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Hybrid Numerical Model of Acceleration of Charged Particles on a Shock Wave Front

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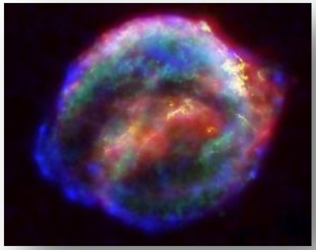

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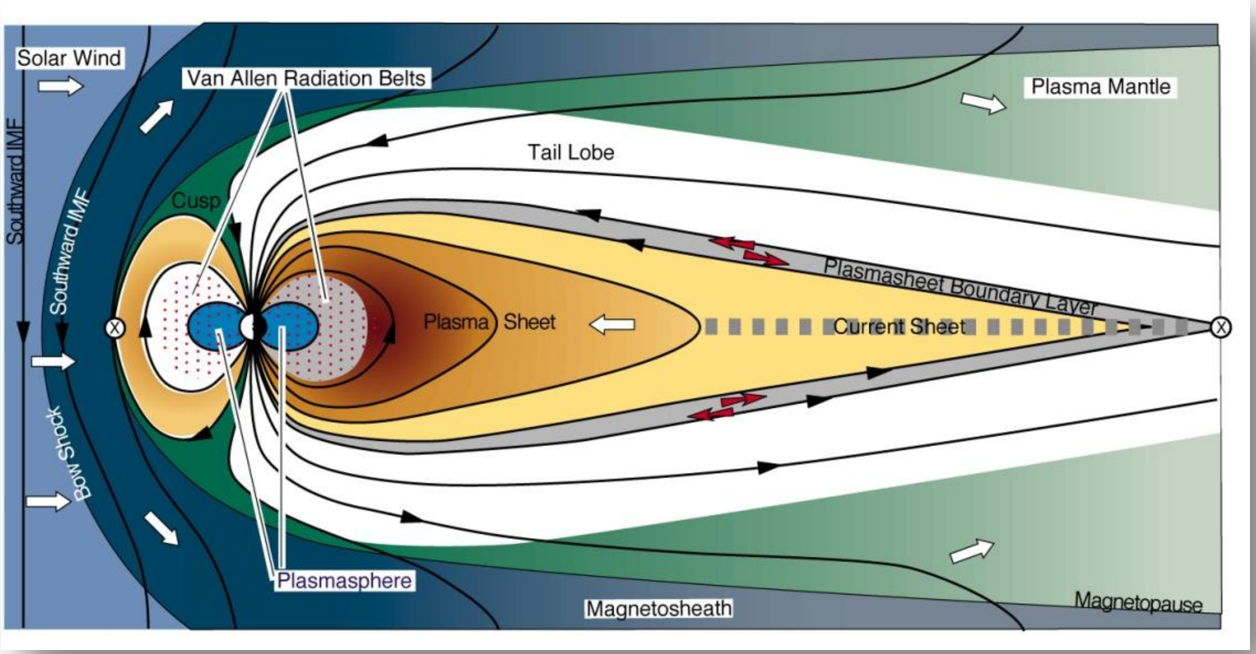

Hybrid models

ions->kinetics
electrons->fluid

Vlasov eq.

MHD

electrons->kinetics
ions->fluid

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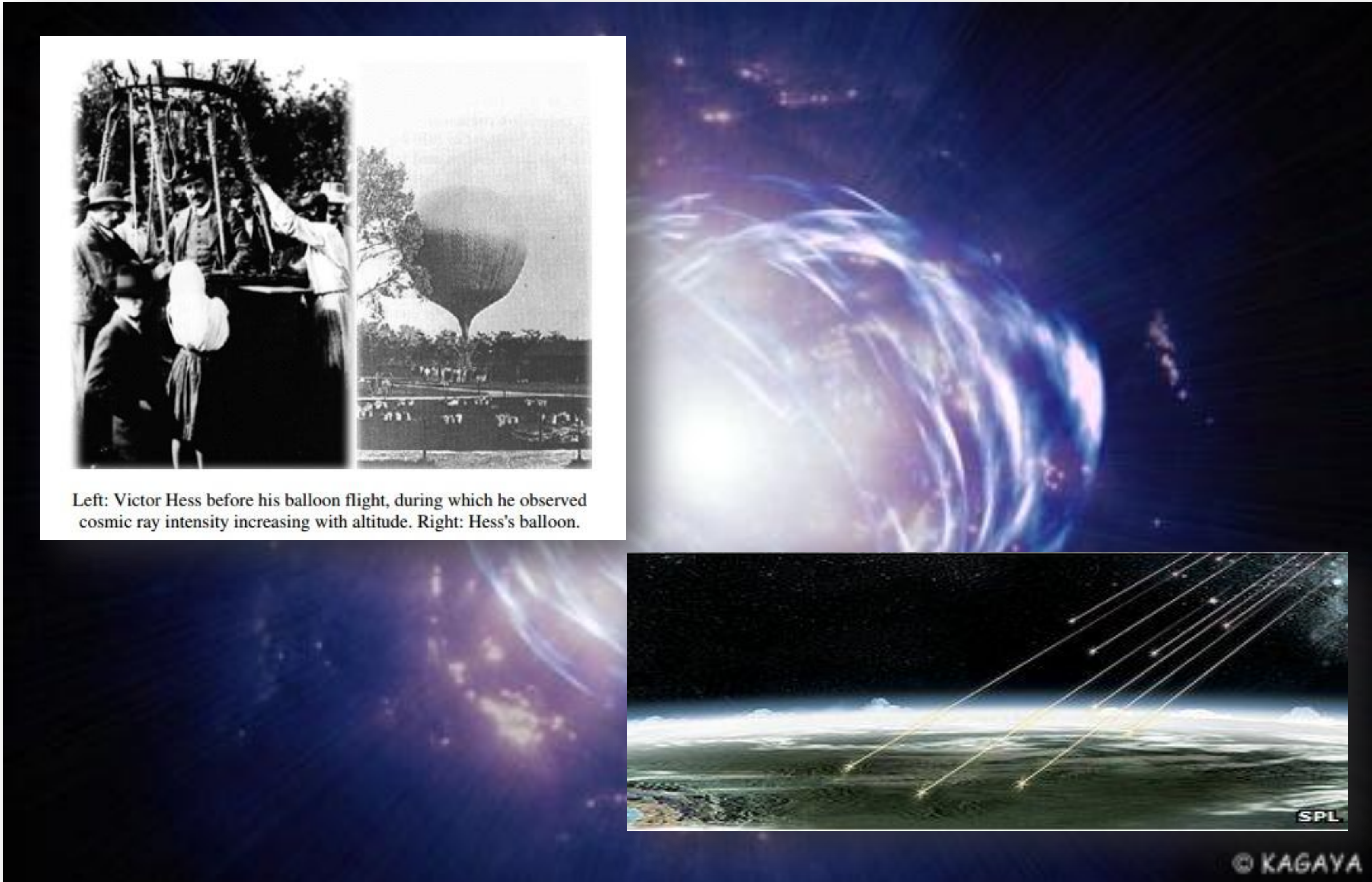
ions	→	kinetics
electrons	→	fluid

Cosmic rays

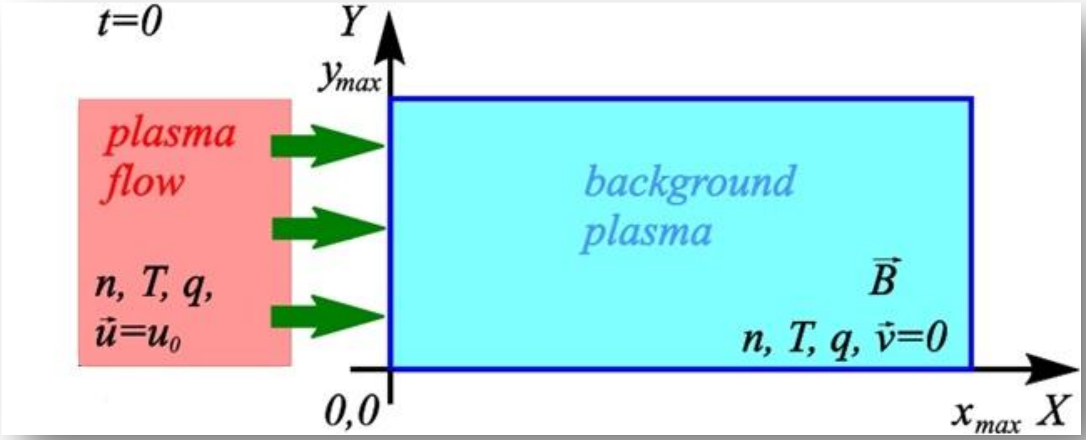


Left: Victor Hess before his balloon flight, during which he observed cosmic ray intensity increasing with altitude. Right: Hess's balloon.

Hybrid numerical model of acceleration of charged particles on a shock wave front

1.1 Geometry and assumptions

- 2D problem in Cartesian coordinates;
- plasma consists of hydrogen ions and electrons;
- uniform magnetic field;
- quasineutral plasma.



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1.2 Equations

Motion equations for ions:

$$\frac{d\vec{r}}{dt} = \vec{u}_\alpha,$$

$$m_\alpha \frac{d\vec{u}_\alpha}{dt} = Z_\alpha e\left(\vec{E} + \frac{1}{c}\vec{u}_\alpha \times \vec{B}\right) + \vec{R}_\alpha.$$

Here Z_α is an ion charge of sort α , $\vec{R}_\alpha$ is a friction force between ions of sort α and electrons.

A density and an average ion velocity:

$$n_\alpha = \int f_\alpha d\vec{u}_\alpha,$$

$$\vec{V}_\alpha = \frac{1}{n_\alpha}\int f_\alpha \vec{u}_\alpha d\vec{u}_\alpha.$$

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1.2 Equations

Motion equations for electrons:

$$m_e\left(\frac{\partial \vec{V}_e}{\partial t}+\left(\vec{V}_e\cdot\nabla\right)\vec{V}_e\right)=-e\left(\vec{E}+\frac{1}{c}\vec{V}_e\times\vec{B}\right)-\frac{\nabla p_e}{n_e}+\vec{R}_e,$$

$$n_e\left(\frac{\partial T_e}{\partial t}+\left(\vec{V}_e\cdot\nabla\right)T_e\right)+\left(\gamma-1\right)p_e\nabla\cdot\vec{V}_e=\left(\gamma-1\right)\left(Q_e-\nabla\cdot\vec{q}_e\right).$$

Maxwell equations:

$$\nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t},\quad \nabla\times\vec{B}=\frac{4\pi}{c}\vec{j},\quad \nabla\cdot\vec{B}=0.$$

Here $\vec{j}$ is a current density which in case of multicomponent plasma is as follows

$$\vec{j}=e\left(\sum_{\alpha}Z_\alpha n_\alpha\vec{V}_\alpha-n_e\vec{V}_e\right).$$

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1.2 Equations

Plasma is quasineutral, i.e. $n_e = \sum_{\alpha} Z_\alpha n_\alpha;$

$\vec{q}_e = -k_1 \nabla T_e$ where k_1 is a coefficient of electron heat conductivity,

$\vec{V} = (V_x, V_y, V_z)$ is an average ion velocity,

$Q_e = \frac{j^2}{\sigma}, \sigma = \frac{n_e e^2}{m_e \nu}$ and friction forces in plasma are

$$\vec{R}_1 = -\frac{Z_1 e}{\sigma}\vec{j},\ \vec{R}_2 = -\frac{Z_2 e}{\sigma}\vec{j},\ \vec{R}_e = -\frac{n_1}{n_e}\vec{R}_1 - \frac{n_2}{n_e}\vec{R}_2.$$

1.3 Normalization

$$n_0,\ B_0,\ V_A = \frac{B_0}{\sqrt{4\pi m_i n_0}},\ T_0 = \frac{B_0^2}{8\pi n_0},\ \omega_{pi} = \sqrt{\frac{4\pi n_0 e^2}{m_i}},\ L = \frac{c}{\omega_{pi}},$$

$$t_0 = \frac{L}{V_A},\quad E_0 = \frac{1}{c} B_0 V_A,\quad \beta = \frac{m_e}{m_i}.$$

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1.4 Equations in dimensionless form

Ion motion equations:

$$\frac{d\vec{r}}{dt} = \vec{u}_\alpha, \quad \frac{M_\alpha}{Z_\alpha}\frac{d\vec{u}_\alpha}{dt} = \vec{E} + \vec{u}_\alpha \times \vec{B} - \zeta \frac{1}{n_e}\nabla \times \vec{B}.$$

Motion equations for electrons:

$$\beta\left(\frac{\partial \vec{V}_e}{\partial t} + \left(\vec{V}_e \cdot \nabla\right)\vec{V}_e\right) = -\vec{E} - \vec{V}_e \times \vec{B} - \frac{1}{2n_e}\nabla p_e + \zeta \frac{1}{n_e}\nabla \times \vec{B},$$

$$n_e\left(\frac{\partial T_e}{\partial t} + \left(\vec{V}_e \cdot \nabla\right)T_e\right) + (\gamma - 1)p_e \nabla \cdot \vec{V}_e = (\gamma - 1)\left(2\zeta \frac{1}{n_e}\left(\nabla \times \vec{B}\right)^2 + \nabla \cdot \left(\zeta_1 \nabla T_e\right)\right).$$

Maxwell equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \nabla \times \vec{B} = \vec{j}, \quad \nabla \cdot \vec{B} = 0.$$

Here $M_\alpha = \frac{m_\alpha}{m_i}, \zeta = \frac{m_e \nu c}{B_0 e}, \zeta_1 = \frac{4\pi e \kappa_1}{B_0 c}.$

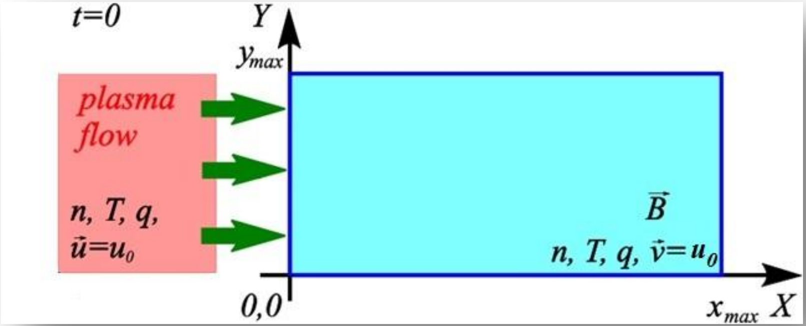
Hybrid numerical model of shock waves in collisionless plasma

1.5 Initial data and boundary conditions

Initial data of the background plasma:

$$t = 0: \quad n = n_0 = const, \quad B_z = B_0 = const, \quad B_x = B_y = 0,$$

$$T = T_0, \quad v_x = u_0, \; v_y = v_z = 0, \quad E_x = E_y = E_z = 0.$$



Solution area:

$$0 \le x \le x_{max}, \quad 0 \le y \le y_{max}.$$

Boundary conditions:

a) <u>for particles:</u> $x = 0 -$ conditions of the incoming flow,

$x = x_{max} -$ conditions of reflection,

$y = 0,\ y = y_{max} -$ periodical conditions;

b) $x = 0,\ x = x_{max}: \; E_x = 0,\ \partial E_y / \partial x = \partial E_z / \partial x = 0,\ \partial n / \partial x = 0;$

$y = 0,\ y = y_{max}:$ periodical conditions.

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1.6 Algorithm

$$\frac{\vec{u}^{m+1}-\vec{u}^{m}}{\tau}=\hat{\vec{E}}^{m}+\vec{u}^{\,m}\times\vec{B}^{m}$$

$$\frac{\vec{r}^{\,m+1}-\vec{r}^{\,m}}{\tau}=\vec{u}^{\,m+1}$$

$$n^{m+1}(\vec{r})=\sum_j q_j\overline{R}\left(\vec{r}-\vec{r}_j^{\,m+1}\right)$$

$$\vec{V}^{m+1}(\vec{r})=\frac{1}{n^{m+1}(\vec{r})}\sum_j q_j\vec{u}_j^{m+1}\overline{R}\left(r-r_j^{m+1}\right)$$

(PIC)

$$\vec{V}_e^{m+1}=\vec{V}^{m+1}-\frac{1}{n^{m+1}}\nabla\times\vec{B}^{m}$$

$$\frac{\vec{B}^{m+1}-\vec{B}^{m}}{\tau}=\nabla\times\left(\vec{V}_e^{\,m}\times\vec{B}^{m+1}\right)-\nabla\times\left(\frac{\zeta}{n_e}\nabla\times\vec{B}^{m+1}\right)+\nabla\times\left[\beta\left(\frac{\partial\vec{V}_e}{\partial t}+\left(\vec{V}_e\cdot\nabla\right)\vec{V}_e\right)+\frac{1}{2n_e}\nabla p_e\right]$$

$$\vec{E}^{m+1}=-\vec{V}_e\times\vec{B}^{m+1}+\frac{\zeta}{n_e}\nabla\times\vec{B}^{m+1}-\beta\left(\frac{\partial\vec{V}_e}{\partial t}+\left(\vec{V}_e\cdot\nabla\right)\vec{V}_e\right)+\frac{1}{2n_e}\nabla p_e$$

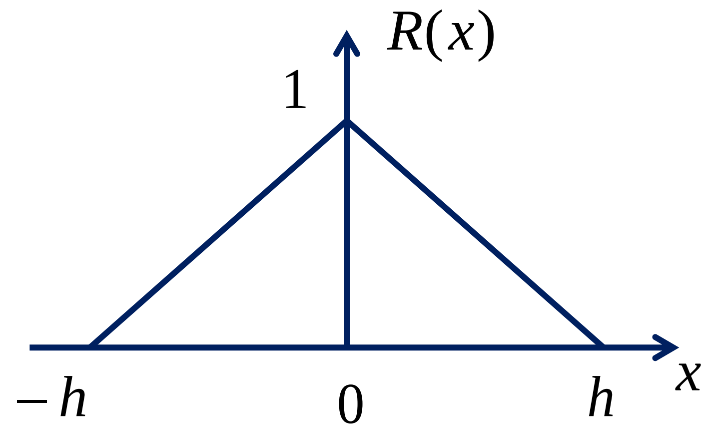
$$\frac{T_e^{\,m+1}-T_e^{\,m}}{\tau}+\left(\vec{V}\cdot\nabla\right)T_e^{\,m+1}+(\gamma-1)n^{m+1}T_e^{\,m+1}\nabla\cdot\vec{V}_e^{\,m+1}=(\gamma-1)\left(Q_e^{m+1}-\nabla\cdot q_e^{\,m+1}\right)$$

Hybrid numerical model of acceleration of charged particles on a shock wave front

1.6 Algorithm

Here $\overline{R}(\vec{r}) = \overline{R}(x)\overline{R}(y)$ and

$$\overline{R}(f) = \begin{cases} 1 - \dfrac{|f|}{h}, & |f| \le h, \\ 0, & |f| > h, \quad where \quad f = \{x, y\}. \end{cases}$$



There is a uniform mesh with steps h_1, h_2 by axis x, y, respectively. Grid functions are set as follows:

$$B_z \Rightarrow \left(x_i = ih_1,\ y_k = kh_2\right),$$
$$V_x, V_{ex}, E_x, B_y \Rightarrow \left(x_i,\ y_{k-1/2} = (k - 0.5)h_2\right),$$
$$V_y, V_{ey}, E_y, B_x \Rightarrow \left(x_{i-1/2} = (i - 0.5)h_1,\ y_k\right),$$
$$n, T_e, V_z, V_{ez}, E_z \Rightarrow \left(x_{i-1/2},\ y_{k-1/2}\right).$$

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1.7 Results

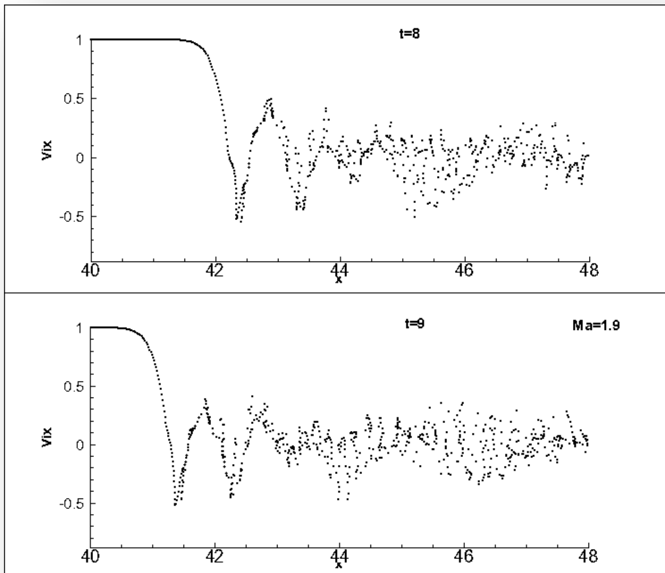


Fig. 1. Phase space of ions at times t=8.0 (a), 9.0 (b).
Here $v_0 = 1,\ M_A = 1.9.$

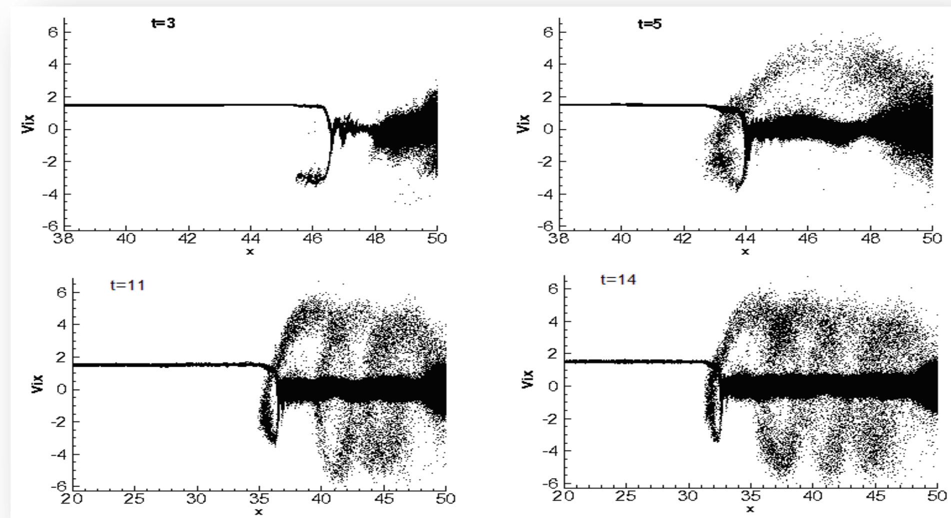


Fig. 2. Phase space of ions at times t=3, 5, 11, 14.
Here $v_0 = 1.5,\ M_A = 2.8$. The ion reflection by a shock wave along with the ion rotation on the Larmor radius and the quasistationary structure formation due to the continuous reflection of the oncoming plasma flow by the shock wave and its rotation in the external magnetic field.

Hybrid numerical model of acceleration of charged particles on a shock wave front

1.7 Results

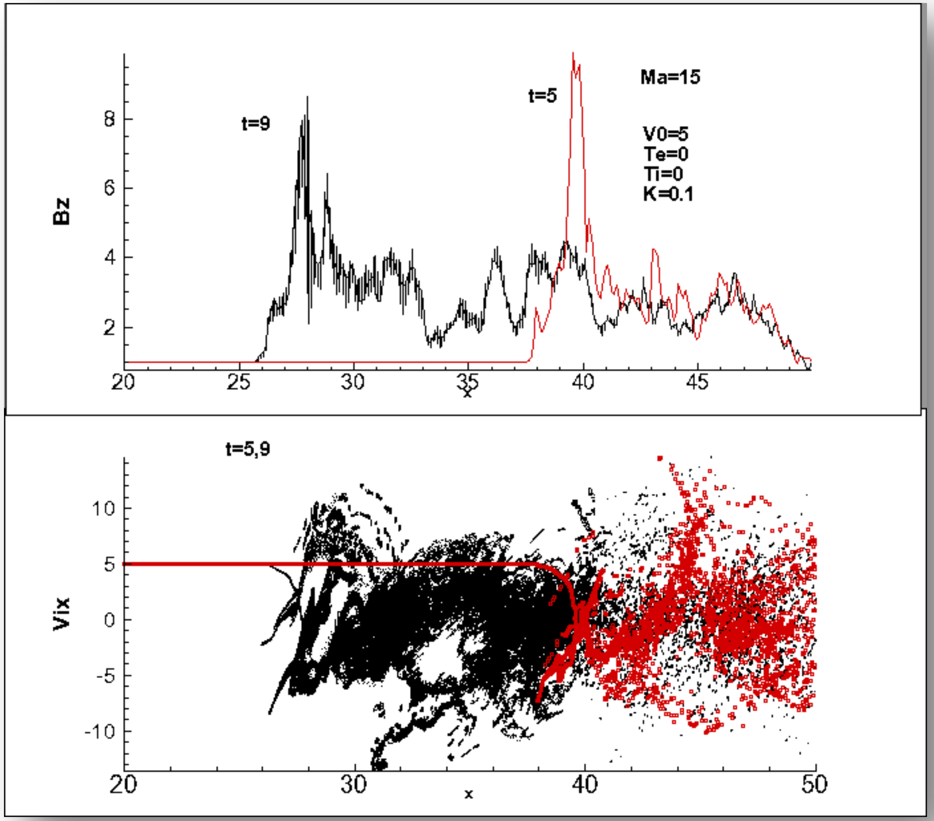


Fig. 3. Magnetic field B_Z and phase space of ions at times $t = 5, 9,\ v_0 = 5,\ M_A = 15.$

Hybrid numerical model of acceleration of charged particles on a shock wave front
1.7 Results

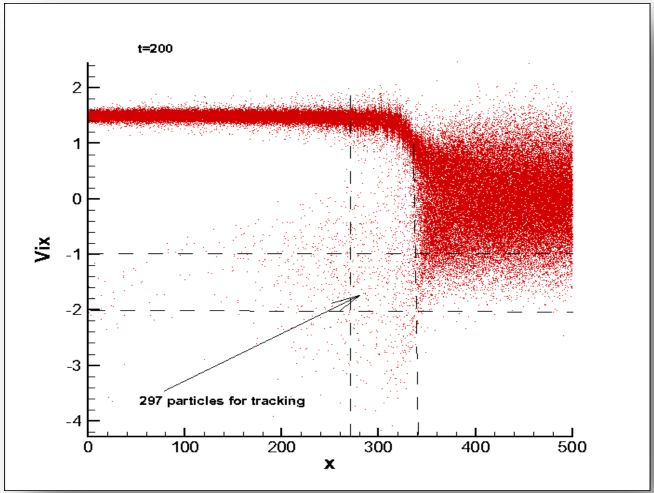


Fig. 4. Phase space of ions at time t=200.

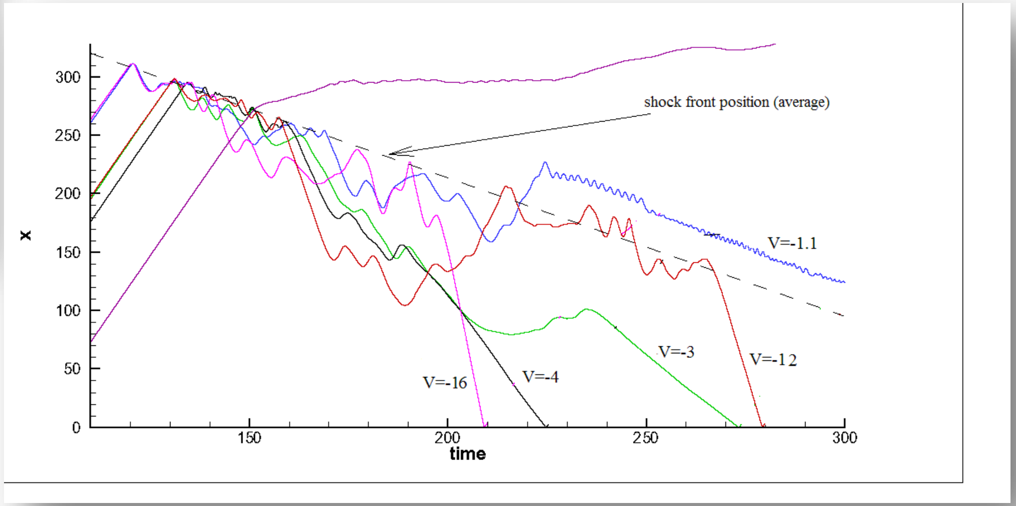


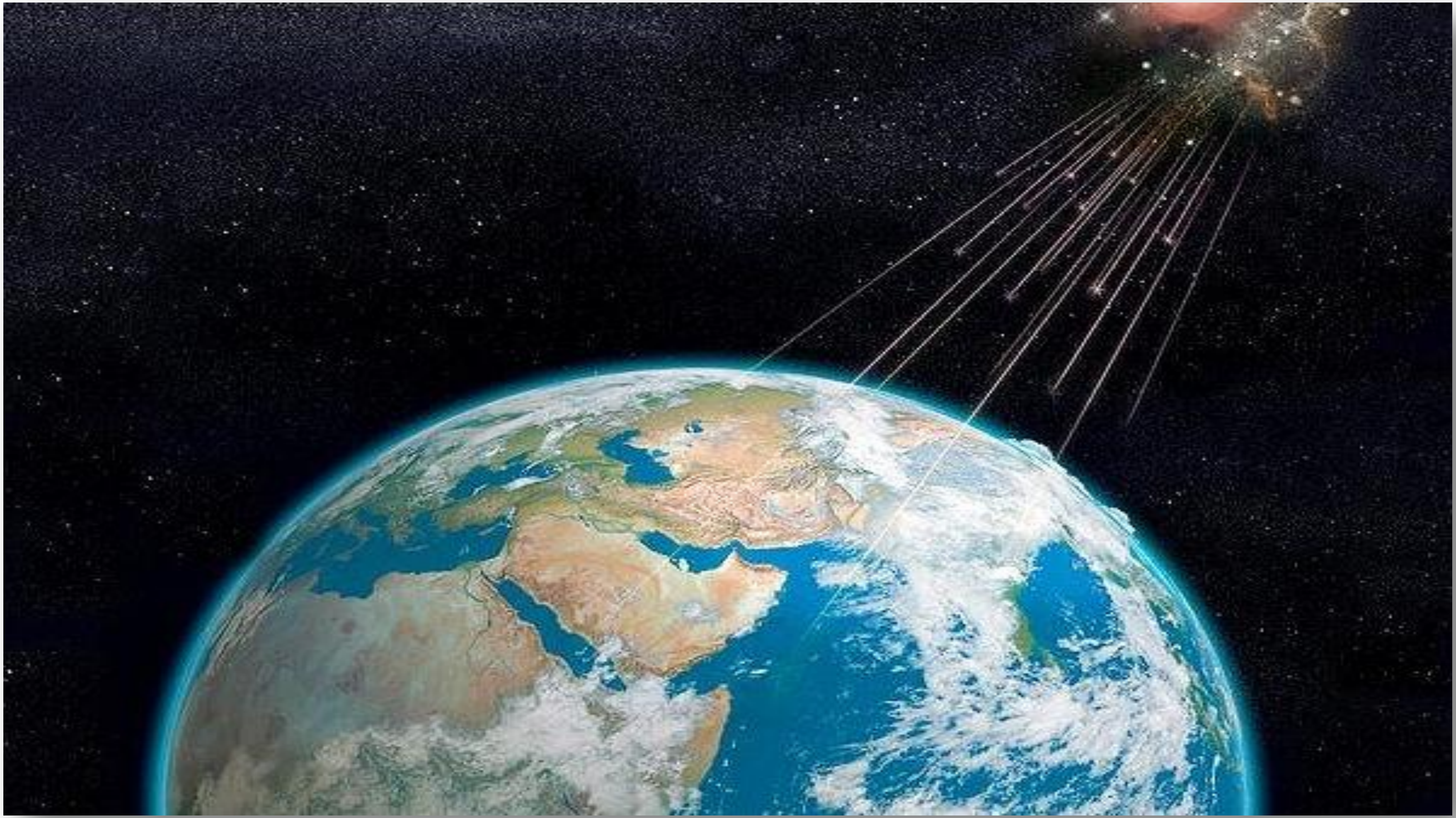
Fig. 5. Particle trajectories.

Conclusion

- the new 2D numerical model of the shock wave generation and structure to study the acceleration mechanism of cosmic ray particles on its front has been made;
- the new numerical algorithm has been developed;

- the further modification of the model to solve problems which are close to the real studied phenomena (acceleration of cosmic rays) is in future plans.

Thank you for your attention!



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1.7 Results