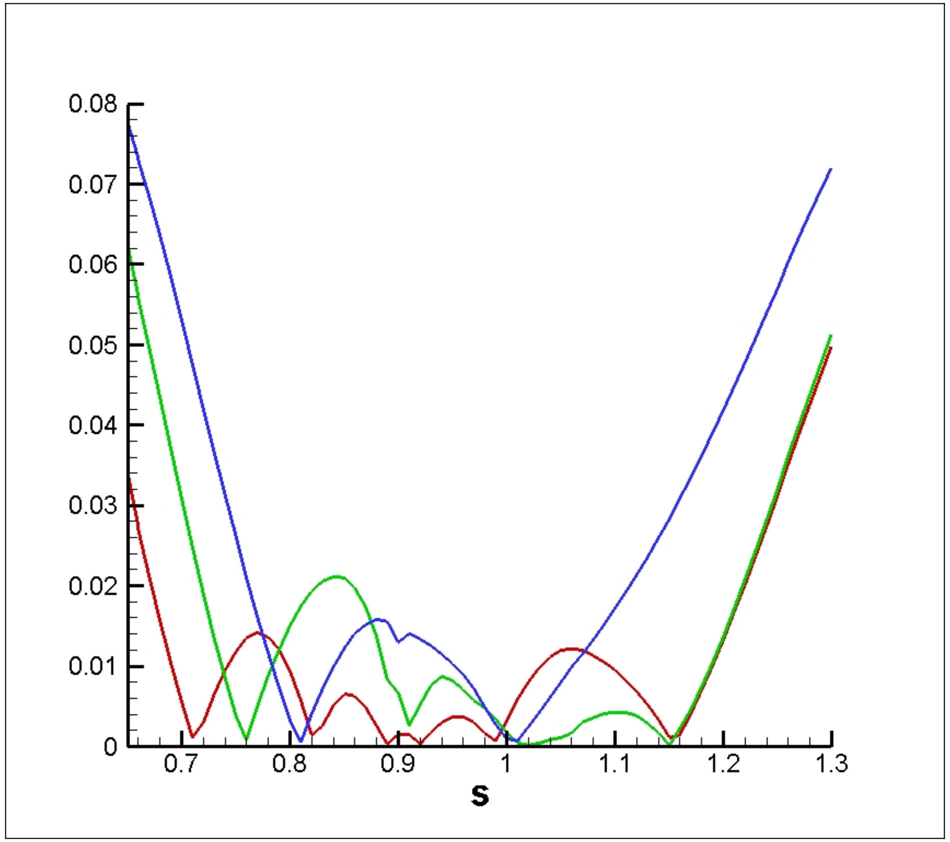


Difference



$$\log\Big(\big(u^{(n)}(\varphi_0(t),T-t)-g_0(t)\big)^2+\big(u^{(n)}(\varphi_{10}(t),T-t)-g_{10}(t)\big)^2 + \big(u^{(n)}(\varphi_{20}(t),T-t)-g_{20}(t)\big)^2+\cdots+\big(u^{(n)}(\varphi_{160}(t),T-t)-g_{160}(t)\big)^2\Big)$$

T, months	K	L	$r(s)$	$\sigma(s)$	τ, days	N_t	h_t	N_s	h_s	Iterations
3	0.9	9	0.05	0.02	0,10,20,30	1200	0.025	900	0.01	5000



Conclusion

1. The more data we apply, the more stable and close to the exact solution we obtain. The functional decrease faster with additional measurements for the same number of iterations.
2. The size of the line segment on which the functions restored depends on the the values of the function changing on the curve, where we measure the inverse problem data. For 3 additional data (we suppose that new option is issued each 10 days) we see that size of the line segment increase from [0.9, 1.05] for 1 data to [0.85, 1.1] for 3 additional measurements, to [0.85, 1.1] for 11 additional measurements [10,20,…,110 days]. If we suppose that new option is issued each 20 days and obtain that for 7 additional measurements [20,40,…,140 days] increase to [0.75, 1.1]. The interval of recovering unknown coefficient are coincide with intervals of variation of the function $\varphi(t)$ (functions $\varphi_\tau(t)$).
3. If we are going to recover the coefficient on the large the line segment as possible we have to set curves for data measurement which should change over a sufficiently large range of its values.

Coefficient inverse problems for pharmacodynamics
Collaboration work with
Sergey Kabanikhin, Ruslan Zhalnin, Yurii Derugin, Andrey Kozelkov

Inverse problems of medicine:
diffusion of drugs, absorption of the drug through the patient's skin, etc.

The skin belongs to the natural barriers that prevent the entry of xenobiotics into the body.

Despite the successes achieved in connection with the development of transdermal therapeutic systems and the study of skin permeability, it is impossible to say with certainty which structural elements of the skin are barriers for various drugs.

Inverse Problem Formulation

Let $u(x,t)$ be a solution of the direct problem ($\sigma(x) > 0$, $q(t) > 0$)

$$q(t)u_t = \operatorname{div}(\sigma(x)\nabla u), \qquad x \in \Omega, \quad t \in (0,T), \tag{12}$$

$$u(x,0) = u_0(x), \qquad x \in \Omega, \tag{13}$$

$$u|_{\partial\Omega} = 0, \qquad t \in (0,T), \tag{14}$$

and we know the nonlocal inverse problem data

$$\int\limits_0^L u(x,t)\mathrm{d}x = f(t), \qquad t \in (0,T). \tag{15}$$

In inverse problem (12)—(15) it is required to find the coefficient $q(t)$.

Cannon, Yin, 1990; Cannon, Rundell, 1991; Bouziani, 1996; Ivanchov, 1998; Dehghan, 2005; Hao, Thanh, Lesnic, Ivanchov, 2014; Hao, Duc, 2015; Hussein, Lesnic, Ismailov, 2016.

Finite-Difference Scheme Inversion

Let us describe the FDSI proposed in [Vabishchevich, Klibanov, 2016] to the inverse problem (12)—(15). Let N_t be a mesh size $h_t = T/N_t$. Let us denote $u^m(x) = u(x, h_t m)$, $q^m = q(h_t m)$. For the direct problem solution (12)—(14) we consider the discrete problem

$$q^{m+1}\frac{u^{m+1}-u^m}{h_t} = \operatorname{div}\left(\sigma(x)\nabla u^{m+1}\right), \quad x \in \Omega,\ m = 0, \ldots, N_t - 1, \tag{16}$$

$$u^0(x) = u_0(x), \qquad x \in \Omega, \tag{17}$$

$$u^m|_{\partial\Omega} = 0. \tag{18}$$

Inverse problem data we rewrite as follows

$$\int\limits_0^L u^m(x)\mathrm{d}x = f^m, \qquad m = 0, \ldots, N_t. \tag{19}$$

The FDSI is based on the linearization $q(t)u(t)$ в точке $t^{m+1/2}$

$$q^{m+1/2}u^{m+1/2} = \frac{1}{2}\Big[q^{m+1}u^{m} + q^{m}u^{m+1}\Big] + O(h_t^2).$$

Then (16) we can rewrite

$$q^{m+1}\frac{u^{m+1}-u^{m}}{h_t} = \frac{q^{m+1}}{2}\Big[\frac{\operatorname{div}\left(\sigma(x)\nabla u^{m+1}\right)}{q^{m}} + \frac{\operatorname{div}\left(\sigma(x)\nabla u^{m}\right)}{q^{m+1}}\Big].$$

Let us suppose that the solution $q(0)$ is known.

$$q^{m}y^{m+1} - \frac{h_t}{2}\operatorname{div}\left(\sigma(x)\nabla y^{m+1}\right) = q^{m}u^{m}, \qquad y^{m+1}|_{\partial\Omega} = 0,$$

$$q^{m}w^{m+1} - \frac{h_t}{2}\operatorname{div}\left(\sigma(x)\nabla w^{m+1}\right) = \frac{h_t}{2}q^{m}\operatorname{div}\left(k(x)\nabla u^{m}\right), \qquad w^{m+1}|_{\partial\Omega} = 0.$$

$$q^{m+1} = \frac{\int\limits_0^L w^{m+1}(x)\mathrm{d}x}{f^{m+1} - \int\limits_0^L y^{m+1}(x)\mathrm{d}x}.$$

Optimization approach

Let us minimize the cost functional

$$J(q) = \int_0^T \left[\int_0^L u(x,t;q)\mathrm{d}x - f(t) \right] \mathrm{d}t \to \min_q .$$

by the gradient method

$$q^{(k+1)}(t) = q^{(k)}(t) - \alpha J'(q^{(k)}(t)).$$

Here J' is the gradient of the functional.

Gradient Method

1. Let $q^{(k)}(t)$ be known and let us determine the $q^{(k+1)}(t)$.
2. $$q^{(k)}(t)u_t^{(k)} = \operatorname{div}\left(\sigma(x)\nabla u^{(k)}\right), \qquad x \in \Omega, \quad t \in (0, T);$$
$$u^{(k)}(x,0) = u_0(x),\ u^{(k)}|_{\partial\Omega} = 0.$$
3. $$\left(q^{(k)}(t)\psi^{(k)}\right)_t = -\operatorname{div}\left(\sigma(x)\nabla\psi^{(k)}\right) - 2\left[\int_0^L u^{(k)}(x,t)\mathrm{d}x - f(t)\right],$$
$$\psi^{(k)}(x,T) = 0, \qquad x \in \Omega,\ \psi^{(k)}|_{\partial\Omega} = 0, \qquad t \in (0,T).$$
4. $$J'(q^{(k)})(t) = \int_0^L u_t^{(k)}(x,t)\psi^{(k)}(x,t)\mathrm{d}x.$$

Nonlocal Data given in discrete time

If we have inverse problem data for a discrete time

$$\int_0^L u(x, t_j)\mathrm{d}x = f(t_j), \qquad t = t_j,\, j = 1, 2, \ldots, K.$$

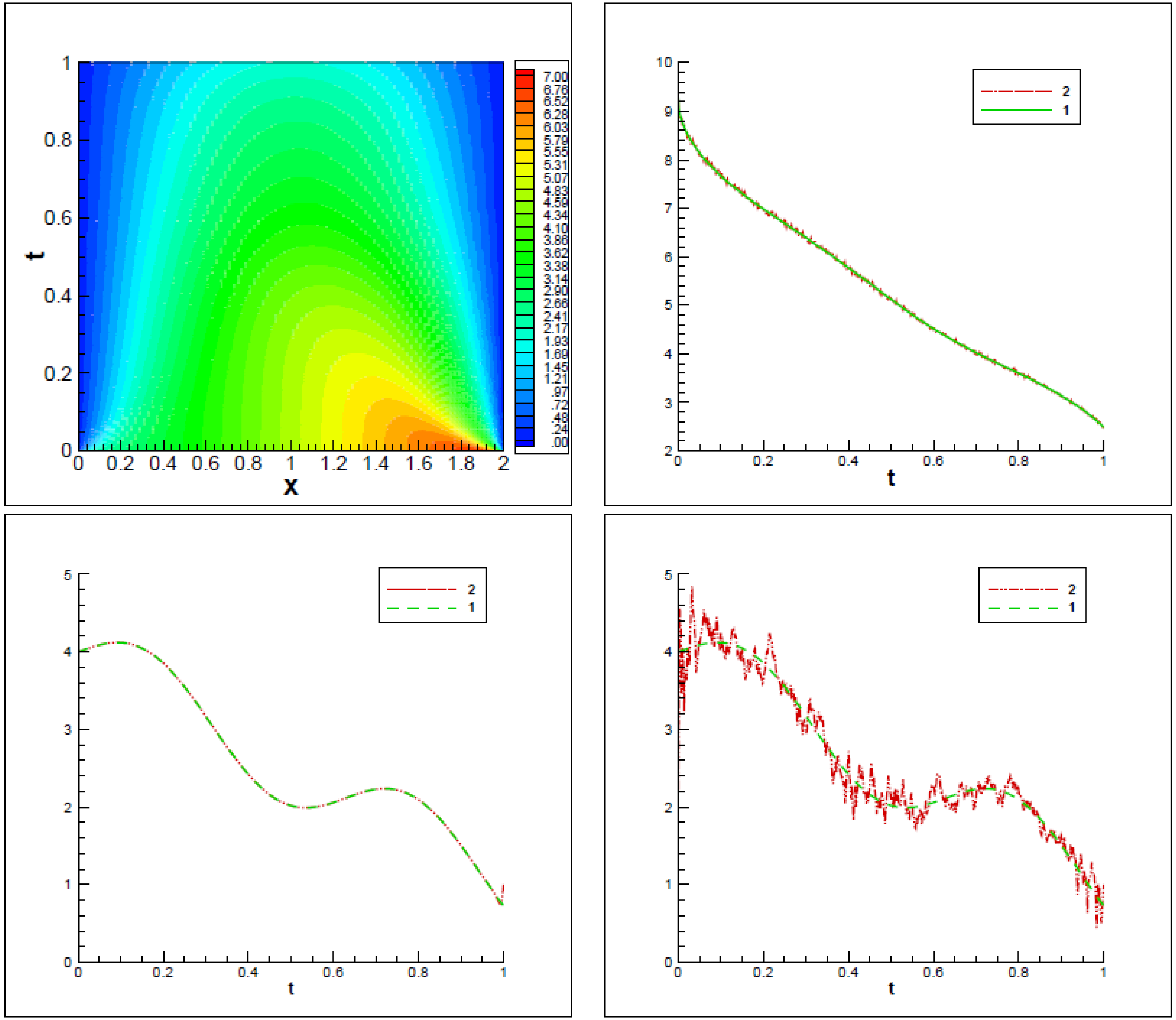
$$\left(q^{(k)}(t)\psi^{(k)}\right)_t = -\operatorname{div}\left(\sigma(x)\nabla\psi^{(k)}\right), \qquad x \in \Omega, \quad t \in (0, T),\, t \neq t_j,\, j =$$

$$\left[q^{(k)}(t)\psi^{(k)}\right]_{t=t_j} = -2\left[\int_0^L u^{(k)}(x, t_j)\mathrm{d}x - f(t_j)\right], \qquad x \in \Omega, \quad j = 1, \ldots,$$

$$\psi^{(k)}(x, T) = 0, \qquad x \in \Omega,$$

$$\psi^{(k)}|_{\partial\Omega} = 0, \qquad t \in (0, T).$$

Here $\left[q^{(k)}(t)\psi^{(k)}\right]_{t=t_j}$ is the jump of the function $q^{(k)}(t)\psi^{(k)}(x, t)$ in the point $t = t_j$.



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2.17
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.97
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.48
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1

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Научные конференции в Академгородке

Международная конференция «**Актуальные проблемы вычислительной и прикладной математики**», 1 - 5 июля.

«**Математика в приложениях**», в честь 90-летия С.К. Годунова, 4 - 10 августа

Eurasian Conference on Applied Mathematics, 26 - 29 августа

Молодежная научная школа-конференция «**Теория и численные методы решения обратных и некорректных задач**», 26 августа - 3 сентября

Thank you for attention!