Numerical methods of solving inverse problems for hyperbolic equations


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Содержание

Problems formulation

Differential: $L_q u = g,\ Pu = f$

Finite-difference: $L_q^h u^h = g^h,\ P^h u^h = f^h$

Operator: $A(q) = f$

Variational: $J(q) = \|A(q) - f\|^2$

Theoretical results: uniqueness theorems, conditional stability estimates.

Kabanikhin S.I., Satybaev A.D., Shishlenin M.A. *Direct Methods of Solving Inverse Hyperbolic Problems*. VSP, The Netherlands, 2004.

Kabanikhin S.I., Iskakov K.T., Bektemesov M.A., Ayapbergenova A.T. *Iterative Methods of Solving Inverse Hyperbolic Problems*. VSP, The Netherlands, 2004.

Содержание

Numerical methods:

Finite-difference scheme inversion;

Linearization $q_1 = [\, A' (q_0)\,]^{-1} f_1$;

Newton-Kantorovich method $q_{n+1} = q_n - [A'(q_n)]^{-1} (A(q_n) - f)$;

Gradient methods:

- Landweber iteration $q_{n+1}= q_n - \alpha [A'(q_n)]^* (A(q_n) - f)$;

- Steepest descent $q_{n+1} = q_n - \alpha_n J'(q_n)$;

Gelfand-Levitan-Krein-Marchenko method;

Boundary control method.

Inverse problems for hyperbolic equations

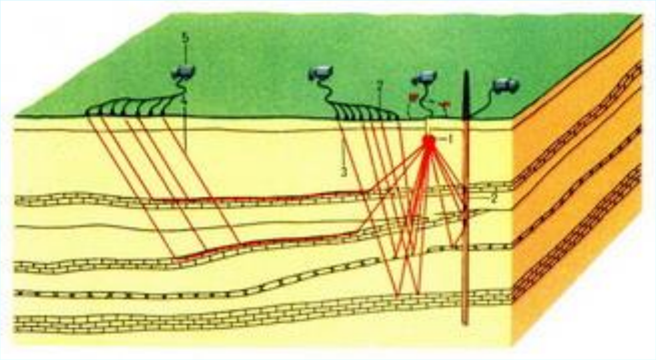
Hyperbolic equations describing the wave processes are of great concern in many domains of applied mathematics.

Waves comes through object and deliver information about its structure to the surface.

Solutions of hyperbolic equations can contain non-smooth and singular components. This leads to easier (compared with elliptic and parabolic cases) inversion of the operator.

Usually inverse problems for hyperbolic equations are solved by minimizing the residual functional. Iterative method of minimizing the functional requires the solution of the direct (and, perhaps, adjoint) problem for every iteration of the method.

In multidimensional case iterative methods for multidimensional inverse problems are very timeconsuming.

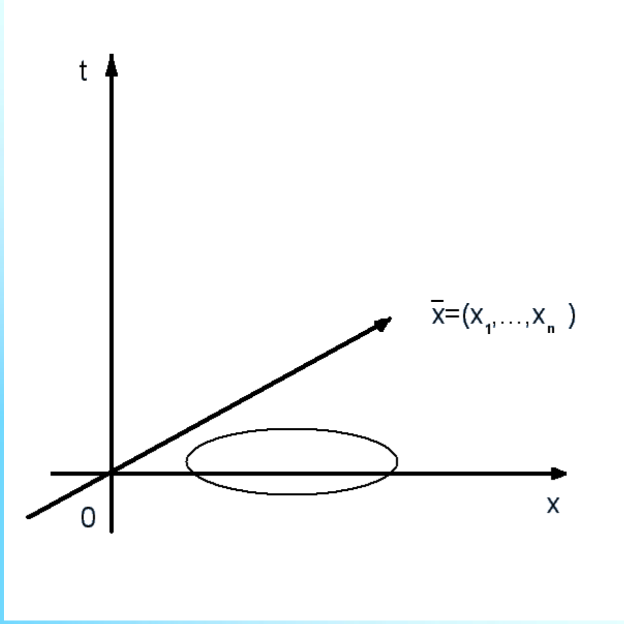


Forward (Direct) Problem

1) $$c^{-2}(x,y)v_{tt} = \Delta v - \nabla \ln \rho(x,y)\cdot\nabla v,$$
$$y \in R^{n-1}, \quad x>0, \quad t>0;$$

2) $$v|_{t<0} \equiv 0;$$

3) $$v_x(+0,y,t) = h(y)\cdot\delta(t), \quad y\in R^n, t\in R$$



$c(x,y) \geq c_0 > 0$ (c_0 = const) is the velocity;
$\rho(x,y) \geq \rho_0 > 0$ (ρ_0 = const) is the density;
$v(x,y,t)$ is the exceeded pressure.

Inverse Problem: find the coefficients of equation (1) using additional information:

(4) $$v(+0,y,t) = f(y,t), \quad y\in R^n, t\in R.$$

Finite-difference scheme inversion

The main idea of the finite-difference scheme inversion consists in the following: the inverse problem is replaced by a finite-difference analogue and reduce to the system of nonlinear algebraic equations.

The method of inversion of finite difference schemes is quite natural from a physical point of view because it uses the theory of characteristics along which extends, as a rule, basic information about the features of the solution of the direct problem and of the investigated medium.
In the computational aspect (number of operations) method for the finite-difference scheme inversion is equivalent to a solution of the corresponding direct problem and allows the parallelization of the procedure of calculations.

Based on the projection method for the finite-difference scheme inversion can be generalized to a wide class of multidimensional inverse problems, in the case of sufficiently smooth with respect to horizontal variables.

The main disadvantage of the method: the inverse problem is not stable with depth in the case of large measurement errors in the data.

Finite-Difference Statement of the Problem

Let us denote $u_i^k = u(ih, kh)$, $q_i = q(ih)$.
Then inverse problem (1) - (4) can be written in discrete form:

$$\frac{u_i^{k+1} - 2u_i^k + u_i^{k-1}}{h^2} = \frac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{h^2} - q_i \cdot \frac{u_{i+1}^k - u_{i-1}^k}{2h} + o\left(h^2\right);$$

$$u_i^i = s_i; \quad u_0^k = f^k;$$

$$u_1^k = \frac{u_0^{k+1} + u_0^{k-1}}{2} + o\left(h^2\right);$$

$$s_{i+1} = \frac{h}{4} \cdot \left(s_0 \cdot a_0 + s_{i+1} \cdot q_{i+1}\right) + \frac{h}{2} \sum_{j=1}^{i} s_j \cdot q_j + o\left(h^2\right).$$

Then we can formulate the inverse problem in finite-difference statement:

$$v_i^{k+1} + v_i^{k-1} = v_{i+1}^k + v_{i-1}^k - Q_i \cdot h \cdot \frac{v_{i+1}^k - v_{i-1}^k}{2};$$

$$v_i^i = p_i; \quad v_0^k = f^k;$$

$$v_1^k = \frac{v_0^{k+1} + v_0^{k-1}}{2};$$

$$p_{i+1} = \frac{h}{4} \cdot \left(p_0 \cdot A_0 + p_{i+1} \cdot Q_{i+1}\right) + \frac{h}{2} \sum_{j=1}^{i} p_j \cdot Q_j .$$

We have to determine v_i^k and p_i.

Finite-Difference Scheme Inversion

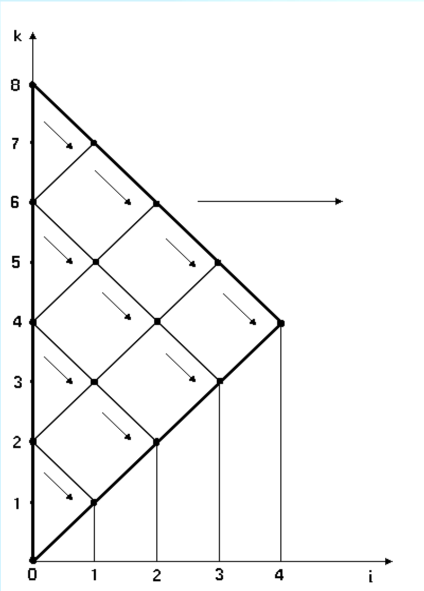
Now we consider the finite-difference statement of the inverse acoustic problem.

$$v_i^{k+1} + v_i^{k-1} = v_{i+1}^k + v_{i-1}^k - A_i \cdot h \cdot \frac{v_{i+1}^k - v_{i-1}^k}{2};$$

$$v_i^i = p_i;$$

$$v_0^k = f^k; \quad v_1^k = \frac{v_0^{k+1} + v_0^{k-1}}{2};$$

$$p_{i+1} = \frac{h}{4} \cdot (p_0 \cdot A_0 + p_{i+1} \cdot A_{i+1}) + \frac{h}{2} \sum_{j=1}^{i} p_j \cdot A_j .$$



The algorithm of finite-difference scheme inversion is following:

1) Find v_0^k and v_1^k using (4) and (5) respectively.
2) Determine p_0 and p_1 from (2). After we put $A_0 = 2(p_1-p_0)/(hp_0)$.

2) Determine A_i for $i \geq 0$: $$A_{i+1} = \frac{4}{h} - 4 \cdot \frac{p_i}{h \cdot p_{i+1}} \cdot \left(1 + \frac{h}{4} A_i\right);$$

3) Find v_{i+1}^k by known A_i: $$v_{i+1}^k = v_i^{k+1} + v_i^{k-1} - v_{i-1}^k + A_i \cdot h \cdot \frac{v_{i+1}^k - v_{i-1}^k}{2};$$

4) Suppose $p_{i+1} = V_{i+1}^{i+1}$;

5) Find A_{i+1} and so on...

Gel’fand-Levitan-Krein-Marchenko method

Advantages:

This method overcomes nonlinearity of the problems – the nonlinear inverse problem reduces to a system of linear integral equations

GLKM method in some sense is the direct method – there is no need to solve the forward problem (no iteration process)

Short history

I.M. Gel’fand and **B.M. Levitan** (1951), **M.G. Krein** (1954) – first results (spectral inverse problems)

V.A. Marchenko (1950-ies) – inverse scattering problem

A.S. Alekseev (1960-ies) – inverse seismic problem (**A.S. Blagoveschenskiy, V.I. Dobrinskiy, B. Gopinath, M. Sondhi, R. Burridge, W.W. Symes, e.t.c.**)

M.I. Belishev (1987), **S.I. Kabanikhin** (1988) – two-dimensional

Gel’fand-Levitan-Krein-Marchenko method

- **Acoustics**

Multidimensional nonlinear acoustic inverse problem (S.I. Kabanikhin, A.D. Satybaev, M.A. Shishlenin, 2004)

- **Seicmics**

Recovering of the Lame parameters and density of the medium

(A.S. Alekseev, 1967; V.S. Belonosov, A.S. Alekseev, 1998)

- **Scattering, tomography, optics, e.t.c.**

Method of inverse scattering problem: integrating nonlinear equations (C. S. Gardner, J. M. Greene, M. D. Kruskal and R. M. Miura, 1967): KdF (1D) and Kadomtcev-Petviashvili (2D, V.E. Zakharov and A.B. Shabat, 1974).

Solving the GLM-equations for obtaining the solution of the nonlinear Schrodinger equation (D.A. Shapiro, 2011; R.G. Novikov, 2014; S.K. Turitsyn, 2015).

Gel'fand-Levitan-Krein method

Let us derive 2D analog of Gel'fand-Levitan-Krein equation (Kabanikhin (1988), Kabanikhin and Lorenzi (1999)).
We consider the family of direct problems ($k \in \boldsymbol{Z}$)

$$u_{tt}^k = \Delta u^k - \nabla \ln \rho(x,y) \nabla u^k, \quad x > 0, \quad y \in R, \quad t > 0;$$

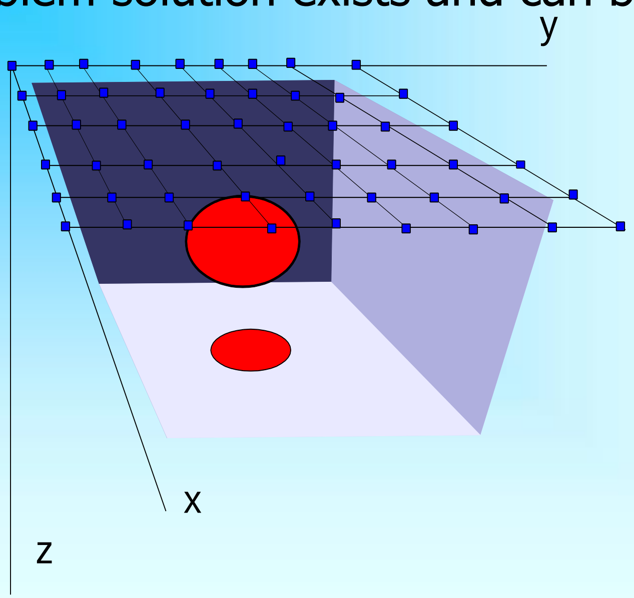
$$u^k |_{t<0} \equiv 0;$$

$$u_x^k(+0, y, t) = e^{iky} \cdot \delta(t);$$

$$u^k |_{y=\pi} = u^k |_{y=-\pi} .$$

We suppose that the trace of forward problem solution exists and can be measured

$$u^k(+0, y, t) = f^k(y, t).$$



Метод Гельфанда-Левитана-Крейна

According to Kabanikhin (1977) we define the auxiliary family of forward problems ($m \in \mathbf{Z}$)

$$w_{tt}^m = \Delta w^m - \nabla \ln \rho(x,y) \nabla w^m, \quad x > 0, \, y \in R, \, t > 0;$$

$$w^m(0,y,t) = e^{imy} \cdot \delta(t); \quad w_x^m(0,y,t) = 0.$$

Solution can be represented in the form:

$$w^m(x,y,t) = S^m(x,y) \cdot \left[\delta(x+t) + \delta(x-t)\right] + \breve{w}^m(x,y,t), \quad S^m(x,y) = \frac{1}{2}\sqrt{\frac{\rho(x,y)}{\rho(0,y)}} \cdot e^{imy}.$$

Solutions of initial direct and auxiliary problems are connected with the following equality

$$u^k(x,y,t) = \sum_m \int_0^t f_m^k(t-s) w^m(x,y,s) ds.$$

Let us denote $\Phi^m(x,t) = \int_{-\pi}^{\pi} \int_{o}^{x} \frac{w^m(\xi,y,t)}{\rho(\xi,y)} d\xi \, dy.$

Therefore initial inverse problem can be reduced to
the multidimensional analog of Gelfand-Levitan-Krein equation

The multidimensional analog of Gelfand-Levitan-Krein (GLK) equation

$$2\Phi^{k}(x,t)-\sum_{m}\int_{-x}^{x} f_{m}^{k\,\prime}(t-s)\Phi^{m}(x,s)\,ds=-\int_{-\pi}^{\pi}\frac{e^{iky}}{\rho(0,y)}dy,$$

$$|t|<x,\quad k=0,\pm1,\pm2,\dots$$

The solution of inverse problem can be obtained from the solution of Gelfand-Levitan-Krein equation by formula

$$\rho(x,y)=-\frac{\pi^{2}}{\rho(0,y)}\left[\sum_{m}\Phi^{(m)}(x,x-0)\,e^{-i(m,y)}\right]^{-2}$$

Therefore in order to find solution $\rho(x,y)$ in the depth x_0 we can solve GLK equation with the fixed parameter x_0 and then calculate $\rho(x_0,y)$.

N- approximation of M.G. Krein equation

$$2\Phi(x,t)-\sum_{|m|\le N}\int_{-x}^{x}F(t-s)\Phi(x,s)ds=G,\qquad t\in(-x,x),k_j=-\overline{N,N},j=1{,}2.$$

Here $\Phi(x,t)=\left(\Phi^{(-N)}(x,t),\dots,\Phi^{(0)}(x,t),\dots,\Phi^{(N)}(x,t)\right)^T$,

$G=\left(G^{(-N)},\dots,G^{(0)},\dots,G^{(N)}\right)^T$ и $G^{(k)}=-\int_{-\pi}^{\pi}\frac{e^{i(k,y)}}{\rho(0,y)}dy$.

$$F(t)=\begin{vmatrix} {f_{-N}^{(-N)}}' & {f_{-N+1}^{(-N)}}' & \dots & {f_{0}^{(-N)}}' & \dots & {f_{N}^{(-N)}}' \\ {f_{-N}^{(-N+1)}}' & {f_{-N+1}^{(-N+1)}}' & \dots & {f_{0}^{(-N+1)}}' & \dots & {f_{N}^{(-N+1)}}' \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ {f_{-N}^{(0)}}' & {f_{-N+1}^{(0)}}' & \dots & {f_{0}^{(0)}}' & \dots & {f_{N}^{(0)}}' \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots \\ {f_{-N}^{(N)}}' & {f_{-N+1}^{(N)}}' & \dots & {f_{0}^{(N)}}' & \dots & {f_{N}^{(N)}}' \end{vmatrix}$$

Boundary control method

The Boundary Control method is an approach to solving inverse problems based on the control theory and systems. Method is justified on the basis of Riemannian geometry, asymptotic methods for PDE, functional analysis and operator equations.

The method was proposed in 1987.

The dynamic variant of the method of boundary control is considered in the time domain, which is a response of the operator (hyperbolic variant of the Dirichlet-to-Neumann map).

The method provides optimal recovery time: the longer the observation time, the larger the area in which the parameters will be restored.

This feature makes it the option most relevant in acoustics and Geophysics. The algorithm was developed to recover the speed of the propagation of the waves.

Boundary Control Method

We consider the family of inverse problems ($k \in \boldsymbol{Z}$)

$$u_{tt}^k = \Delta u^k - \nabla \ln \rho(x) \nabla u^k, \quad x > 0, \quad y \in R, \quad t > 0;$$

$$u^k \,|_{t<0} \equiv 0;$$

$$u_x^k(+0, y, t) = e^{iky} \cdot \delta(t);$$

$$u^k \,|_{y=\pi} = u^k \,|_{y=-\pi};$$

$$u^k \,|_{x=0} = f^k(y,t).$$

The solution of forward problem can be connected with solution to the forward problem with arbitrary source $u_x^g \,|_{x=0} = g(y,t)$ is following:

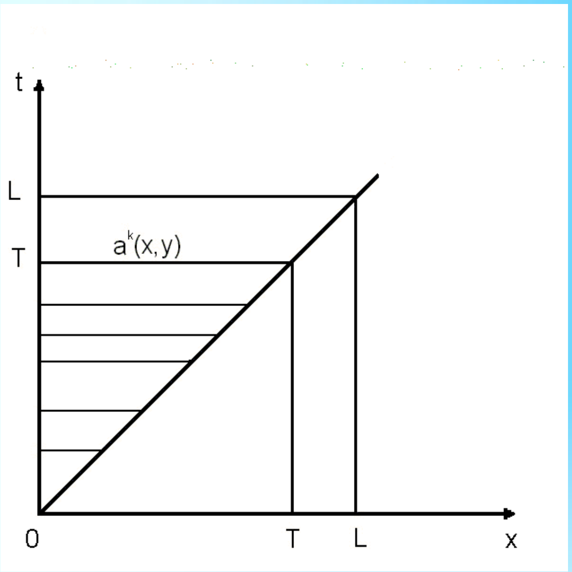
$$u^g(x,y,t) = \sum_k \int_0^t u^k(x,y,t-s) \cdot g_k(s) ds.$$

Boundary Control Method

Let us consider the arbitrary function *a(x,y)* on $x\in[0,T]$, $y\in[-\pi, \pi]$ supposing $a(x,y)=0$ for $x\in(T,L]$, $y\in[-\pi, \pi]$.

The problem of control: find the source *g(y,t)* such that

$$u^g(x,y,T)=a(x,y),\quad x\in[0,T],\quad y\in[-\pi,\pi].$$



Sources *g(y,t)* are considered as functions of $L_2([0,T]\times[-\pi, \pi])$ and functions defined on wave – as elements of space *H*, where inner product in *H* is defined

$$\left(u^g(x,y,T),u^h(x,y,T)\right)_H=\int_{-\pi}^{\pi}\int_0^L\frac{u^g(x,y,T)\cdot u^h(x,y,T)}{\rho(x,y)}dx\,dy.$$

Boundary Control Method

Let $\{g_p(y,t)\}$, $p=1,\ldots,\infty$ is basis in $L_2([0,T]\times[-\pi,\pi])$. Then any sources can be represented uniquely in series form. Further let $u_p(x,y,T)=u^{g_p}(x,y,T)$.
Then $u_p(x,y,T)$ is basis in H, i.e. any function in H can be presented

$$a(x,y)=\sum_{p=1}^{\infty}\alpha_p u_p(x,y,T).$$

Approximate solution of boundary control problem with defined function $a(x,y)$ is calculated with using finite system of sources $\{g_p(y,t)\}$, $p=1,\ldots,N$.

We consider the minimization problem of the discrepancy with respect $\alpha_1,\ldots,\alpha_N$

$$\left\| a-\sum_{p=1}^{N}\alpha_p u_p(x,y,T)\right\|$$

The minimizer $\alpha=(\alpha_1,\ldots,\alpha_N)$ is solution to the system of algebraic equations

$$\sum_{n=1}^{N}\Gamma_{jn}\alpha_n=b_j,\quad j=\overline{1,N};$$

where $\Gamma_{jn}=\left(u_j(x,y,T),u_n(x,y,T)\right)_H,\quad b_j=\left(a(x,y),u_j(x,y,T)\right)_H.$

We define the approximate solution of boundary control problem

$$f_N^k(y,t)=\sum_{p=1}^{N}\alpha_p g_p(y,t).$$

Boundary Control Method

Then we write the following relations

$$\|a\|_H^2 \approx \|a_N\|_H^2 = \left(\sum_{j=1}^{N} \alpha_j u_j(x,y,T), \sum_{p=1}^{N} \alpha_p u_p(x,y,T) \right)_H = \sum_{j=1}^{N} \alpha_j b_j,$$

Let us consider the quantity $w^{hg}(t,s) = \int_{-\pi}^{\pi} \int_{0}^{L} \frac{u^h(x,y,t) \cdot u^g(x,y,s)}{\rho(x,y)} dxdy \quad (30)$

For *0<t, s<L* we obtain (31):

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial s^2} \right] w^{hg}(t,s) = -\int_{-\pi}^{\pi} \frac{1}{\rho(0,y)} \left[g(y,s) \int_0^t \sum_m f^m(y,t-\eta) h_m(\eta)\, d\eta - h(y,t) \int_0^s \sum_m f^m(y,s-\eta) g_m(\eta)\, d\eta \right]$$

From initial data it follows that $w^{hg}(0,s) = 0, \quad w_t^{hg}(0,s) = 0, \quad w^{hg}(t,0) = 0. \quad (32)$

Using D'Alambert formula we obtain the solution to (31), (32). Then for *t=s=T*

$$w^{hg}(T,T) = -\int_{-\pi}^{\pi} \frac{1}{\rho(0,y)} \sum_m f^m(y,+0) \int_0^T \int_0^{T-\tau} h(y,\xi) d\xi \int_0^{T-\tau} g(y,\eta) d\eta d\tau dy \quad (33)$$

$$- \int_{-\pi}^{\pi} \frac{1}{\rho(0,y)} \int_0^T \int_0^{T-\tau} g(y,\eta') d\eta' \int_0^T \sum_m \left[f^{m'}(y,\tau+\eta) + f^{m'}(y,|\tau-\eta|) \right] \int_0^{T-\tau} h(y,\xi) d\xi d\tau d\eta dy.$$

Boundary Control Method

Let $\quad a^k(x,y) = e^{iky}\cdot\theta(T-x) \quad \Rightarrow \quad \frac{d}{dT}\left\|a^k\right\|_H^2 = \int_{-\pi}^{\pi}\frac{e^{iky}dy}{\rho(T,y)}.$

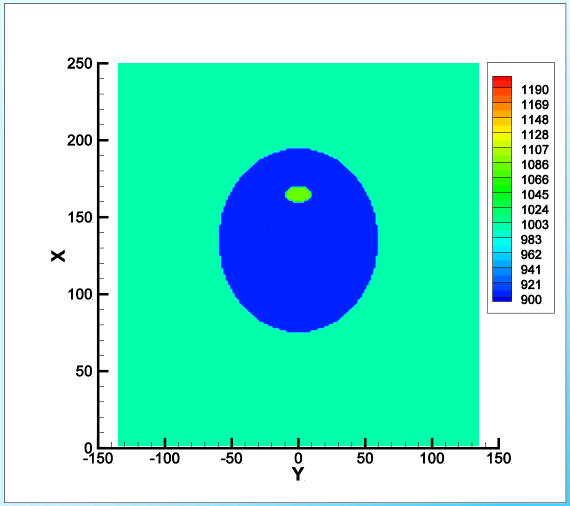
One can show that coefficients of matrix Γ and components of vector b are defined *via* $\{ g_k \}$:

$$\Gamma_{jn} = -\int_{-\pi}^{\pi}\frac{1}{\rho(0,y)}\sum_m f^m(y,+0)\int_0^T\int_0^{T-\tau} g_j(y,\xi)d\xi\int_0^{T-\tau} g_n(y,\eta)d\eta d\tau dy \qquad (34)$$

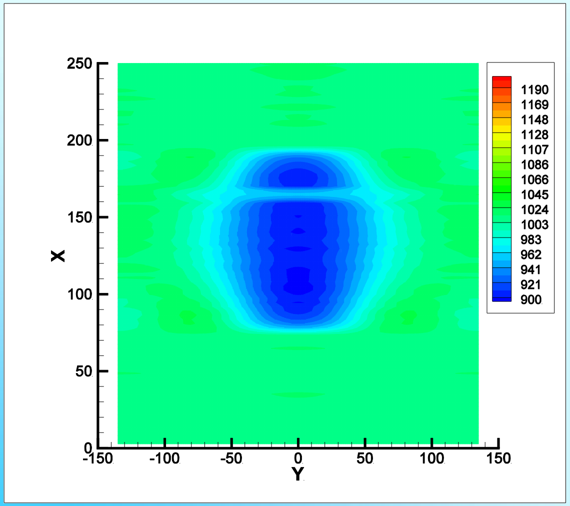
$$-\int_{-\pi}^{\pi}\frac{1}{\rho(0,y)}\int_0^T\int_0^{T-\tau} g_j(y,\eta')d\eta'\int_0^T\sum_m\left[f^{m'}(y,\tau+\eta)+f^{m'}(y,|\tau-\eta|)\right]\int_0^{T-\tau} g_n(y,\xi)d\xi d\tau d\eta dy.$$

$$b_j = \left(a,u_j\right)_H = -\frac{1}{2\pi}\int_{-\pi}^{\pi}\int_0^T (T-t)\frac{g_j(y,t)}{\rho(0,y)}dtdy.$$

$$\int_{-\pi}^{\pi}\frac{e^{iky}dy}{\rho(T,y)} \approx \left(\frac{d}{dT}\left[\sum_{n=1}^N \alpha_n^k\cdot b_n^k\right]\right)^{-1}.$$



Exact



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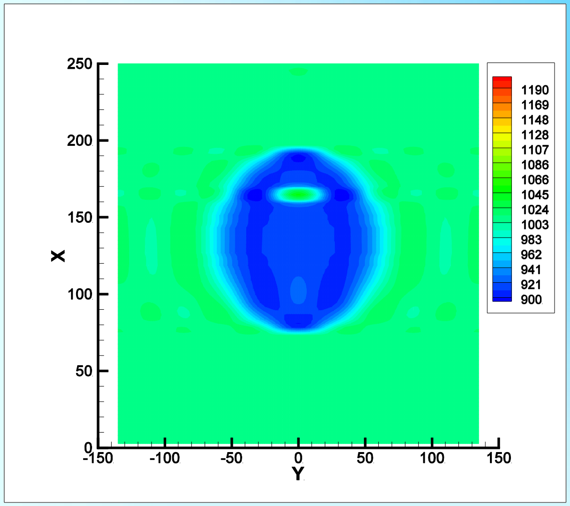

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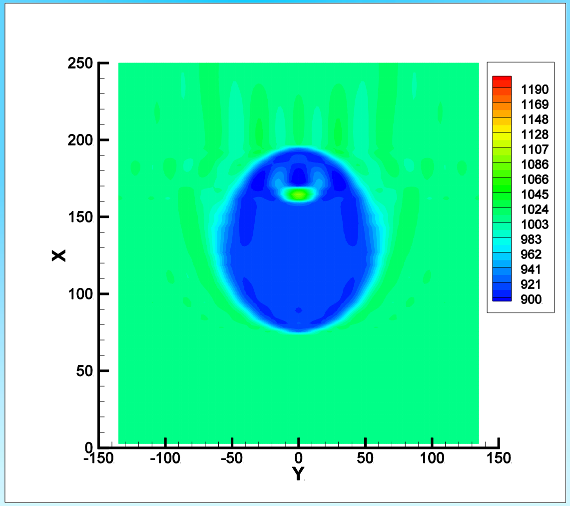
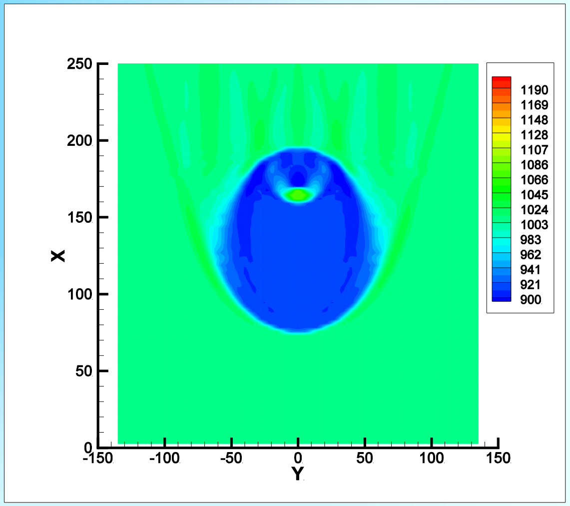


10 observers



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Thank you for your attention!

Conclusion and Remarks

- FDSI gives exact solution for exact data. The method is not sufficiently stable in the case of large measurement errors in the data of the inverse problem.
- NK converges very fast but the initial approximation has to be in the close neighborhood of the exact solution.
- The BC-method and GLK method determine the solution of inverse problem in particular point x_0 in depth without any special calculations of unknown coefficients on the interval $(0, x_0)$.
- BC method allows to define unknoun velocity of wave propagation $c(x,y)$ if the density $\rho(x,y)$ is known.
- In the case of large measurement errors in the data or for sufficiently large domain:

1) find the solution in points $x_1,\dots,x_n$ using direct methods (BC and GLK);
2) calculate solution by NK, O, SD with fixed $x_1,\dots,x_n$.

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Introduction

В настоящее время, благодаря площадным системам наблюдений, удалось создать принципиально новый метод решения трехмерных обратных задач, в котором используются:

- трехмерные аналоги уравнений Гельфанда-Левитана-Крейна,
- параллельные вычисления на высокопроизводительных кластерах,
- методы Монте-Карло,
- супербыстрые алгоритмы обращения блочно-теплицевых матриц больших размерностей.

Основной проблемой исследования трехмерных упругих сред является большой размер области, в которой необходимо производить высокоточные вычисления.

Даже для сравнительно небольшого участка 2 км × 2 км × 2 км решение прямой задачи сейсморазведки является очень сложной проблемой, а если учесть, что большинство современных методов решения обратных задач основаны на итерационных процедурах, то даже количество операций, требуемых для проведения нескольких итераций, может привести к неконтролируемым ошибкам.

Это обстоятельство осложняется сильной некорректностью обратных задач, которое заключается в неединственности решения, а также в неустойчивости, которая сильно возрастает с глубиной.