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CONSTRUCTION OF DECISION OF CHARACTERISTIC CAUCHIE PROBLEM WITH INITIAL DATA DESCRIBING TWISTING OF GAS

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Plan report

1) Raising of task
2) Method of decision
3) Conclusion

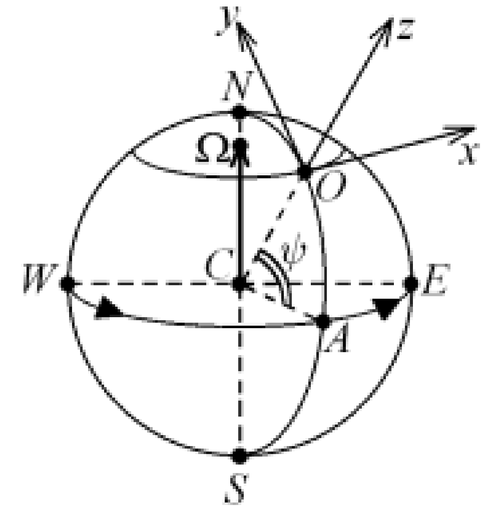
Introduction

In the monograph [The characteristic Cauchy problem and its applications in gas dynamics. Bautin SP], a theorem on the uniqueness of solutions of the characteristic Cauchy problem was proved under the necessary conditions for the solvability of this problem and the specification of additional conditions.

Taking into account this proof, in the book [Destructive atmospheric vortices: theorems, calculations, experiments: monograph / S.P. Bautin, I.Yu. Krutova, A.G. Obukhov, K.V. Bautin] considered the stationary case of the gas flow problem in the bottom region of an ascending swirling flow.

This problem will be considered in the nonstationary case.

System of equalizations of gas dynamics in the conditions of gravities and Coriolis



$$\begin{cases} c_t + uc_r + \frac{v}{r}c_\varphi + wc_z + \frac{(\gamma-1)}{2}c\left(u_r + \frac{u}{r} + \frac{v_\varphi}{r} + w_z\right) = 0 \\ u_t + uu_r + \frac{v}{r}u_\varphi - \frac{v^2}{r} + wu_z + \frac{2}{(\gamma-1)}cc_r = av - bw\cos\varphi \\ v_t + uv_r + \frac{uv}{r} + \frac{v}{r}v_\varphi + wv_z + \frac{2}{(\gamma-1)}\frac{c}{r}c_\varphi = -au + bw\sin\varphi \\ w_t + uw_r + \frac{v}{r}w_\varphi + ww_z + \frac{2}{(\gamma-1)}cc_z = bu\cos\varphi - bv\sin\varphi - g \end{cases} \qquad (1)$$

t. r,φ, z - independent variables *(φ, z* - cylindrical coordinates*)*
c, u. v. w - sought after functions
a,b - coefficients for the account of acceleration of Coriolis
g - gravity

The initial and boundary conditions in the problem under consideration

$$\begin{cases} c(t,r,\varphi,z)|_{z=0} = c_0(t,r,\varphi) \\ u(t,r,\varphi,z)|_{z=0} = u_0(t,r,\varphi) \\ v(t,r,\varphi,z)|_{z=0} = v_0(t,r,\varphi) \\ w(t,r,\varphi,z)|_{z=0} = 0 \end{cases} \quad (2)$$

- condition of gas flow through the horizontal plane z = 0

The Cauchy characteristic problem is obtained, where z = 0 is a contact characteristic of multiplicity 2.

$$\begin{cases} u(t,r,\varphi,z)|_{r=r_{in}} = u^o(t,\varphi,z); \\ v(t,r,\varphi,z)|_{r=r_{in}} = v^o(t,\varphi,z); \quad r_{in} = \text{const} > 0 \end{cases} \quad (3)$$

Therefore, we must specify two additional boundary conditions for the construction of a unique solution when they are compatible.

$$\begin{cases} u_0(t,r,\varphi)|_{r=r_{in}} = u^o(t,\varphi,z)|_{z=0} \\ v_0(t,r,\varphi)|_{r=r_{in}} = v^o(t,\varphi,z)|_{z=0} \end{cases} \quad (4)$$

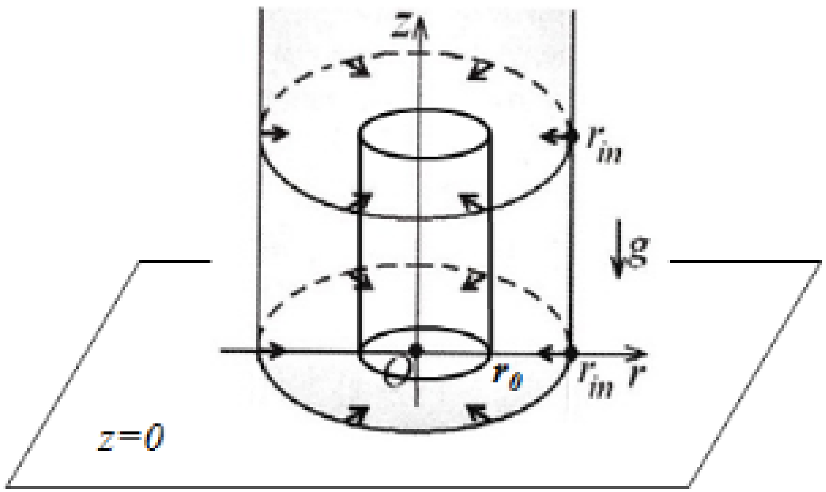
Theorem 1. Problem (1), (2) is a characteristic Cauchy problem of the standard form with data on the characteristic of multiply two which, in the case of analyticity of all input data, has a unique analytic solution when the necessary conditions for the solvability of the characteristic Cauchy problem are satisfied and two additional conditions are agreed with Initial conditions.

A constant radial inflow is considered when $r = r_{in}$, that is,

$$u|_{r=r_{in}} = u_* = \text{const} < 0$$

$$v|_{r=r_{in}} = 0$$

Chart of flow, exposing gas-dynamic sense of problem



$$u(t,r,\varphi,z)|_{r=r_{in}} = u_{in} = \text{const} < 0; \;\; v(t,r,\varphi,z)|_{r=r_{in}} = 0$$

The decision of problem (1)-(4)

$$\mathbf{U} = \begin{pmatrix} v \\ w \\ c \\ u \end{pmatrix}$$

can be presented in a kind

$$\mathbf{U}(t,r,\varphi,z) = \sum_{k=0}^{\infty} \mathbf{U}_k(t,r,\varphi)\frac{z^k}{k!}; \quad \mathbf{U}_k(t,r,\varphi) = \left.\frac{\partial^k \mathbf{U}(t,r,\varphi,z)}{\partial z^k}\right|_{z=0} \tag{5}$$

Where series 5 meets in some vinicity of point of M with coordinates

$$t = 0, r = r_0, \varphi = \varphi_0, z = z_0, \text{где } 0 \le \varphi_0 \le 2\pi.$$

It is necessary to define coefficients $\mathbf{U_0}, \mathbf{U_1}, \mathbf{U_2}, \mathbf{U_3}$

$$\mathbf{U} \approx \mathbf{U}_0(t,r) + \mathbf{U}_1(t,r,\varphi) + \mathbf{U}_2(t,r,\varphi)\frac{z^2}{2} + \mathbf{U}_3(t,r,\varphi)\frac{z^3}{6}$$

We put z = 0 in the SEGD, we take into account the initial conditions

$$\begin{cases} c_{0t} + u_0 c_{0r} + \frac{v_0}{r} c_{0\varphi} + \frac{(\gamma-1)}{2} c_0 \left(u_{0r} + \frac{u_0}{r} + \frac{v_{0\varphi}}{r} + w_1 \right) = 0, \\ u_{0t} + u_0 u_{0r} + \frac{v_0}{r} u_{0\varphi} - \frac{v_0^2}{r} + \frac{2}{(\gamma-1)} c_0 c_{0r} = a v_0, \\ v_{0t} + u_0 v_{0r} + \frac{u_0 v_0}{r} + \frac{v_0}{r} v_{0\varphi} + \frac{2}{(\gamma-1)} \frac{c_0}{r} c_{0\varphi} = -a u_0, \\ \frac{2}{(\gamma-1)} c_0 c_1 = b u_0 \cos\varphi - b v_0 \sin\varphi - g. \end{cases} \qquad (6)$$

We obtain from (6)

$$\begin{aligned} w_1 &= -\frac{2}{(\gamma-1)c_0} \left(c_{0t} + u_0 c_{0r} + \frac{v_0}{r} c_{0\varphi} \right) - u_{0r} - \frac{u_0}{r} - \frac{v_{0\varphi}}{r} \\ c_1 &= \frac{(\gamma-1)}{2c_0} \left(b u_0 \cos\varphi - b v_0 \sin\varphi - g \right) \end{aligned} \qquad (7)$$

$$\begin{cases} u_{0t} + u_0 u_{0r} + \frac{v_0}{r} u_{0\varphi} - \frac{v_0^2}{r} + \frac{2}{(\gamma-1)} c_0 c_{0r} = a v_0 \\ v_{0t} + u_0 v_{0r} + \frac{u_0 v_0}{r} + \frac{v_0}{r} v_{0\varphi} + \frac{2}{(\gamma-1)} \frac{c_0}{r} c_{0\varphi} = -a u_0 \end{cases}$$

- necessary conditions for the solvability of the characteristic Cauchy problem

We put $c_0 = c_0(r)$; $u_0 = u_0(r)$; $v_0 = v_0(r)$, that is, they do not depend on φ, and we add the first equation from (6) taking into account the fact that we put $w_1 = 0$

$$\begin{cases} \underline{c_{0t} + u_0 c_{0r}} + \frac{(\gamma-1)}{2} c_0 \left(u_{0r} + \frac{u_0}{r} \right) = 0 \\ \underline{u_{0t} + u_0 u_{0r}} - \frac{v_0^2}{r} + \frac{2}{(\gamma-1)} c_0 c_{0r} = a v_0 \\ \underline{v_{0t} + u_0 v_{0r}} + \frac{u_0 v_0}{r} + \frac{2}{(\gamma-1)} \frac{c_0}{r} c_{0\varphi} = -a u_0 \end{cases} \qquad (8)$$

In the future, the system will be solved numerically by the method of characteristics.

$$c_0(r)|_{r=r_{in}} = 1; \;\; u_0(r)|_{r=r_{in}} = u_{in} = \text{const} < 0; \;\; v_0(r)|_{r=r_{in}} = 0$$

After this, the coefficients c_1 are recovered from the second equation (7) in the form

$$c_1 = c_1(t, r, \varphi) \equiv c_{10}(r) + c_{11}(r) \cos\varphi + c_{12}(r) \sin\varphi$$

$$c_{10} = -g \frac{(\gamma-1)}{2} \frac{1}{c_0(r)}; \;\; c_{11} = b \frac{(\gamma-1)}{2} \frac{u_0(r)}{c_0(r)}; \;\; c_{12} = -b \frac{(\gamma-1)}{2} \frac{v_0(r)}{c_0(r)}$$

The construction of the coefficients c_2, u_1, v_1, w_2 of the power series

$\frac{\partial \mathrm{SEGD}}{\partial z}\Big|_{z=0}$, are taken into account initial conditions and the first coefficients found

$$c_{1t}+u_0c_{1r}+u_1c_{0r}+\frac{v_0}{r}c_{1\varphi}+\frac{(\gamma-1)}{2}c_1\left(u_{0r}+\frac{u_0}{r}\right)+$$
$$+\frac{(\gamma-1)}{2}c_0\left(u_{1r}+\frac{u_1}{r}+\frac{v_{1\varphi}}{r}+w_2\right)=0;$$
$$u_{1t}+u_1u_{0r}+u_0u_{1r}+\frac{v_0}{r}u_{1\varphi}-\frac{2v_0v_1}{r}+\frac{2}{(\gamma-1)}c_1c_{0r}+\frac{2}{(\gamma-1)}c_0c_{1r}=av_1; \qquad (9)$$
$$v_{1t}+u_1v_{0r}+u_0v_{1r}+\frac{u_1v_0}{r}+\frac{u_0v_1}{r}+\frac{v_0}{r}v_{1\varphi}+\frac{2}{(\gamma-1)}\frac{c_0}{r}c_{1\varphi}=-au_1;$$
$$\frac{2}{(\gamma-1)}c_1^2+\frac{2}{(\gamma-1)}c_0c_2=bu_1\cos\varphi-bv_1\sin\varphi.$$

From (9) it follows that

$$w_2 = -\frac{2}{(\gamma-1)}\frac{1}{c_0}\left\{c_{1t} + u_0 c_{1r} + u_1 c_0'(r) + \frac{v_0(r)}{r}c_{1\varphi} + \right.$$

$$\left. + \frac{(\gamma-1)}{2}c_1\left[u_0'(r) + \frac{u_0(r)}{r}\right] + \frac{(\gamma-1)}{2}c_0(r)\left(u_{1r} + \frac{u_1}{r} + \frac{v_{1\varphi}}{r}\right)\right\}$$

$$c_2 = \frac{(\gamma-1)}{2}\frac{1}{c_0}\left[bu_1\cos\varphi - bv_1\sin\varphi - \frac{2}{(\gamma-1)}c_1^2\right]$$

Then u_1, v_1 is sought in the form

$$u_1 = u_{10}(t,r) + u_{11}(t,r)\cos\varphi + u_{12}(t,r)\sin\varphi$$

$$v_1 = v_{10}(t,r) + v_{11}(t,r)\cos\varphi + v_{12}(t,r)\sin\varphi$$

For u_{1i}, v_{1i} we obtain the system

$$u_{10t} + u_{0r}u_{10} + u_0u_{10r} - \frac{2v_0}{r}v_{10} + \frac{2}{(\gamma-1)}c_{0r}c_{10} + \frac{2}{(\gamma-1)}c_0c_{10r} = av_{10};$$

$$u_{11t} + u_{0r}u_{11} + u_0u_{11r} + \frac{v_0}{r}u_{12} - \frac{2v_0}{r}v_{11} + \frac{2}{(\gamma-1)}c_{0r}c_{11} + \frac{2}{(\gamma-1)}c_0c_{11r} = av_{11};$$

$$u_{12t} + u_{0r}u_{12} + u_0u_{12r} - \frac{v_0}{r}u_{11} - \frac{2v_0}{r}v_{12} + \frac{2}{(\gamma-1)}c_{0r}c_{12} + \frac{2}{(\gamma-1)}c_0c_{12r} = av_{12};$$

$$v_{10t} + v_{0r}u_{10} + u_0v_{10r} + \frac{v_0}{r}u_{10} + \frac{u_0}{r}v_{10} = -au_{10};$$

$$v_{11t} + v_{0r}u_{11} + u_0v_{11r} + \frac{v_0}{r}u_{11} + \frac{u_0}{r}v_{11} + \frac{v_0}{r}v_{12} + \frac{2}{(\gamma-1)}\frac{c_0}{r}c_{12} = -au_{11};$$

$$v_{12t} + v_{0r}u_{12} + u_0v_{12r} + \frac{v_0}{r}u_{12} + \frac{u_0}{r}v_{12} - \frac{v_0}{r}v_{11} - \frac{2}{(\gamma-1)}\frac{c_0}{r}c_{11} = -au_{12}.$$

Then it will be solved numerically by the method of characteristics.

General methodology:

1) SEGD is differentiated with respect to z k times;
2) assume z = 0, consider the form of all previous coefficients;
3) w_{k+1}, c_{k+1} explicitly through the previous ones, and u_k, v_k from the system of partial differential equations, in which the variables are partially separated;

$$u_k(t,r,\varphi) = u_{k0}(t,r) + \sum_{i=1}^{k}[\,U_{ki}(t,r)\cos(k\varphi) + U_{ki}(t,r)\sin(k\varphi)]$$

4) for $u_{ki;\,1,2}$ is a linear system with partial derivatives of hyperbolic type.

Conclusion

1) A problem is formulated that has a meaningful gas dynamic meaning
2) The problem is considered in the case of t, r, φ, z
3) The solution is constructed in the form of infinite convergent series, the method of partial separation of variables
4) Systems of hyperbolic equations for the coefficients of series in z^4 inclusive
5) In the future, calculations will be made

Thank you for attention!

Система уравнений для искомых u_{3j}, v_{3j}, $j = 0, 1, ..., 8$:

$$
\begin{gathered}
u_{30t} + u_0 u_{30r} + u_{0r} u_{30r} + 3u_{20} u_{10r} + \frac{3}{2} u_{21} u_{11r} + \frac{3}{2} u_{22} u_{12r} + \\
+ 3u_{10} u_{20r} + \frac{3}{2} u_{11} u_{21r} + \frac{3}{2} u_{12} u_{22r} + \frac{3}{2r} v_{21} u_{12} - \frac{3}{2r} v_{22} u_{11} + \frac{3}{2r} v_{11} u_{22} - \frac{3}{2r} v_{12} u_{21} + \\
+ \frac{6}{r} v_{10} v_{20} + \frac{3}{r} v_{11} v_{21} + \frac{3}{r} v_{12} v_{22} - \frac{2}{r} v_0 v_{30} + w_{30} u_{10} + \frac{1}{2} w_{31} u_{11} + \frac{1}{2} w_{32} u_{12} + \\
+ 3w_{20} u_{20} + \frac{3}{2} w_{21} u_{21} + \frac{3}{2} w_{22} u_{22} + \frac{3}{2} w_{23} u_{23} + \frac{3}{2} w_{24} u_{24} + \frac{2}{(\gamma - 1)} c_{30} c_{0r} + \\
+ \frac{6}{(\gamma - 1)} c_{20} c_{10r} + \frac{3}{(\gamma - 1)} c_{21} c_{11r} + \frac{3}{(\gamma - 1)} c_{22} c_{12r} + \frac{6}{(\gamma - 1)} c_{10} c_{20r} + \\
+ \frac{3}{(\gamma - 1)} c_{11} c_{21r} + \frac{3}{(\gamma - 1)} c_{12} c_{22r} + \frac{2}{(\gamma - 1)} c_0 c_{30r} = a v_{30} - \frac{1}{2} b w_{31};
\end{gathered}
$$

$$
\begin{aligned}
&u_{31t} + u_{31}u_{0r} + u_0u_{31r} + 3u_{20}u_{11r} + 3u_{21}u_{10r} + \frac{3}{2}u_{23}u_{11r} + \frac{3}{2}u_{24}u_{12r}+ \\
&+3u_{10}u_{21r} + 3u_{11}u_{20r} + \frac{3}{2}u_{11}u_{23r} + \frac{3}{2}u_{12}u_{24r} + \frac{3}{r}v_{20}u_{12} + \frac{3}{2r}v_{23}u_{12} - \frac{3}{2r}v_{24}u_{11}+ \\
&+\frac{3}{r}v_{10}u_{22} + \frac{3}{r}v_{11}u_{24} - \frac{3}{r}v_{12}u_{23} + \frac{1}{r}v_0u_{32} - \frac{6}{r}v_{10}u_{21} - \frac{6}{r}v_{11}u_{20} - \frac{3}{r}v_{11}u_{23}- \\
&-\frac{3}{r}v_{12}u_{24} - \frac{2}{r}v_0u_{31} + w_{30}u_{11} + w_{31}u_{10} + \frac{1}{2}w_{33}u_{11} + \frac{1}{2}w_{34}u_{12} + 3w_{20}u_{21} + 3w_{21}u_{20}+ \\
&+\frac{3}{2}w_{21}u_{23} + \frac{3}{2}w_{22}u_{24} + \frac{3}{2}w_{23}u_{21} + \frac{3}{2}w_{24}u_{22} + \frac{2}{(\gamma-1)}c_0c_{31r} + \frac{2}{(\gamma-1)}c_{31}c_{0r}+ \\
&+\frac{6}{(\gamma-1)}c_{21}c_{10r} + \frac{6}{(\gamma-1)}c_{20}c_{11r} + \frac{3}{(\gamma-1)}c_{23}c_{11r} + \frac{3}{(\gamma-1)}c_{24}c_{11r}+ \\
&+\frac{6}{(\gamma-1)}c_{10}c_{21r} + \frac{6}{(\gamma-1)}c_{11}c_{20r}+ \\
&+\frac{3}{(\gamma-1)}c_{11}c_{23r} + \frac{3}{(\gamma-1)}c_{12}c_{24r} = av_{31} - bw_{30} - \frac{1}{2}bw_{33};
\end{aligned}
$$

$$
\begin{aligned}
&u_{32t} + u_{32}u_{0r} + u_0u_{32r} + 3u_{20}u_{12r} + 3u_{22}u_{10r} - \frac{3}{2}u_{23}u_{12r} + \frac{3}{2}u_{24}u_{11r} + \\
&+3u_{10}u_{22r} + 3u_{11}u_{20r} + \frac{3}{2}u_{11}u_{24r} - \frac{3}{2}u_{12}u_{23r} - \frac{3}{r}v_{20}u_{11} + \frac{3}{2r}v_{23}u_{11} + \frac{3}{2r}v_{24}u_{12} - \\
&-\frac{3}{r}v_{10}u_{21} - \frac{3}{r}v_{11}u_{23} - \frac{3}{r}v_{12}u_{24} - \frac{1}{r}v_0u_{31} - \frac{6}{r}v_{10}u_{22} - \frac{6}{r}v_{12}u_{20} - \frac{3}{r}v_{11}u_{24} + \frac{3}{r}v_{12}u_{23} - \\
&-\frac{2}{r}v_0u_{32} + w_{30}u_{12} + w_{32}u_{10} - \frac{1}{2}w_{33}u_{12} + \frac{1}{2}w_{34}u_{11} + 3w_{20}u_{22} + 3w_{22}u_{20} + \frac{3}{2}w_{21}u_{24} - \\
&-\frac{3}{2}w_{22}u_{23} + \frac{3}{2}w_{24}u_{21} + \frac{2}{(\gamma-1)}c_{32}c_{0r} + \frac{2}{(\gamma-1)}c_0c_{32r} + \frac{6}{(\gamma-1)}c_{20}c_{12r} + \\
&+\frac{6}{(\gamma-1)}c_{22}c_{10r} - \frac{3}{(\gamma-1)}c_{23}c_{12r} + \frac{3}{(\gamma-1)}c_{24}c_{11r} + \frac{6}{(\gamma-1)}c_{10}c_{22r} + \\
&+\frac{6}{(\gamma-1)}c_{12}c_{20r} + \frac{3}{(\gamma-1)}c_{11}c_{24r} - \frac{3}{(\gamma-1)}c_{12}c_{23r} = av_{32} - bw_{34};
\end{aligned}
$$

$$
\begin{aligned}
&u_{33t}+u_{33}u_{0r}+u_0u_{33r}+3u_{23}u_{10r}+\frac{3}{2}u_{21}u_{11r}-\frac{3}{2}u_{22}u_{12r}+3u_{10}u_{23r}+\frac{3}{2}u_{11}u_{21r}-\\
&-\frac{3}{2}u_{12}u_{22r}+\frac{3}{2r}v_{21}u_{12}+\frac{3}{2r}v_{22}u_{11}+\frac{3}{r}v_{10}u_{24}+\frac{3}{2r}v_{11}u_{22}++\frac{3}{2r}v_{12}u_{21}+\frac{2}{r}v_0u_{34}-\\
&-\frac{6}{r}v_{10}u_{23}-\frac{3}{r}v_{11}u_{21}+\frac{3}{r}v_{12}u_{22}-\frac{2}{r}v_0v_{33}+w_{33}u_{10}+\frac{1}{2}w_{31}u_{11}-\frac{1}{2}w_{32}u_{12}+\\
&+\frac{1}{2}w_{35}u_{11}+\frac{1}{2}w_{36}u_{12}+3w_{20}u_{23}+3w_{23}u_{20}+\frac{3}{2}w_{21}u_{21}-\frac{3}{2}w_{22}u_{22}+\frac{2}{(\gamma-1)}c_{33}c_{0r}+\\
&+\frac{2}{(\gamma-1)}c_0c_{33r}+\frac{6}{(\gamma-1)}c_{23}c_{10r}+\frac{3}{(\gamma-1)}c_{21}c_{11r}-\frac{3}{(\gamma-1)}c_{22}c_{12r}+\\
&+\frac{6}{(\gamma-1)}c_{10}c_{23r}+\frac{3}{(\gamma-1)}c_{11}c_{21r}-\\
&-\frac{3}{(\gamma-1)}c_{12}c_{22r}=av_{33}-\frac{1}{2}bw_{35}-\frac{1}{2}bw_{31};
\end{aligned}
$$

$$u_{34t}+u_{34}u_{0r}+u_0u_{34r}+3u_{24}u_{10r}+\frac{3}{2}u_{21}u_{12r}+\frac{3}{2}u_{22}u_{11r}+3u_{10}u_{24r}+\frac{3}{2}u_{11}u_{22r}+$$

$$+\frac{3}{2}u_{12}u_{21r}-\frac{3}{2r}v_{21}u_{11}+\frac{3}{2r}v_{22}u_{12}-\frac{3}{r}v_{10}u_{23}-\frac{3}{2r}v_{11}u_{21}+\frac{3}{2r}v_{12}u_{22}-\frac{2}{r}v_0u_{33}-$$

$$-\frac{6}{r}v_{10}u_{24}-\frac{3}{r}v_{11}u_{22}-\frac{3}{r}v_{12}u_{21}-\frac{2}{r}v_0v_{34}+w_{34}u_{10}+\frac{1}{2}w_{31}u_{12}+\frac{1}{2}w_{32}u_{11}-$$

$$-\frac{1}{2}w_{35}u_{12}+\frac{1}{2}w_{36}u_{11}+3w_{20}u_{24}+3w_{24}u_{20}+\frac{3}{2}w_{21}u_{22}+\frac{3}{2}w_{22}u_{21}+\frac{2}{(\gamma-1)}c_{34}c_{0r}+$$

$$+\frac{2}{(\gamma-1)}c_0c_{34r}+\frac{6}{(\gamma-1)}c_{24}c_{10r}+\frac{3}{(\gamma-1)}c_{21}c_{12r}+\frac{3}{(\gamma-1)}c_{22}c_{11r}+$$

$$+\frac{6}{(\gamma-1)}c_{10}c_{24r}+\frac{3}{(\gamma-1)}c_{11}c_{22r}+\frac{3}{(\gamma-1)}c_{12}c_{21r}=av_{34}-\frac{1}{2}bw_{36}-\frac{1}{2}bw_{32};$$

$$
\begin{gathered}
u_{35t}+u_{35}u_{0r}+u_0u_{35r}+\frac{3}{2}u_{23}u_{11r}-\frac{3}{2}u_{24}u_{12r}+\frac{3}{2}u_{11}u_{23r}-\frac{3}{2}u_{12}u_{24r}+\frac{3}{2r}v_{23}u_{12}+\\
\frac{3}{2r}v_{24}u_{11}+\frac{3}{r}v_{11}u_{24}+\frac{3}{r}v_{12}u_{23}+\frac{3}{r}v_0u_{36}-\frac{3}{r}v_{11}v_{23}+\frac{3}{r}v_{12}v_{24}-\frac{2}{r}v_0v_{35}+w_{35}u_{10}+\\
+\frac{1}{2}w_{33}u_{11}-\frac{1}{2}w_{34}u_{12}+\frac{3}{2}w_{21}u_{23}-\frac{3}{2}w_{22}u_{24}+\frac{3}{2}w_{23}u_{21}-\frac{3}{2}w_{24}u_{22}+\frac{2}{(\gamma-1)}c_{35}c_{0r}+\\
+\frac{2}{(\gamma-1)}c_0c_{35r}+\frac{3}{(\gamma-1)}c_{23}c_{11r}-\frac{3}{(\gamma-1)}c_{24}c_{12r}+\\
+\frac{3}{(\gamma-1)}c_{11}c_{23r}-\frac{3}{(\gamma-1)}c_{12}c_{24r}=av_{35}-\frac{1}{2}bw_{33};
\end{gathered}
$$

$$u_{36t}+u_{36}u_{0r}+u_0u_{36r}+\frac{3}{2}u_{23}v_{12r}+\frac{3}{2}u_{24}v_{11r}+\frac{3}{2}u_{12}v_{24r}+\frac{3}{2}u_{12}v_{23r}+\frac{u_{36}v_0}{r}+$$
$$+\frac{3}{2r}u_{23}v_{11}+\frac{3}{2r}u_{24}v_{11}+\frac{3}{2r}u_{11}v_{24}+\frac{3}{2r}u_{12}v_{23}+\frac{1}{r}u_0v_{36}-\frac{3}{2r}v_{23}v_{11}+\frac{3}{2r}v_{24}v_{12}-$$
$$-\frac{3}{r}v_{11}v_{23}+\frac{3}{r}v_{24}v_{12}-\frac{3}{r}v_{35}v_0+w_{36}v_{10}+\frac{1}{2}w_{33}v_{12}+\frac{1}{2}w_{34}v_{11}+\frac{3}{2}w_{23}v_{12}+\frac{3}{2}w_{24}v_{11}-$$
$$-\frac{6}{(\gamma-1)r}c_{23}c_{11}+\frac{6}{(\gamma-1)r}c_{24}c_{12}-\frac{6}{(\gamma-1)r}c_{35}c_0=-au_{36}+\frac{1}{2}bw_{33};$$
$$u_{37t}+u_0u_{37r}=\frac{w_{35}u_{11}}{2}-\frac{w_{36}u_{12}}{2}+\frac{3}{2}w_{23}u_{23}-\frac{3}{2}w_{24}u_{24}-\frac{1}{2}bw_{35};$$
$$u_{38t}+u_0u_{38r}=\frac{w_{35}u_{12}}{2}+\frac{w_{36}u_{11}}{2}+\frac{3}{2}w_{23}u_{24}+\frac{3}{2}w_{24}u_{23}-\frac{1}{2}bw_{36}.$$

$$
v_{30t}+u_0v_{30r}+u_{30}v_{0r}+3u_{20}v_{10r}+\frac{3}{2}u_{21}v_{11r}+\frac{3}{2}u_{22}v_{12r}+3u_{10}v_{20r}+\frac{3}{2}u_{11}v_{21r}+
$$

$$
+\frac{3}{2}u_{12}v_{22r}+\frac{u_{30}v_0}{r}+\frac{3}{r}u_{20}v_{10}+\frac{3}{2r}u_{21}v_{11}+\frac{3}{2r}u_{22}v_{12}+\frac{3}{r}u_{10}v_{20}+\frac{3}{2r}u_{11}v_{21}+
$$

$$
+\frac{3}{2r}u_{12}v_{22}+\frac{u_0v_{30}}{r}+\frac{3}{2r}v_{21}v_{12}-\frac{3}{2r}v_{22}v_{11}+\frac{3}{2r}v_{11}v_{22}-\frac{2}{2r}v_{12}v_{21}+w_{30}v_{10}+
$$

$$
+\frac{1}{2}w_{31}v_{11}+\frac{1}{2}w_{32}v_{12}+3w_{20}v_{10}+\frac{3}{2}w_{21}v_{11}+\frac{3}{2}w_{22}v_{12}+\frac{3}{(\gamma-1)r}c_{21}c_{12}-
$$

$$
-\frac{3}{(\gamma-1)r}c_{22}c_{11}+\frac{3}{(\gamma-1)r}c_{11}c_{22}+\frac{3}{(\gamma-1)r}c_{12}c_{21}=-au_{30}+\frac{1}{2}bw_{32};
$$

$$
\begin{aligned}
& v_{31t} + u_0 v_{31r} + u_{31} v_{0r} + 3u_{20} v_{11r} + 3u_{21} v_{10r} + \frac{3}{2} u_{23} v_{11r} + \frac{3}{2} u_{24} v_{12r} 3u_{10} v_{21r} + \\
& + 3u_{11} v_{20r} + \frac{3}{2} u_{11} v_{23r} + \frac{3}{2} u_{12} v_{24r} + \frac{u_{31} v_0}{r} + \frac{3}{r} u_{20} v_{11} + \frac{3}{r} u_{21} v_{10} + \frac{3}{2r} u_{23} v_{11} + \\
& + \frac{3}{2r} u_{24} v_{12} + \frac{3}{r} u_{10} v_{21} + \frac{3}{r} u_{11} v_{20} + \frac{3}{2r} u_{11} v_{23} + \frac{3}{2r} u_{12} v_{24} + \frac{u_0 v_{31}}{r} + \frac{3}{r} v_{20} v_{12} + \\
& + \frac{3}{2r} v_{23} v_{12} - \frac{3}{2r} v_{24} v_{11} + \frac{3}{r} v_{10} v_{22} + \frac{3}{r} v_{11} v_{24} - \frac{3}{r} v_{12} v_{23} + \frac{1}{r} v_{32} v_0 + w_{30} v_{11} + \\
& + \frac{1}{2} w_{33} v_{11} + \frac{1}{2} w_{34} v_{12} + w_{31} v_{10} + 3w_{20} v_{11} + 3w_{21} v_{10} + \frac{3}{2} w_{23} v_{11} + \frac{3}{2} w_{24} v_{12} + \\
& + \frac{6}{(\gamma-1)r} c_{20} c_{12} + \frac{3}{(\gamma-1)r} c_{23} c_{12} - \frac{3}{(\gamma-1)r} c_{24} c_{11} + \frac{6}{(\gamma-1)r} c_{10} c_{22} + \\
& + \frac{6}{(\gamma-1)r} c_{11} c_{24} - \frac{6}{(\gamma-1)r} c_{12} c_{23} - \frac{2}{(\gamma-1)r} c_{32} c_0 = -a u_{31} + \frac{1}{2} b w_{34};
\end{aligned}
$$

$$v_{32t}+u_0v_{32r}+u_{32}v_{0r}+3u_{20}v_{12r}+3u_{22}v_{10r}-\frac{3}{2}u_{23}v_{12r}+\frac{3}{2}u_{24}v_{11r}+3u_{10}v_{22r}+$$
$$+3u_{12}v_{20r}+\frac{3}{2}u_{11}v_{24r}-\frac{3}{2}u_{12}v_{23r}+\frac{u_{32}v_0}{r}+\frac{3}{r}u_{20}v_{12}+\frac{3}{r}u_{22}v_{10}+\frac{3}{2r}u_{24}v_{11}-$$
$$-\frac{3}{2r}u_{23}v_{12}++\frac{3}{r}u_{10}v_{22}+\frac{3}{r}u_{12}v_{20}+\frac{3}{2r}u_{11}v_{24}-\frac{3}{2r}u_{12}v_{23}+\frac{u_0v_{32}}{r}-\frac{3}{r}v_{20}v_{11}+$$
$$+\frac{3}{2r}v_{23}v_{11}+\frac{3}{2r}v_{24}v_{12}-\frac{3}{r}v_{10}v_{21}+\frac{3}{r}v_{12}v_{23}-\frac{3}{r}v_{24}v_{12}-\frac{v_0v_{31}}{r}+w_{30}v_{12}+w_{32}v_{10}+$$
$$+\frac{1}{2}w_{34}v_{11}-\frac{1}{2}w_{33}v_{12}+3w_{20}v_{12}+3w_{22}v_{10}-\frac{3}{2}w_{23}v_{12}+\frac{3}{2}w_{24}v_{11}-\frac{6}{(\gamma-1)r}c_{20}c_{11}+$$
$$+\frac{3}{(\gamma-1)r}c_{23}c_{11}+\frac{3}{(\gamma-1)r}c_{24}c_{12}-\frac{6}{(\gamma-1)r}c_{10}c_{21}-\frac{6}{(\gamma-1)r}c_{11}c_{23}-$$
$$-\frac{6}{(\gamma-1)r}c_{12}c_{24}-\frac{2}{(\gamma-1)r}c_{31}c_0=-au_{32}+bw_{30}-\frac{1}{2}bw_{33};$$

$$v_{33t}+u_0v_{33r}+u_{33}v_{0r}+3u_{23}v_{10r}+\frac{3}{2}u_{21}v_{11r}-\frac{3}{2}u_{22}v_{12r}+3u_{10}v_{23r}+\frac{3}{2}u_{11}v_{21r}-$$

$$-\frac{3}{2}u_{12}v_{22r}+\frac{3}{r}u_{23}v_{10}+\frac{3}{2r}u_{21}v_{11}-\frac{3}{2r}u_{22}v_{12}+\frac{u_{33}v_0}{2}+\frac{3}{r}u_{10}v_2+\frac{3}{2r}u_{11}v_{21}-$$

$$-\frac{3}{2r}u_{12}v_{22}+\frac{u_0v_{33}}{r}+\frac{3}{2r}v_{21}v_{12}+\frac{3}{2r}v_{22}v_{11}+\frac{6}{r}v_{10}v_{24}+\frac{3}{2r}v_{11}v_{23}+\frac{3}{2r}v_{12}v_{21}+$$

$$+\frac{2v_{34}v_0}{r}+\frac{1}{2}w_{31}v_{11}-\frac{1}{2}w_{32}v_{12}+w_{33}v_{10}+\frac{1}{2}w_{35}v_{11}+\frac{1}{2}w_{36}v_{12}+3w_{23}v_{10}+\frac{3}{2}w_{21}v_{11}-$$

$$-\frac{3}{2}w_{22}v_{12}+\frac{3}{(\gamma-1)r}c_{21}c_{12}+\frac{3}{(\gamma-1)r}c_{22}c_{11}+\frac{12}{(\gamma-1)r}c_{10}c_{24}+\frac{3}{(\gamma-1)r}c_{11}c_{22}+$$

$$+\frac{3}{(\gamma-1)r}c_{12}c_{21}+\frac{4}{(\gamma-1)r}c_{34}c_0=-au_{33}+\frac{1}{2}bw_{36}-\frac{1}{2}bw_{32};$$

$$
\begin{gathered}
v_{34t}+u_0v_{34r}+u_{34}v_{0r}+3u_{24}v_{10r}+\frac{3}{2}u_{21}v_{12r}+\frac{3}{2}u_{22}v_{11r}+3u_{10}v_{24r}+\frac{3}{2}u_{11}v_{22r}+\\
+\frac{3}{2}u_{12}v_{21r}+\frac{u_{34}v_0}{r}+\frac{3}{r}u_{24}v_{10}+\frac{3}{2r}u_{21}v_{12}+\frac{3}{2r}u_{22}v_{11}+\frac{3}{r}u_{10}v_{24}+\frac{3}{2r}u_{11}v_{22}+\\
+\frac{3}{2r}u_{12}v_{21}+\frac{u_0v_{34}}{r}-\frac{3}{2r}v_{21}v_{11}+\frac{3}{2r}v_{22}v_{12}-\frac{6}{r}v_{10}v_{23}-\frac{3}{2r}v_{11}v_{21}+\frac{3}{2r}v_{12}v_{22}-\\
-\frac{2}{r}v_{33}v_0+\frac{1}{2}w_{31}v_{12}+\frac{1}{2}w_{32}v_{11}+\frac{1}{2}w_{34}v_{10}-\frac{1}{2}w_{35}v_{12}+\frac{1}{2}w_{36}v_{11}+3w_{24}v_{10}+\\
+\frac{3}{2}w_{21}v_{12}+\frac{3}{2}w_{22}v_{11}-\frac{3}{(\gamma-1)r}c_{21}c_{11}+\frac{3}{(\gamma-1)r}c_{22}c_{12}-\\
-\frac{12}{(\gamma-1)r}c_{10}c_{23}--\frac{3}{(\gamma-1)r}c_{11}c_{21}+\frac{3}{(\gamma-1)r}c_{12}c_{22}-\\
-\frac{4}{(\gamma-1)r}c_{33}c_0=-au_{34}+\frac{1}{2}bw_{31}-\frac{1}{2}bw_{35};
\end{gathered}
$$

$$v_{35t}+u_0v_{35r}+u_{35}v_{0r}+\frac{3}{2}u_{23}v_{11r}-\frac{3}{2}u_{24}v_{12r}+\frac{3}{2}u_{11}v_{23r}-\frac{3}{2}u_{12}v_{24r}+\frac{3}{2r}v_{11}u_{23}-$$
$$-\frac{3}{2r}v_{12}u_{24}+\frac{3}{2r}u_{11}v_{23}+\frac{3}{2r}u_{12}v_{24}+\frac{1}{r}v_0u_{35}+\frac{3}{2r}v_{12}v_{23}+\frac{3}{2r}v_{11}v_{24}+\frac{1}{r}u_0v_{35}+$$
$$+\frac{3}{r}v_{11}v_{24}+\frac{3}{r}v_{12}v_{23}+\frac{3}{r}v_{36}v_0+w_{35}v_{10}+\frac{1}{2}w_{33}v_{11}-\frac{1}{2}w_{34}v_{12}+\frac{3}{2}w_{23}v_{11}-$$
$$-\frac{3}{2}w_{24}v_{12}+\frac{3}{(\gamma-1)r}c_{23}c_{12}-\frac{3}{(\gamma-1)r}c_{24}c_{11}+\frac{3}{(\gamma-1)r}c_{11}c_{24}+$$
$$+\frac{3}{(\gamma-1)r}c_{12}c_{23}+\frac{6}{(\gamma-1)r}c_{36}c_0=av_{35}-\frac{1}{2}bw_{33};$$

$$v_{36t}+u_{36}v_{0r}+u_0v_{36r}+\frac{3}{2}u_{23}v_{12r}+\frac{3}{2}u_{24}v_{11r}+\frac{3}{2}u_{12}v_{24r}+\frac{3}{2}u_{12}u_{23r}-\frac{3}{2r}v_{23}u_{11}+$$
$$+\frac{3}{2r}v_{24}u_{12}-\frac{3}{r}v_{11}u_{23}+\frac{3}{2r}v_{12}u_{24}-\frac{3}{r}v_0u_{35}-\frac{3}{r}v_{11}v_{24}+\frac{3}{r}v_{12}v_{23}-\frac{2}{r}v_0v_{36}+$$
$$+w_{36}u_{10}+\frac{1}{2}w_{33}u_{12}+\frac{1}{2}w_{34}u_{11}+\frac{3}{2}w_{21}u_{24}+\frac{3}{2}w_{22}u_{23}+\frac{3}{2}w_{23}u_{22}+\frac{3}{2}w_{24}u_{21}+$$
$$+\frac{2}{(\gamma-1)}c_{36}c_{0r}+\frac{2}{(\gamma-1)}c_0c_{36r}+\frac{3}{(\gamma-1)}c_{23}c_{12r}-\frac{3}{(\gamma-1)}c_{24}c_{11r}+$$
$$+\frac{3}{(\gamma-1)}c_{11}c_{24r}-\frac{3}{(\gamma-1)}c_{12}c_{23r}=av_{36}-\frac{1}{2}bw_{34};$$
$$v_{37t}+u_0v_{37r}=\frac{1}{2}w_{35}v_{11}-\frac{1}{2}w_{36}v_{12}-\frac{1}{2}bw_{36};$$
$$v_{38t}+u_0v_{38r}=\frac{1}{2}w_{35}v_{12}+\frac{1}{2}w_{36}v_{11}-\frac{1}{2}bw_{35}.$$

Построение коэффициентов c_3, u_2, v_2, w_3 степенного ряда $\frac{\partial^2 \text{СУГД}}{\partial z^2}, z = 0$, учитываются начальные условия и найденные первые коэффициенты

$$
\begin{gathered}
c_{2t} + u_2 c_{0r} + 2u_1 c_{1r} + u_0 c_{2r} + \frac{1}{r}(v_2 c_{0\varphi} + 2v_1 c_{1\varphi} + v_0 c_{2\varphi}) + w_2 c_1 + \\
+\frac{(\gamma-1)}{2} c_2 (u_{0r} + \frac{u_0}{r} + \frac{v_{0\varphi}}{r}) + (\gamma-1) c_1 (u_{1r} + \frac{u_1}{r} + \frac{v_{1\varphi}}{r} + w_2) + \\
+\frac{(\gamma-1)}{2} c_0 (u_{2r} + \frac{u_2}{r} + \frac{v_{2\varphi}}{r} + w_3) = 0; \\
u_{2t} + u_2 u_{0r} + 2u_1 u_{1r} + u_0 u_{2r} + \frac{v_2}{r} u_{0\varphi} + \frac{2v_1}{r} u_{1\varphi} + \frac{v_0}{r} u_{2\varphi} - \\
-\frac{2v_1^2}{r} - \frac{2v_0 v_2}{r} + w_2 u_1 + 2w_1 u_2 + w_0 u_3 + \\
+\frac{2}{(\gamma-1)} c_2 c_{0r} + \frac{4}{(\gamma-1)} c_1 c_{1r} + \frac{2}{(\gamma-1)} c_0 c_{2r} = a v_2 - b w_2 \cos\varphi; \\
v_{2t} + u_2 v_{0r} + 2u_1 v_{1r} + u_0 v_{2r} + \frac{u_2 v_0}{r} + 2\frac{u_1 v_1}{r} + \frac{u_0 v_2}{r} + \\
+\frac{v_2}{r} v_{0\varphi} + 2\frac{v_1}{r} v_{1\varphi} + \frac{v_0}{r} v_{2\varphi} + w_2 v_1 + 2w_1 v_2 + w_0 v_3 + \\
+\frac{2}{(\gamma-1)} \frac{c_2}{r} c_{0\varphi} + \frac{4}{(\gamma-1)} \frac{c_1}{r} c_{1\varphi} + \frac{2}{(\gamma-1)} \frac{c_0}{r} c_{2\varphi} = -a u_2 + b w_2 \sin\varphi; \\
u_0 w_{2r} + \frac{1}{2} v_0 w_{2r} + \frac{2}{(\gamma-1)} (3c_1 c_2 + c_0 c_3) = b u_2 \cos\varphi - b v_2 \sin\varphi.
\end{gathered}
\tag{10}
$$

Из (10) следует

$$w_3 = -\frac{2}{(\gamma-1)}(c_{2t}+u_2c_{0r}+2u_1c_{1r}+u_0c_{2r}+\frac{1}{r}(v_2c_{0\varphi}+2v_1c1\varphi+v_0c_{2\varphi})+w_2c_1+$$

$$+\frac{\gamma-1}{2}c_2(u_{0r}+\frac{u_0}{r}+\frac{v_{0\varphi}}{r})+(\gamma-1)c_1(u_{1r}+\frac{u_1+v_{1\ varphi}}{r}+w_2)+$$

$$+\frac{\gamma-1}{2}c_0(u_{2r}+\frac{u_2}{r}+\frac{v_{2\varphi}}{r}))$$

$$c_3 = \frac{(\gamma-1)}{2}\frac{1}{c_0}\left[bu_2\cos\varphi - bv_2\sin\varphi - u_0w_{2r} - \frac{v_0w_{2\varphi}}{r} - \frac{6}{(\gamma-1)}c_1c_2\right]$$

С учетом вида уже найденных коэффициентов искомые функции u_2 и v_2 представляются в виде:

$$u_2 = u_{20} + u_{21}\cos\varphi + u_{21}\sin\varphi + u_{23}\cos 2\varphi + u_{24}\sin 2\varphi$$

$$v_2 = v_{20} + v_{21}\cos\varphi + v_{21}\sin\varphi + v_{23}\cos 2\varphi + v_{24}\sin 2\varphi$$

Построение коэффициентов c_4, u_3, v_3, w_4 степенного ряда

$\frac{\partial^3 \text{СУГД}}{\partial z^3}\big|_{z=0}$, учитываются начальные условия и найденные первые коэффициенты

$$c_{3t}+u_3c_{0r}+3u_2c_{1r}+3u_1c_{2r}+u_0c_{3r}+\frac{3v_2}{r}c_{1\varphi}+\frac{3v_1}{r}c_{2\varphi}+\frac{v_0}{r}c_{3\varphi}+w_3c_1+3w_2c_2$$
$$+\frac{\gamma-1}{2}c_3(u_{0r}+\frac{u_0}{r})+3\frac{\gamma-1}{2}c_2(u_{1r}+\frac{u_1}{r}+\frac{v_{1\varphi}}{r})+3\frac{\gamma-1}{2}c_1(u_{2r}+\frac{u_{2r}}{r}+\frac{v_{2\varphi}}{r})$$
$$+\frac{\gamma-1}{2}c_0(u_{3r}+\frac{u_3}{r}+\frac{v_{3\varphi}}{r}+w_4)=0;$$

$$u_{3t}+u_3u_{0r}+3u_2u_{1r}+3u_1u_{2r}+u_0u_{3r}$$
$$+\frac{3v_2}{r}u_{1\varphi}+\frac{3v_1}{r}u_{2\varphi}+\frac{v_0}{r}u_{3\varphi}-\frac{6}{r}v_1v_2-\frac{2}{r}v_0v_3+w_3u_1+3w_2u_2+$$
$$+\frac{2}{(\gamma-1)}(c_3c_{0r}+3c_2c_{1r}+3c_1c_{2r}+c_0c_{3r})=av_3-bw_3\sin\varphi;$$

$$v_{3t} + u_3 v_{0r} + 3u_2 v_{1r} + 3u_1 v_{2r} + u_0 v_{3r} + \frac{u_3 v_0}{r} + \frac{3u_2 v_1}{r} + \frac{3u_1 v_2}{r} + \frac{u_0 v_3}{r} +$$
$$+ \frac{3v_2}{r} v_{1\varphi} + \frac{3v_1}{r} v_{2\varphi} + \frac{v_0}{r} v_{3\varphi} + w_3 v_1 + 3w_2 v_1 + \frac{6}{(\gamma - 1)} \frac{c_2}{r} c_{1\varphi} +$$
$$+ \frac{6}{(\gamma - 1)} \frac{c_1}{r} c_{2\varphi} + \frac{2}{(\gamma - 1)} \frac{c_0}{r} c_{3\varphi} = -au_3 + bw_3 \sin\varphi; \tag{11}$$

$$w_{3t} + 3u_1 w_{2r} + u_0 w_{3r} + \frac{3v_1}{r} w_{2\varphi} + \frac{v_0}{r} w_{3\varphi} +$$
$$+3w_2 w2 + \frac{2c_3 c_1}{\gamma - 1} + \frac{6}{\gamma - 1} c_2 c_2 + \frac{6}{\gamma - 1} c_1 c_3 +$$
$$+ \frac{2}{\gamma - 1} c_0 c_4 = bu_3 \cos\varphi - bv_3 \sin\varphi;$$

Из (11) следует

$$w_4 = -\frac{2}{(\gamma-1)c_0}\Big(c_{3t} + u_3 c_{0r} + 3u_2 c_{1r} + 3u_1 c_{2r} + u_0 c_{3r} + \frac{3v_2}{r} c_{1\varphi} +$$

$$+\frac{3v_1}{r} c_{2\varphi} + \frac{v_0}{r} c_{3\varphi} + w_3 c_1 + 3w_2 c_2 +$$

$$+\frac{(\gamma-1)}{2} c_3 (u_{0r} + \frac{u_0}{r}) + 3\frac{(\gamma-1)}{2} c_2 (u_{1r} + \frac{u_1}{r} + \frac{v_{1\varphi}}{r} + w_2) +$$

$$+3\frac{(\gamma-1)}{2} c_1 (u_{2r} + \frac{u_2}{r} + \frac{v_{2\varphi}}{r} + w_3) + \frac{(\gamma-1)}{2} c_0 (u_{3r} + \frac{u_3}{r} + \frac{v_{3\varphi}}{r})\Big)$$

$$c_4 = \frac{(\gamma-1)}{2c_0}\Big(b u_3 \cos\varphi - b v_3 \sin\varphi - w_{3t} - 3u_1 w_{2r} - 3u_0 w_{3r} - \frac{3v_1}{r} w_{2\varphi} -$$

$$-\frac{v_0}{r} w_{3\varphi} - 3w_2 w_2 - \frac{2}{(\gamma-1)} c_3 c_1 - \frac{6}{(\gamma-1)} c_2 c_2 - \frac{6}{(\gamma-1)} c_1 c_3\Big)$$

Теперь u_3, v_3 ищутся в виде

$$u_3 = u_{30} + u_{31}\cos\varphi + u_{32}\sin\varphi + u_{33}\cos 2\varphi + u_{34}\sin 2\varphi + u_{35}\cos 3\varphi + u_{36}\sin 3\varphi$$

$$v_3 = v_{30} + v_{31}\cos\varphi + v_{32}\sin\varphi + v_{33}\cos 2\varphi + v_{34}\sin 2\varphi + v_{35}\cos 3\varphi + v_{36}\sin 3\varphi$$

Система уравнений для искомых u_{2j}, v_{2j}, $j = 0, 1, ..., 4$:

$$u_{20t} + u_{0r}u_{20} + u_{11}u_{11r} + u_{12}u_{12r} + u_0u_{20r} + \frac{1}{r}v_{11}u_{12} -$$

$$-\frac{1}{r}v_{12}u_{11} - \frac{1}{r}v_{11}^2 + \frac{1}{r}v_{12}^2 - \frac{2}{r}v_0v_{20} +$$

$$+\frac{1}{2}w_{21}u_{11} + \frac{1}{2}w_{22}u_{12} + \frac{2}{(\gamma-1)}c_{0r}c_{20} + \frac{4}{(\gamma-1)}c_{10}c_{10r} +$$

$$+\frac{2}{(\gamma-1)}c_{11}c_{11r} + \frac{2}{(\gamma-1)}c_{12}c_{12r} + \frac{2}{(\gamma-1)}c_0c_{20r} = av_{20} - \frac{1}{2}bw_{21};$$

$$u_{21t} + u_{0r}u_{21} + u_0u_{21r} + \frac{1}{r}v_0u_{22} - \frac{2}{r}v_0v_{21} + w_{20}u_{11} +$$

$$+\frac{2}{(\gamma-1)}c_{0r}c_{21} + \frac{4}{(\gamma-1)}c_{10}c_{11r} + \frac{4}{(\gamma-1)}c_{11}c_{10r} +$$

$$+\frac{2}{(\gamma-1)}c_0c_{21r} = av_{21} - bw_{20};$$

$$u_{22t} + u_{0r}u_{22} + u_0u_{22r} - \frac{1}{r}v_0u_{21} - \frac{2}{r}v_0v_{22} + w_{20}u_{12} +$$

$$+\frac{2}{(\gamma-1)}\left(c_{0r}c_{22} + c_0c_{22r}\right) + \frac{4}{(\gamma-1)}\left(c_{10}c_{12r} + c_{12}c_{10r}\right) = av_{22};$$

$$u_{23t}+u_{0r}u_{23}+u_{11}u_{11r}-u_{12}u_{12r}+u_0u_{23r}+\frac{1}{r}v_{11}u_{12}+\frac{1}{r}v_{12}u_{11}+\frac{2}{r}v_0u_{24}-$$

$$-\frac{1}{r}v_{11}^2-\frac{1}{2}v_{12}^2-\frac{2}{r}v_0v_{23}+\frac{1}{2}w_{21}u_{11}-\frac{1}{2}w_{22}u_{12}+$$

$$+\frac{2}{(\gamma-1)}c_{0r}c_{23}+\frac{2}{(\gamma-1)}c_{11}c_{11r}-\frac{2}{(\gamma-1)}c_{12}c_{12r}+$$

$$+\frac{2}{(\gamma-1)}c_0c_{23r}=av_{23}-\frac{1}{2}bw_{21};$$

$$u_{24t}+u_{0r}u_{24}+u_{11}u_{12r}+u_{12}u_{11r}+u_0u_{24r}-\frac{1}{r}v_{11}u_{11}+$$

$$+\frac{1}{r}v_{12}u_{12}-\frac{2}{r}v_0u_{23}-\frac{2}{r}v_{11}v_{12}-\frac{2}{r}v_0v_{24}+$$

$$+\frac{1}{2}w_{21}u_{12}+\frac{1}{2}w_{22}u_{11}+\frac{2}{(\gamma-1)}c_{0r}c_{24}+\frac{2}{(\gamma-1)}c_{11}c_{12r}+$$

$$+\frac{2}{(\gamma-1)}c_{12}c_{11r}+\frac{2}{(\gamma-1)}c_0c_{24r}=av_{24}-\frac{1}{2}bw_{22};$$

$$v_{20t} + v_{0r}u_{20} + u_{12}v_{12r} + u_0 v_{20r} + \frac{1}{r}v_0 u_{20} +$$
$$+ \frac{1}{r}u_{11}v_{11} + \frac{1}{r}u_{12}v_{12} + \frac{1}{r}u_0 v_{20} + \frac{1}{r}v_{11}v_{12} - \frac{1}{r}v_{12}v_{11} +$$
$$+ \frac{1}{2}\left(w_{21}v_{11} + w_{22}v_{12}\right) + \frac{2}{(\gamma-1)}\frac{1}{r}\left(c_{11}c_{12} - c_{12}c_{11}\right) = -au_{20} + \frac{1}{2}bw_{22};$$
$$v_{21t} + v_{0r}u_{21} + u_0 v_{21r} + \frac{1}{r}v_0 u_{21} + \frac{1}{r}u_0 v_{21} +$$
$$+ \frac{1}{r}v_0 v_{22} + w_{20}v_{11} + \frac{4}{(\gamma-1)}\frac{1}{r}c_{10}c_{12} + \frac{2}{(\gamma-1)}\frac{1}{r}c_0 c_{22} = -au_{21};$$
$$v_{22t} + v_{0r}u_{22} + u_0 v_{22r} + \frac{1}{r}v_0 u_{22} + \frac{1}{r}u_0 v_{22} - \frac{1}{r}v_0 v_{21} +$$
$$+ w_{20}v_{12} - \frac{4}{(\gamma-1)}\frac{1}{r}c_{10}c_{11} - \frac{2}{(\gamma-1)}\frac{1}{r}c_0 c_{21} = -au_{22} + bw_{20};$$
$$v_{23t} + v_{0r}u_{23} + u_{11}v_{11r} - u_{12}v_{12r} + u_0 v_{23r} +$$
$$+ \frac{1}{r}v_0 u_{23} + \frac{1}{r}u_{11}v_{11} - \frac{1}{r}u_{12}v_{12} + \frac{1}{r}u_0 v_{23} + \frac{1}{r}v_{11}v_{12} +$$
$$+ \frac{1}{r}v_{12}v_{11} + \frac{2}{r}v_0 v_{24} + \frac{1}{2}w_{21}v_{11} - \frac{1}{2}w_{22}v_{12} + \frac{2}{(\gamma-1)}\frac{1}{r}c_{11}c_{12} +$$
$$+ \frac{2}{(\gamma-1)}\frac{1}{r}c_{12}c_{11} + \frac{4}{(\gamma-1)}\frac{1}{r}c_0 c_{24} = -au_{23} - \frac{1}{2}bw_{22};$$

$$
\begin{gathered}
v_{24t} + v_{0r} u_{24} + u_{11} v_{12r} + u_{12} v_{11r} + \\
+ u_0 v_{24r} + \frac{1}{r} v_0 u_{24} + \frac{1}{r} u_{11} v_{12} + \frac{1}{r} u_{12} v_{11} + \frac{1}{r} u_0 v_{24} - \\
- \frac{1}{r} v_{11}^2 + \frac{1}{r} v_{12}^2 - \frac{2}{r} v_0 v_{23} + \frac{1}{2} w_{21} v_{12} + \frac{1}{2} w_{22} v_{11} - \\
- \frac{2}{(\gamma - 1)} \frac{1}{r} c_{11}^2 + \frac{2}{(\gamma - 1)} \frac{1}{r} c_{12}^2 - \frac{4}{(\gamma - 1)} \frac{1}{r} c_0 c_{23} = -a u_{24} + \frac{1}{2} b w_{21};
\end{gathered}
$$

Уравнения для u_{2j}, v_{2j}, $j = 0, 1, ..., 4$ можно записать в таком виде:

$$
u_{2jt} + u_0 u_{2jr} = F_{u2j}; \;\; v_{2jt} + u_0 v_{2jr} = F_{v2j}; \;\; j = 0, 1, ..., 4
$$