Institute of Theoretical and Mathematical Physics

Russian Federal Nuclear Center -

VNIIEF

Analytical and Numerical Solutions of the Gravitational Light Layer Mixing Problem Using the k- ε Turbulence Model

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On the k-ε Model in EGAK code

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_k}(\rho k u_k)=(G_1+G_2)-\rho\varepsilon+\frac{\partial}{\partial x_k}\left(\frac{\rho D}{\sigma_k}\cdot\frac{\partial k}{\partial x_k}\right)$$

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\frac{\partial}{\partial x_k}(\rho\varepsilon u_k)=\frac{\varepsilon}{k}(c_{\varepsilon1}G_1+c_{\varepsilon3}G_2-c_{\varepsilon2}\rho\varepsilon)+\frac{\partial}{\partial x_k}\left(\frac{\rho D}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial x_k}\right)$$

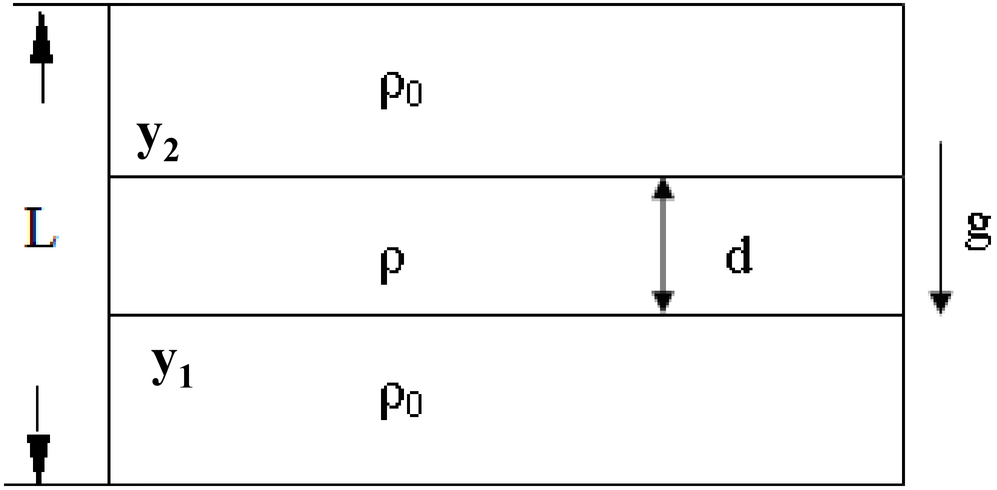
$$D=c_D\frac{k^2}{\varepsilon}$$

Coefficients: $c_D, \sigma_m, \sigma_h, \sigma_k, \sigma_\varepsilon, c_{\varepsilon1}, c_{\varepsilon2}, c_{\varepsilon3}$

	c_D	σ_k	σ_ε	$c_{\varepsilon1}$	$c_{\varepsilon3}$	σ_h	σ_m	$c_{\varepsilon2}$
Standard (1-4)	0.09	1	1.3	1.44	1.44	0.9	0.9	1.92
EGAK (5)	**0.12**	**3/4**	**3/4**	**1.15**	**1**	**1/1.7**	**1/1.7**	**1.7**

1. Launder, B.E., and Spalding, D.B. // Comp. Meth. In Appl. Mech. And Eng. 1974.
2. Rodi W. // Proc. 2nd Symp. on Turbulent Shear Flows, 1979.
3. Tahry, S.H. //AIAA, J. Energy. 1983.
4. Llor A. Lect. Notes Phys., 2005.
5. Guzhova A.R., Pavlunin A.S., Statsenko V.P. // VANT, Series Theoretical and Applied Physics, 2005.

Problem Statement



$L \equiv \Delta y = 2$

d=0.1

g =1

$y_2 \equiv y_1 + d, \;\; y_1 = 0.994$

Initial pressure: $$P = P_0\left(1 - \frac{(\gamma - 1)\,y}{\gamma\Delta}\right)^{\frac{\gamma}{\gamma-1}}$$ where $$\Delta \equiv \frac{P_0}{\rho_0 g}$$

Initial gas density: $$\rho = \rho_0\left(1 - \frac{(\gamma - 1)\,y}{\gamma\Delta}\right)^{\frac{1}{\gamma-1}}, \;\; при \;\; 0<y<y_1, \;\; 2>y>y_2,$$

$\rho = \rho_1 = 1, \;\;$ *при* $\;\; y_1<y<y_2$ *(легкий слой).*

$\rho_0 = 3, \;\; P_0 = 16, \;\; 106$

Analytical Solution for the Self-Similar Flow Stage

Experiments

In experiments*, TMZ growth rate was measured at the self-similar stage (constant growth rate)

$$b \equiv \frac{dL^*}{dt}; \qquad L^* \equiv \frac{L_t}{\sqrt{d \cdot \left| g_y \cdot (1 - \rho_1 / \rho_0) \right|}} \equiv \frac{L_t}{B} \qquad \textbf{(1)}$$

The experimental data : **b=0.35-0.37.**

Analytical Solution of k-ε Model Equations

$$b = \left(c_{\varepsilon 2} - 1\right)\sqrt{\frac{2 \cdot c_D}{c_{\varepsilon 2} \cdot \sigma_h}} \qquad \textbf{(2)}$$

Analytical solution:	for our coefficients	**b=0.343**
	for standard coefficients	**b=0.164**

* *Kucherenko Yu.A., Balabin S.I., Pylaev A.P. // 4th International Workshop on The Physics of Compressible Turbulent Mixing, 1993.*

1D k-ε Simulation Setup

. Number of cells: 1000:

Background values of turbulent energy and its dissipation rates:

$$k_{ph} = \varepsilon_{ph} = 10^{-11}$$

Initial values of turbulent energy and its dissipation rate were defined in the light layer:

$$k_0 = 10^{-3} \qquad \varepsilon_0 = 0.025$$

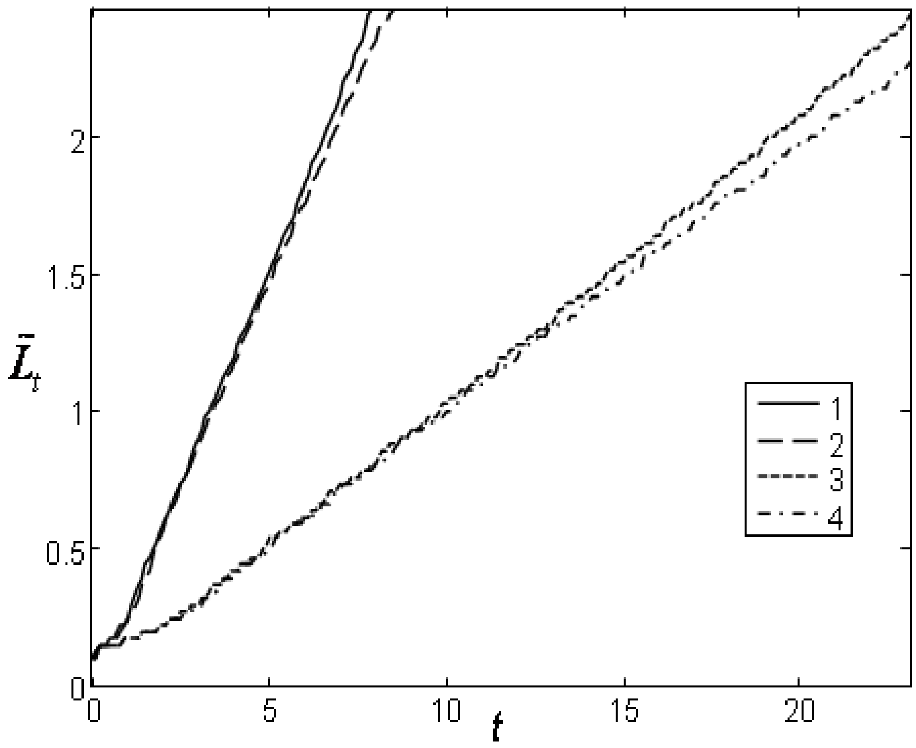
.

Results of 1D k-ε Simulations

Variant #	Model coefficients	Initial pressure P_0	b
1	Standard	16	0.1
2	Our	16	0.3
3	Our	106	0.3
4	Standard	106	0.1

Results of 1D k-ε Simulations

TMZ Width



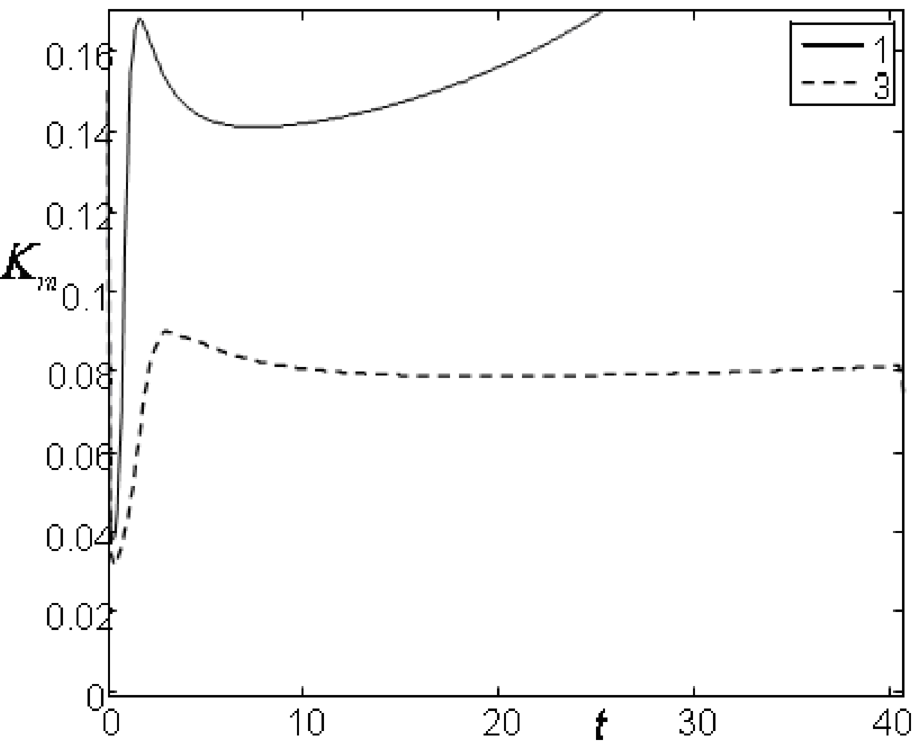
$$L_t^* = L_t \Big/ \sqrt{d \cdot g \cdot (1 - \rho_1 / \rho_0)}$$

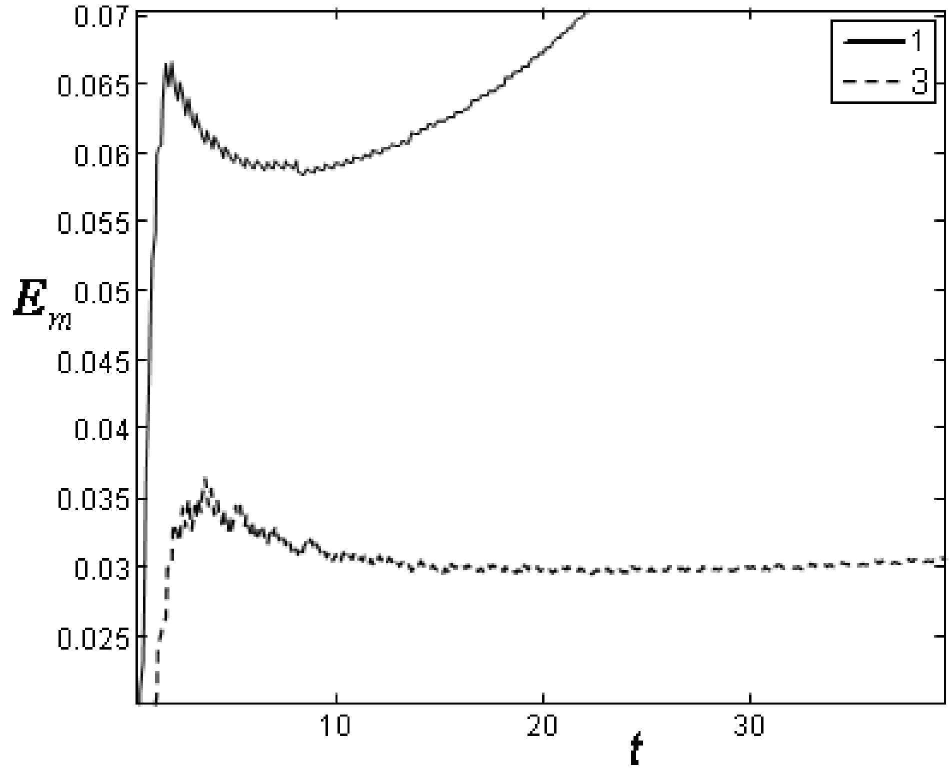
1, 2 – simulations with our coefficients

3, 4 – simulations with "standard" coefficients

Results of 1D k-ε Model Simulations

Maximum Values of k and ε in TMZ





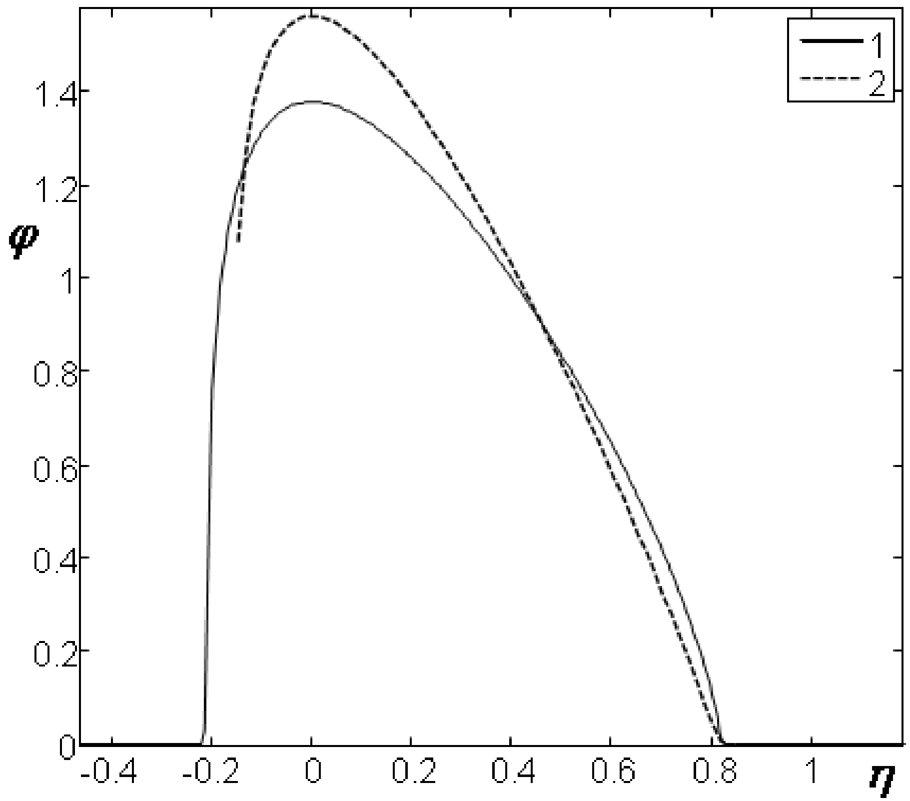
$$K_m = \max(k) \cdot \left[d \cdot g \cdot (1 - \rho_1 / \rho_0)\right]^{-1} \qquad E_m = L_t \cdot \max(\varepsilon) \cdot \left[d \cdot g \cdot (1 - \rho_1 / \rho_0)\right]^{-1}$$

1 – simulations with our coefficients
3 – simulation with "standard" coefficients

Results of 1D k-ε Simulations

Self-Similar Density Profile

$$\varphi(\eta) = \frac{(1 - \rho/\rho_0)}{(1 - \rho_1/\rho_0)} \cdot \frac{L(t)}{d}$$

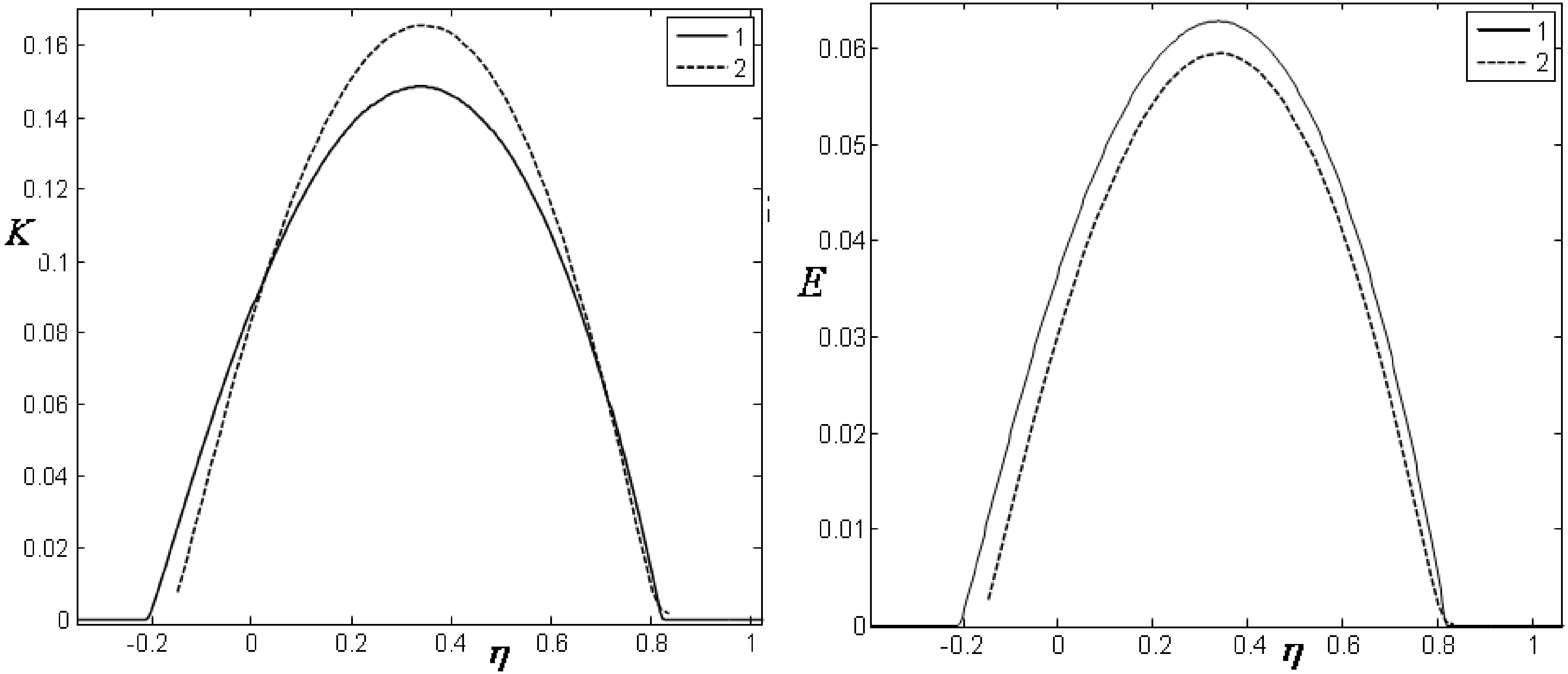


$$\eta \equiv \frac{y - y_d}{L_t}$$

1 – simulation, 2 – analytical solution

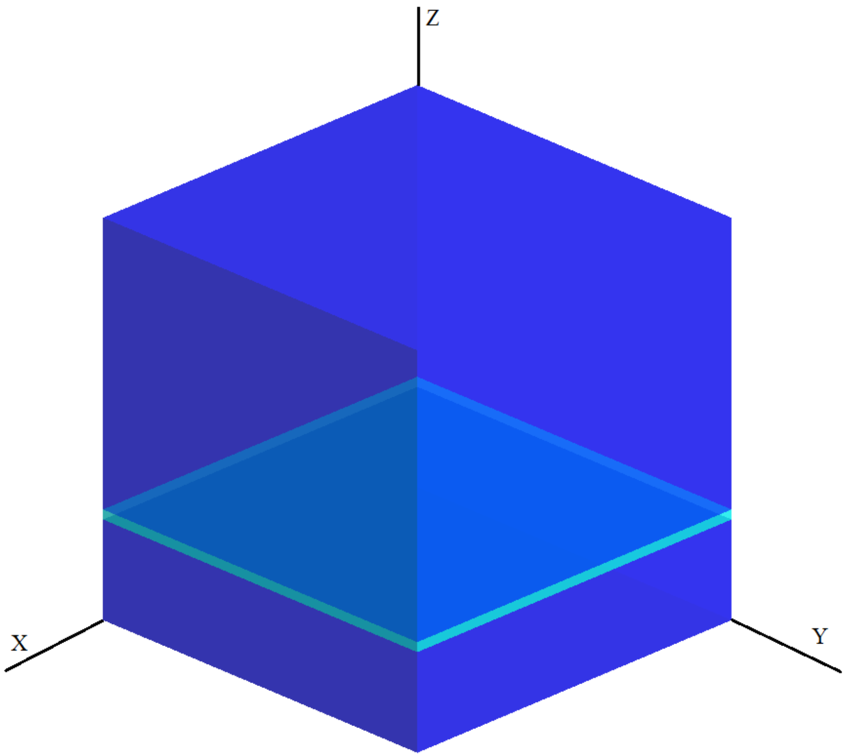
Results of 1D k-ε Model Simulations

Self-Similar Profiles of k and ε



1 – simulation, 2 – analytical solution

3D Simulation Setup



$L = 2$

In the light layer, at $y_1 < y < y_2$:

$\rho = \rho_1 = 0.5,$

$y_2 \equiv y_1 + d,$

$y_1 = 0.5,$

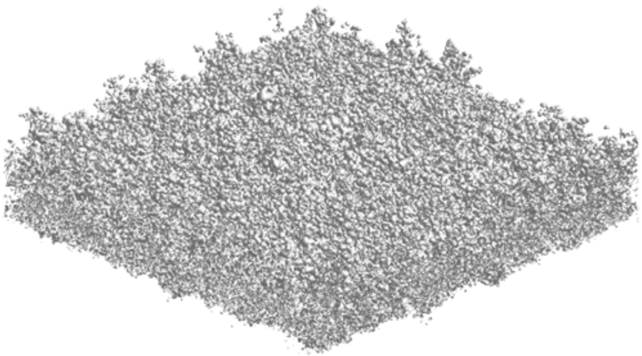
$d = 0.05,$

$P_0 = 30$ **and 100.**

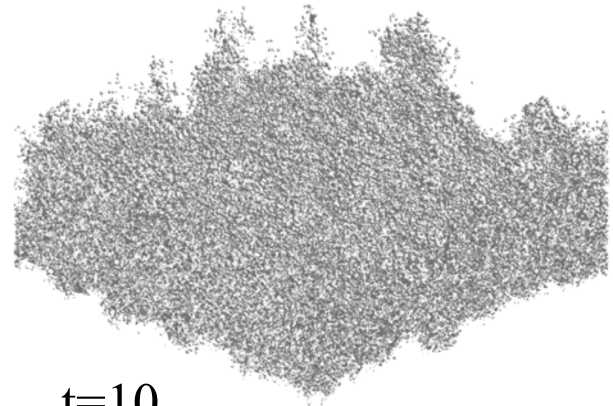
3D Simulation Variants and TMZ Width

№ варианта	$N_x \times N_y \times N_z$	P_0	**b**
5	400 x 400 x 400	30	**0.43**
6	400 x 400 x 400	100	**0.435**
7	200 x 200 x 200	30	**0.38**
8	200 x 200 x 200	100	**0.38**

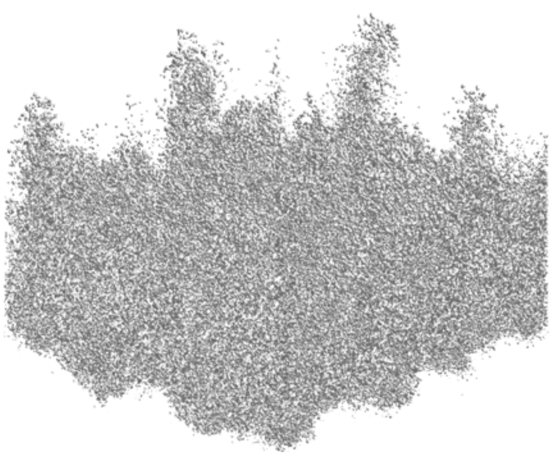
Distributions of volume fraction of light layer material (simulation 6)



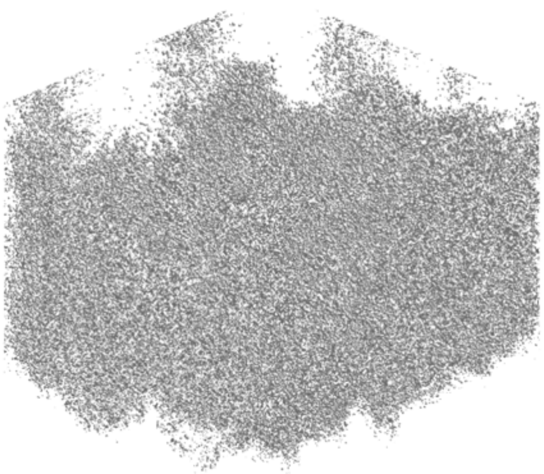
t=5



t=10

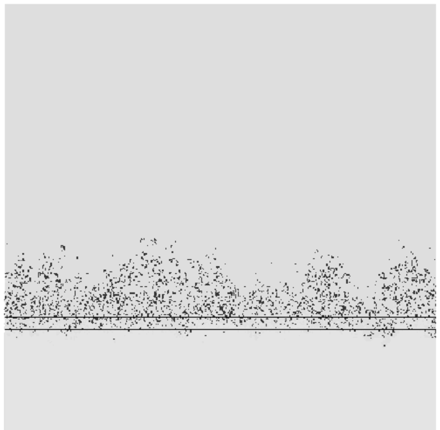


t=15

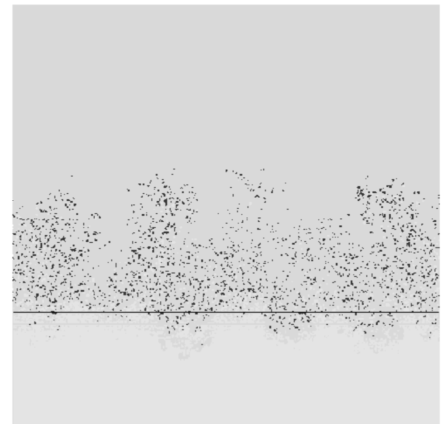


t=20

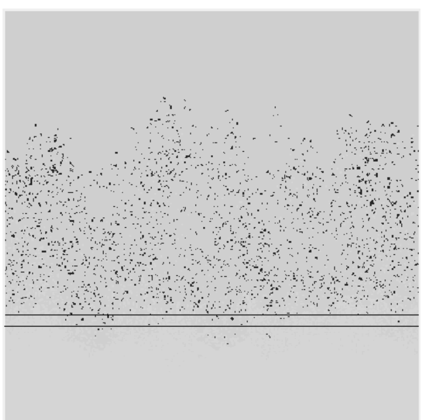
Distributions of volume fraction of light layer material in 2D section x=1



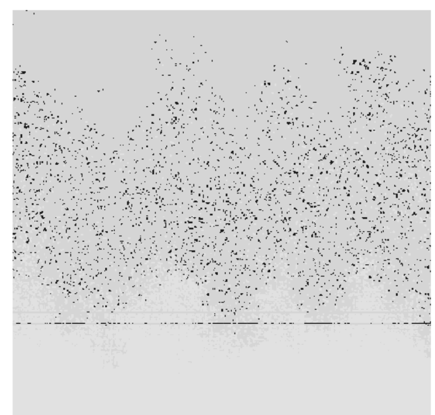
t=5



t=10



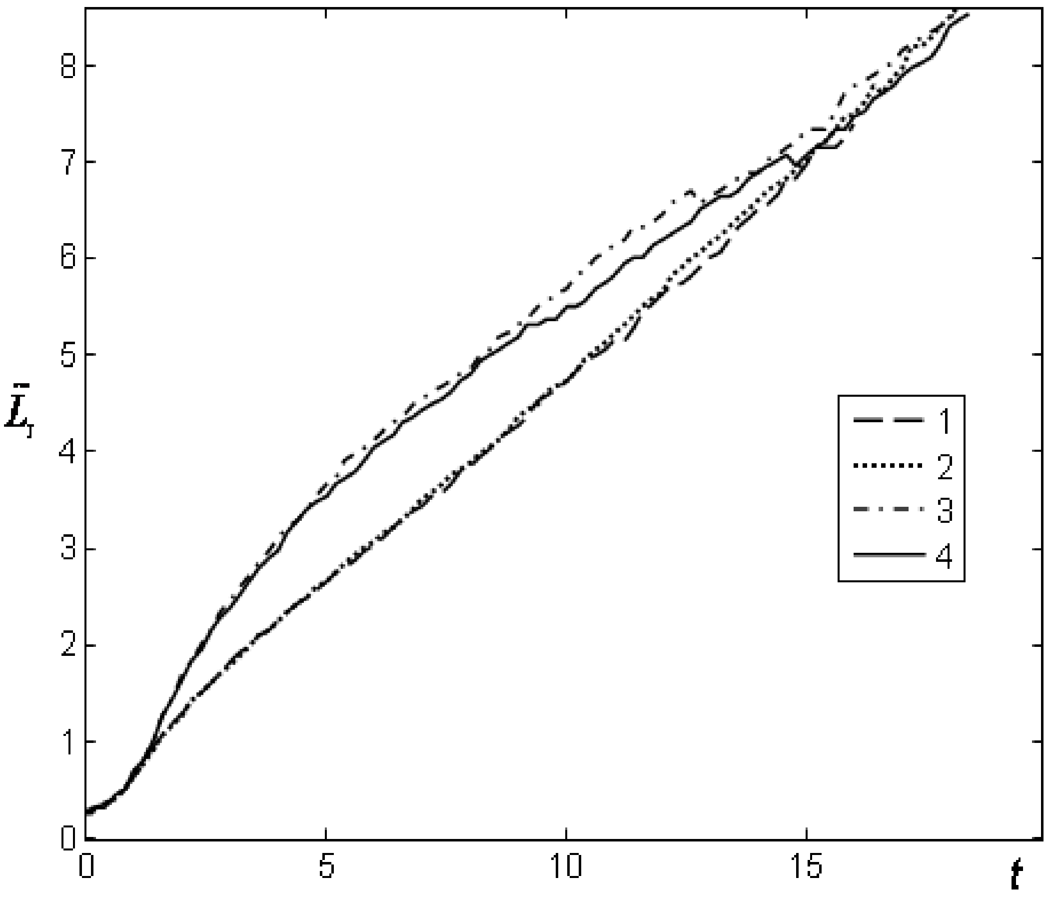
t=15



t=20

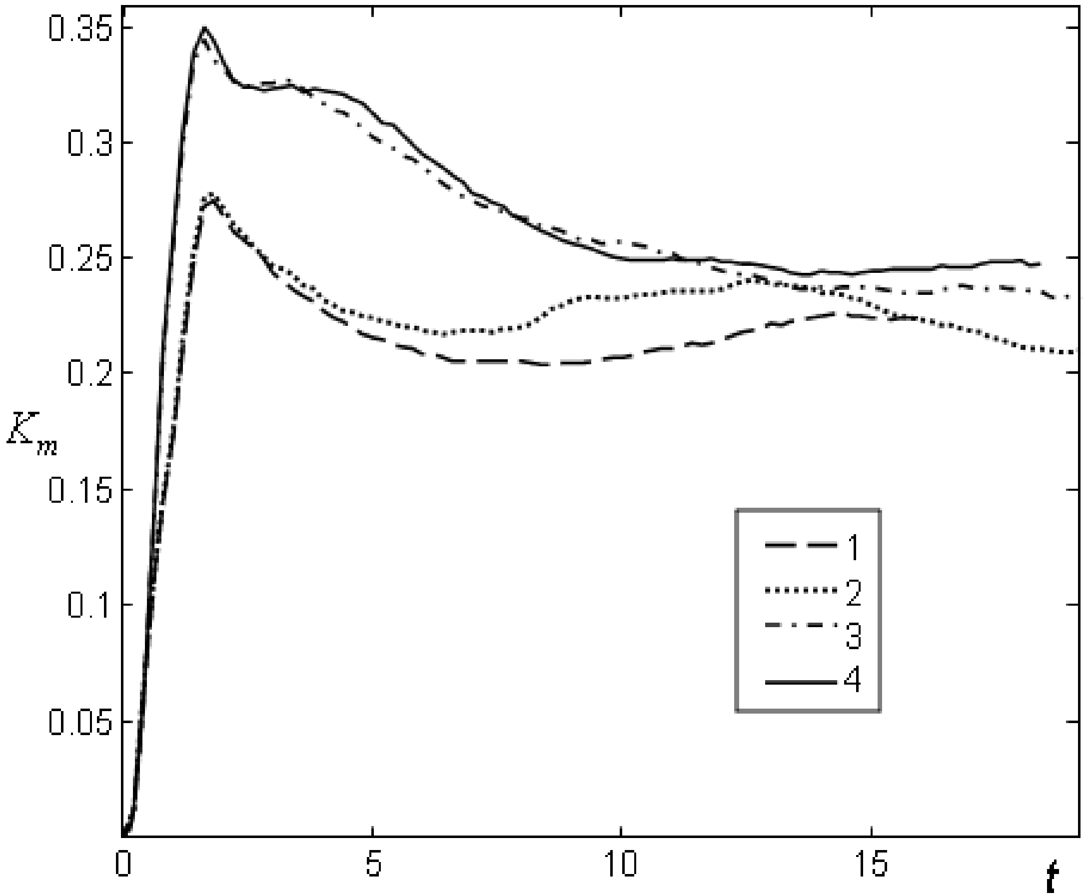
Straight lines are initial locations of the interface layer.

TMZ Width in 3D Simulations

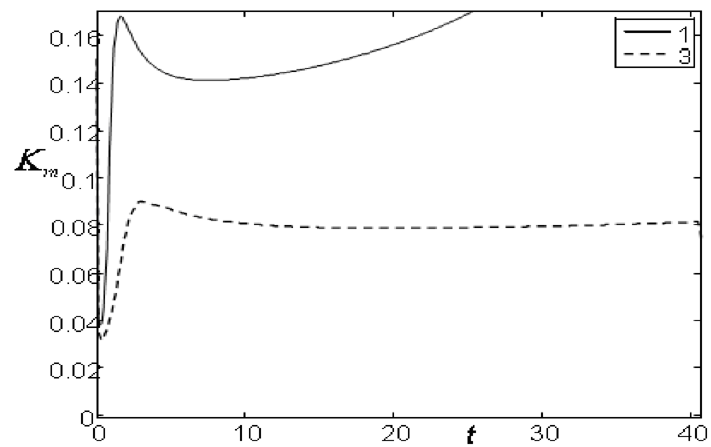


1, 2 – simulations 5, 6 (N=400); 3, 4 – simulations 7, 8 (N=200)

TMZ Maximum Turbulent Energy
3D Simulations vs k-ε

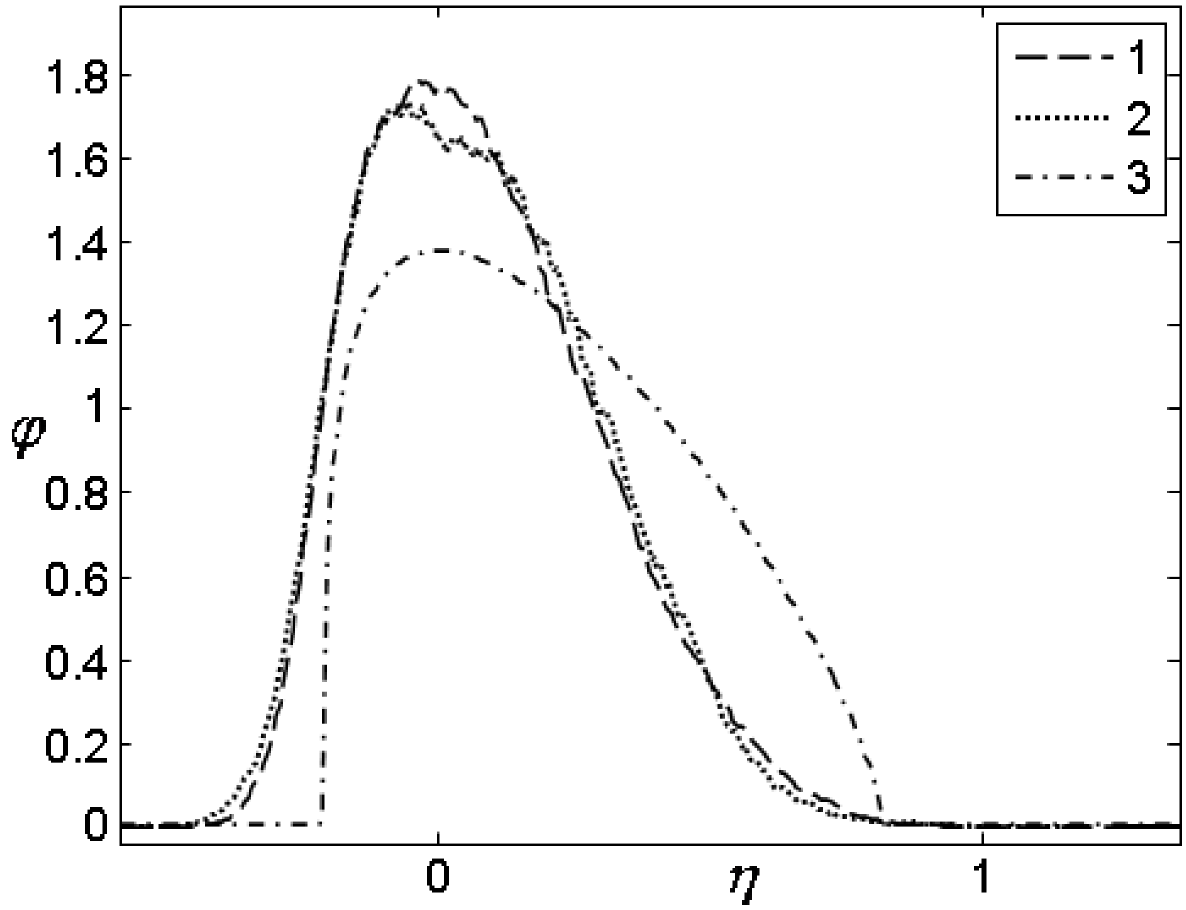


k-ε



1, 2 – simulations 5, 6;   3, 4 – simulations 7, 8

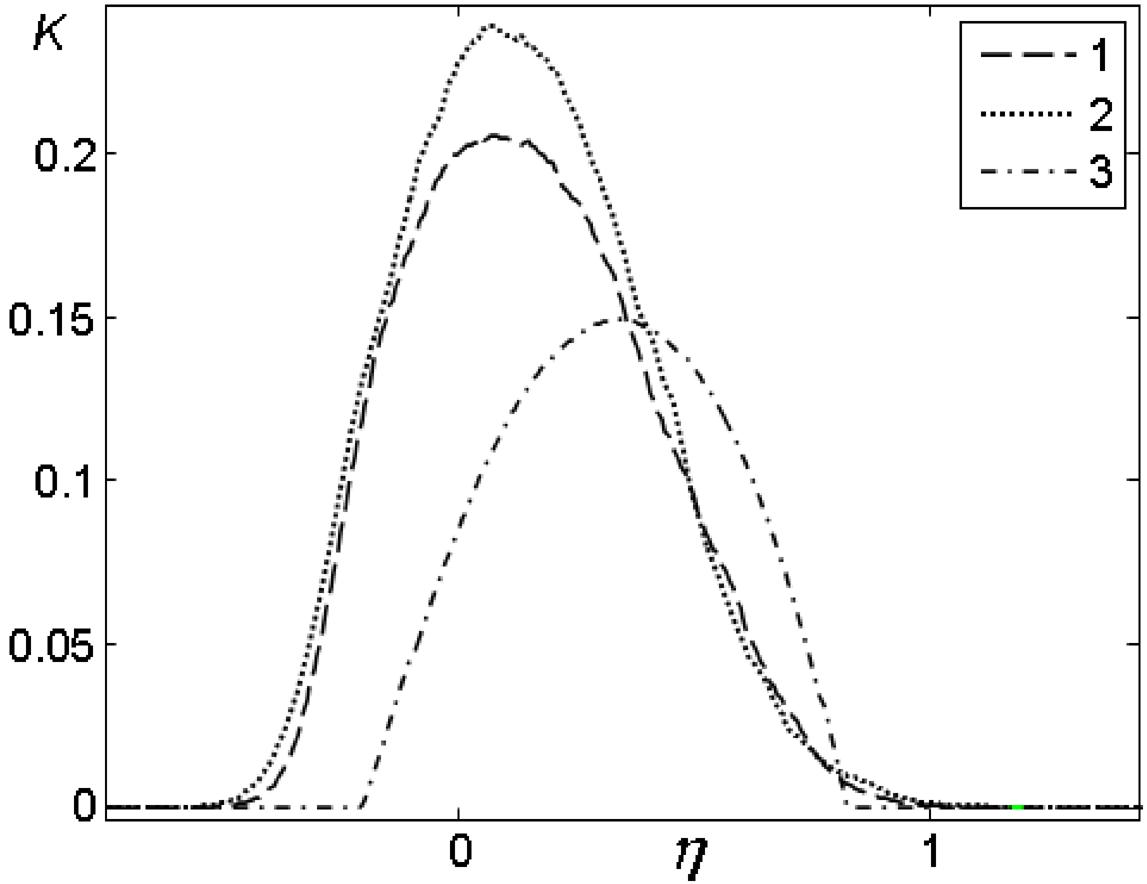
Self-Similar Density
3D Simulations vs k-ε



1 – 3D simulation 5;   2 – 3D simulation 6;   3 – simulation 3 (k-ε)

Turbulent Energy

3D Simulations vs k-ε



1 – 3D simulation 5; 2 – 3D simulation 6; 3 – simulation 3 (k-ε)

Summary of Results and Conclusions

In all calculations there are transition to self-similar regime that is characterized by linear law of TMZ growth with time. This regime is also characterized by time-invariant TMZ maximum values of turbulent energy and its dissipation rate. .

	coefficients	mesh number	b
3D (DNS)	-	**400^3**	**0.43**
1D (к-ε model)	**standart**	**1000**	**0.1**
1D (к- ε model)	**our**	**1000**	**0.3**
Analitical solution	**standart**	**-**	**0.164**
Analitical solution	**our**	**-**	**0.343**
experiments			**0.35÷0.37**

$$b = \left(c_{\varepsilon 2} - 1\right)\sqrt{\frac{2 \cdot c_D}{c_{\varepsilon 2} \cdot \sigma_h}}$$

Thank you for attention!