Institute of Theoretical and Mathematical Physics

Russian Federal Nuclear Center -

VNIIEF

3D Numerical Simulations of Vortex Ring Formation by a Thermal Plume Rising in the Atmosphere

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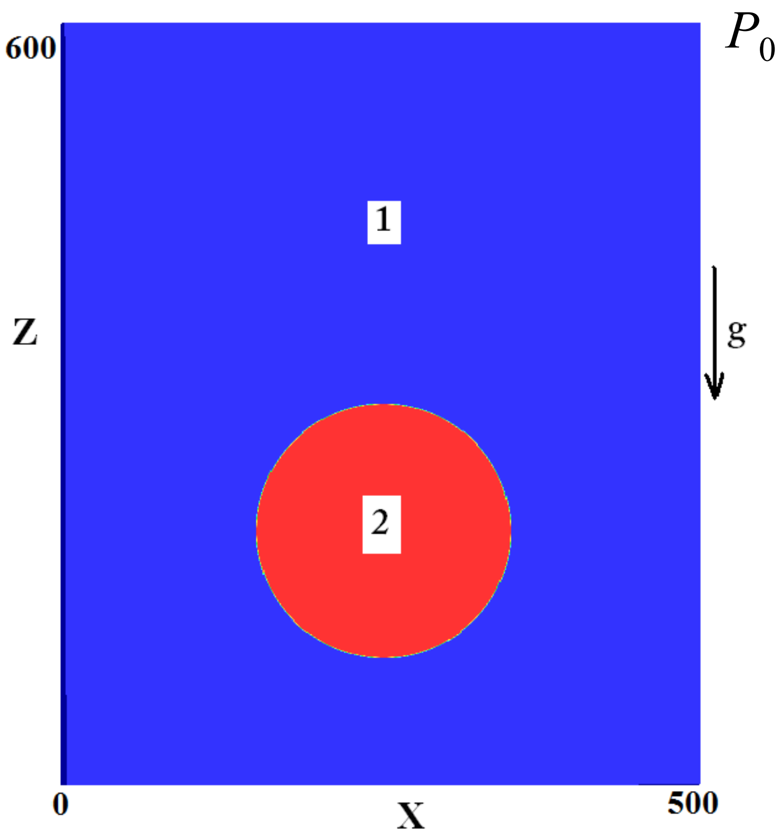
Problem Statement and 3D Simulation Setup



ρ_1=1.25 kg/m3 , ρ_2=0.125 kg/m3 , γ =1.4

R_2=100 m , O_{CENTER} = (250,250,200)

z_v = 600 m: $P = P_0 - \rho_0 \cdot g \cdot (z_v - z)$ (1)

P_0= 1 atm g=9.8 m/s^2

$\xi \equiv \frac{P_0}{\rho_1 g R_2}$ = 81.6 - degree of incompressibility

Re=4*N_Z

	N_x* N_y*N_z
Variant 1	500*500*600 h=1
Variant 2	1000*1000*1200 h=0.5

Background

1. *Тарасов В.Ф.* О движении всплывающего вихревого кольца // Динамика сплошной среды, Новосибирск, вып.23, 1975. (**experiment**)

2. *Жидов И.Г, Мешков Е.Е., Попов В.В., Рогачев В.Г., Толшмяков А.И.* Образование вихревого кольца при всплывании большого воздушного пузыря в воде // ПМТФ, 1977, № 3. (**experiment**)

3. *Глаголева Ю.П., Жмайло В.А., Мальшаков В.Д., Нестеренко Л.В., Стаценко Софронов И.Д.* Образование кольцевого вихря при подъёме лёгкого газа в тяжёлом// ЧММСС, Новосибирск, 1974, т.5, N1 (**2D simulation of initial stage**)

4. Махвиладзе Г.М., Якуш С.Е. Подъём турбулентного осесимметричного термика в неоднородной сжимаемой атмосфере // ПМТФ, 1989, № 1, с.62-68**.** (**2D simulation**)

5. Чуприн И.А., Щербин М.Д. Численное моделирование турбулентного термика // ПМТФ, 2003, т.44, № 3, с.64-75. (**2D simulation**)

Novelty of our work:

1) We conducted 3D numerical problem simulation using the code EGAK.

2) Simulations were run up to significantly late times.

3) Simulations were done on an considerably finer mesh that allows modeling of turbulent mixing at the unstable plume boundary.

4) Theoretical model is presented for gravitational formation and motion of vortex rings, which (as distinct from previous models) is free of empirical coefficients and is based on the known analytical studies of circular vortices [6], [7].

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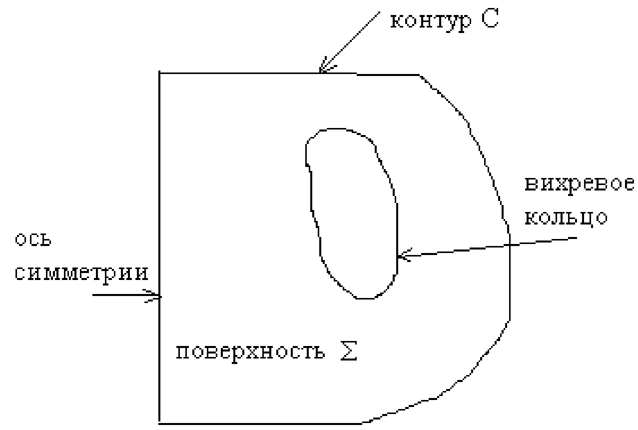
6. *Valters J.K., and Davidson J.F*. The initial motion of a gas bubble formed in an inviscid liquid // J. of Fluid Mech., 1963, v.17, part 3. **(analytical studies)**
7. *Lamb G*. Hydrodynamics, OGIZ – Gostekhizdat, 1947 (in Russian) **(analytical studies)**

Analytical Problem Solution

Integration domain:



We assume that:

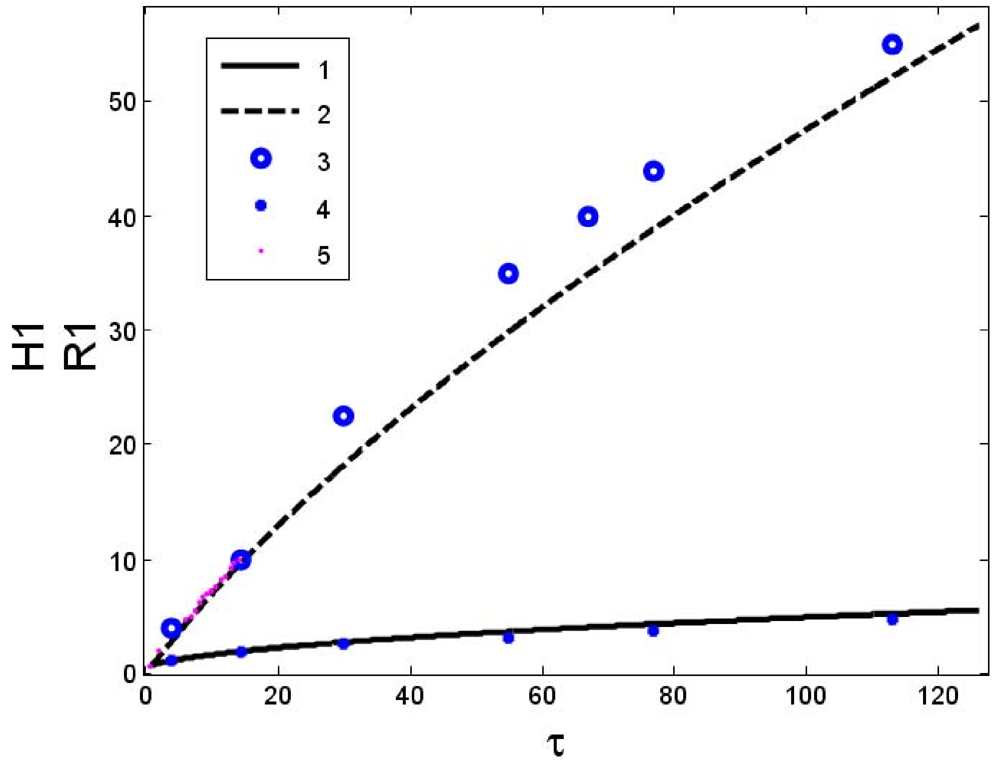
- density in the spherical volume at rest, $\Omega_0 = \frac{4}{3}\pi \cdot R_0^3$, is much smaller than ambient density ρ_a
- vorticity $\vec{\omega} = rot\vec{u}$ stays localized on the surface of the buoyant volume
- the vortex ring has a regular circular section of radius $a_1 < R_1$

for rising velocity we obtain: $U_1 = \frac{1}{\Gamma \cdot r_1^2} \cdot \left[\frac{T}{2\pi\rho_a} + \frac{3r_1\Gamma^2}{8\pi} \right]$, where $\Gamma = \int\limits_{\Sigma} \vec{\omega} \cdot d\vec{\Sigma}$ (1)

for radius we obtain: $R_1 = \sqrt{\frac{4}{3\Gamma_1} \cdot (\tau + u_{10})}$, where $\Gamma_1 = \frac{\Gamma}{R_0 \cdot \sqrt{R_0 \cdot g}}$ (2)

In what follows we use dimensionless time and radius: $\tau = \frac{t}{\sqrt{R_0 / g}}$, R/R_0

Analytical Solution vs Experiment



thermal plume rising height *H1* : analytical / measurements 1* / measurements 2*

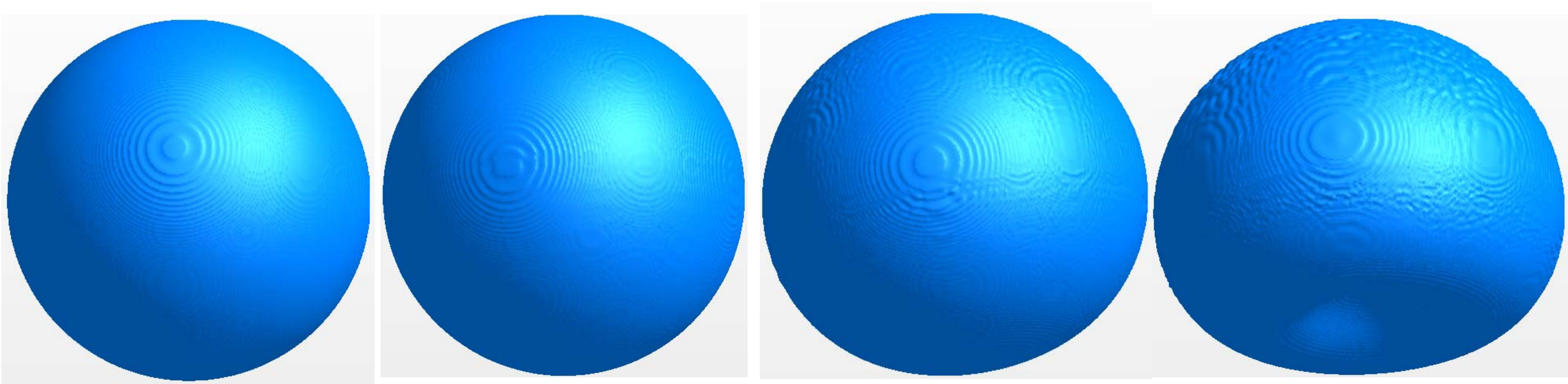
thermal plume radius *R1*: analytical / measurements 1*

1* Tarasov V.F. On the Motion of a Rising Vortex Ring. 1975.

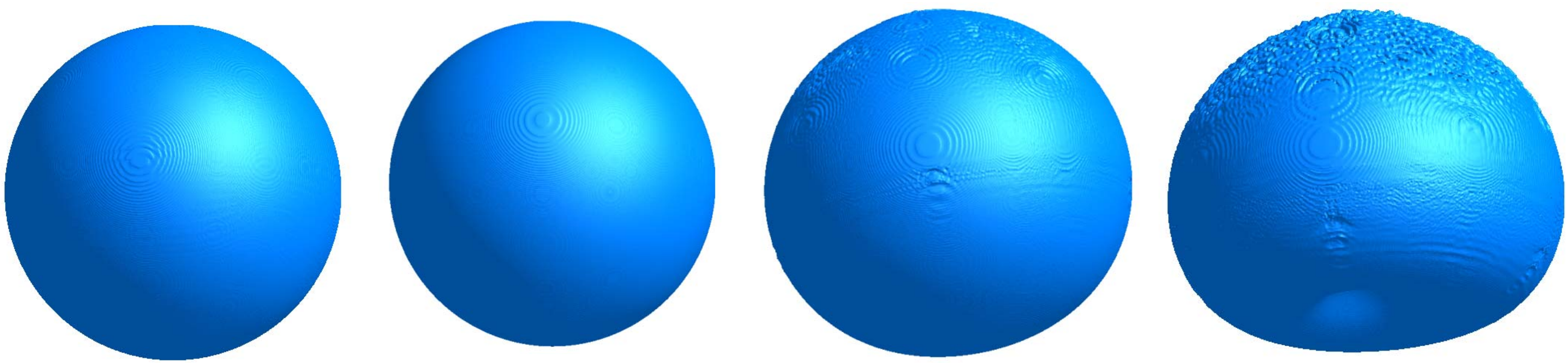
2* Zhidov I.G., et al. Vortex Ring Formation by a Large Air Bubble Rising in Water. 1977.

Isosurfaces of volume fraction β=0.5 of thermal plume material, side view

coarse grid



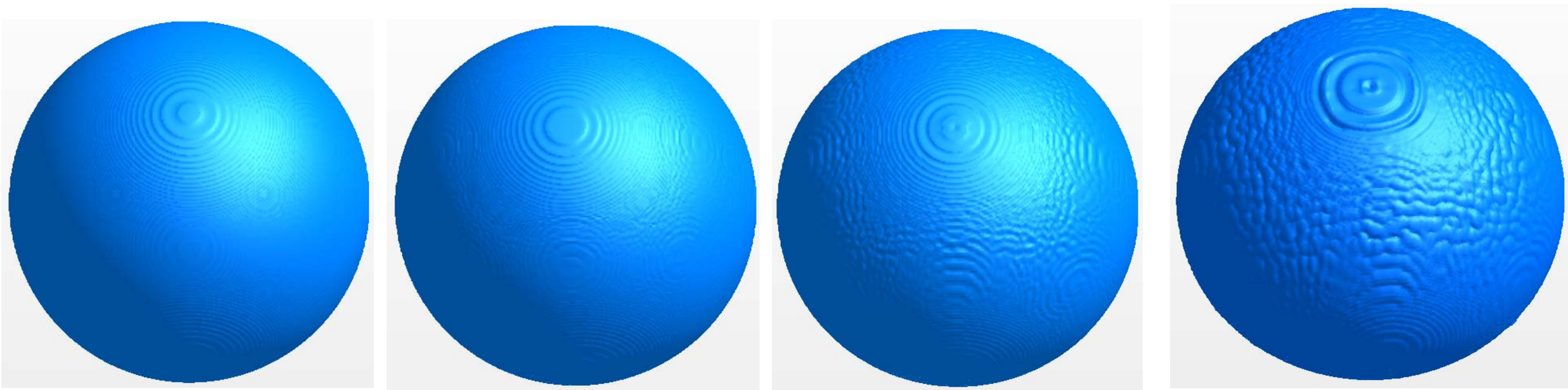
fine grid



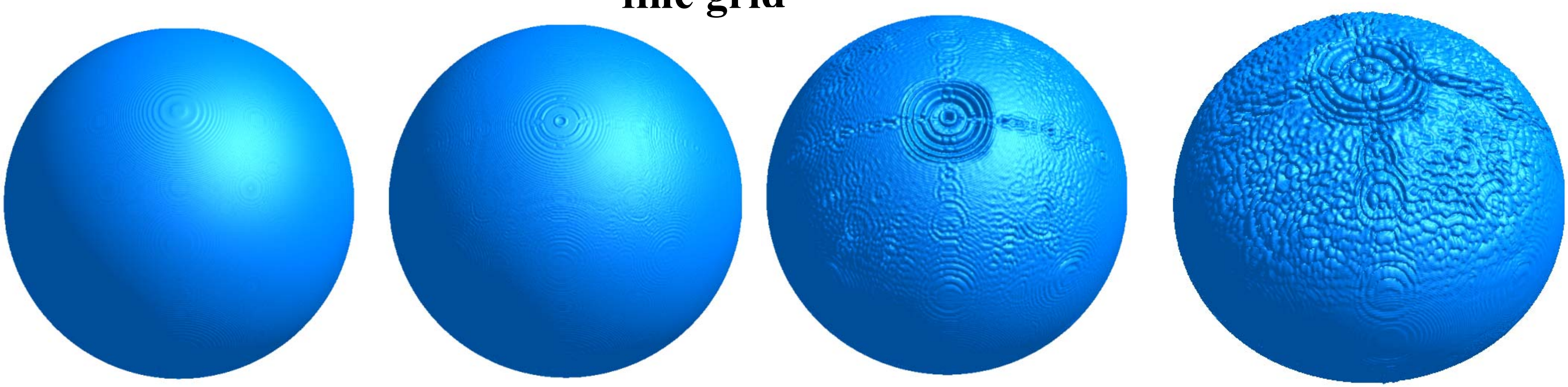
$\tau = 0$ $\tau = 0.313$ $\tau = 0.626$ $\tau = 0.939$

Isosurfaces of volume fraction β=0.5 of thermal plume material, top view

coarse grid



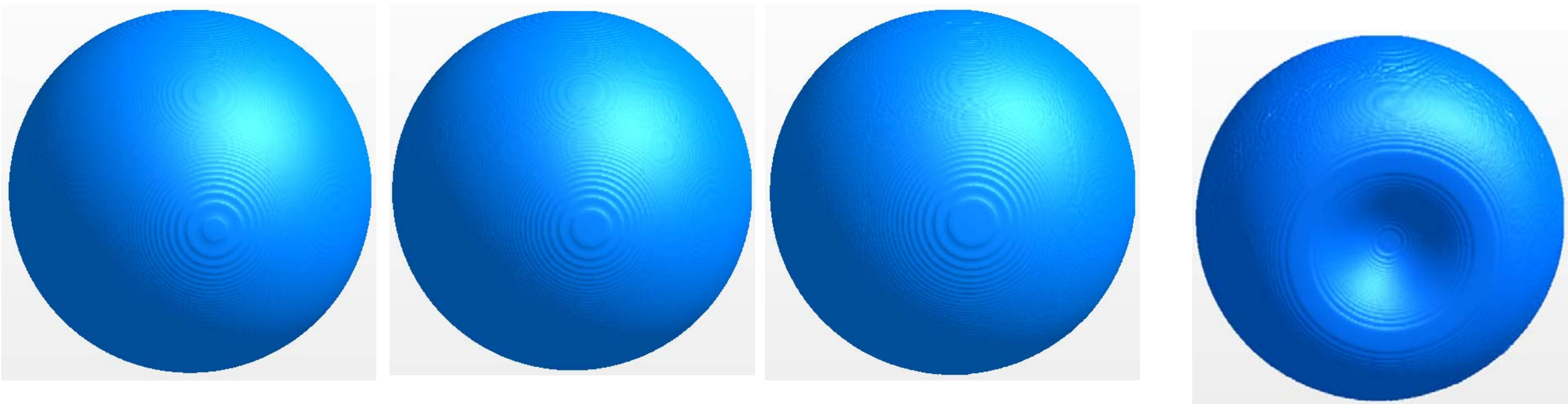
fine grid



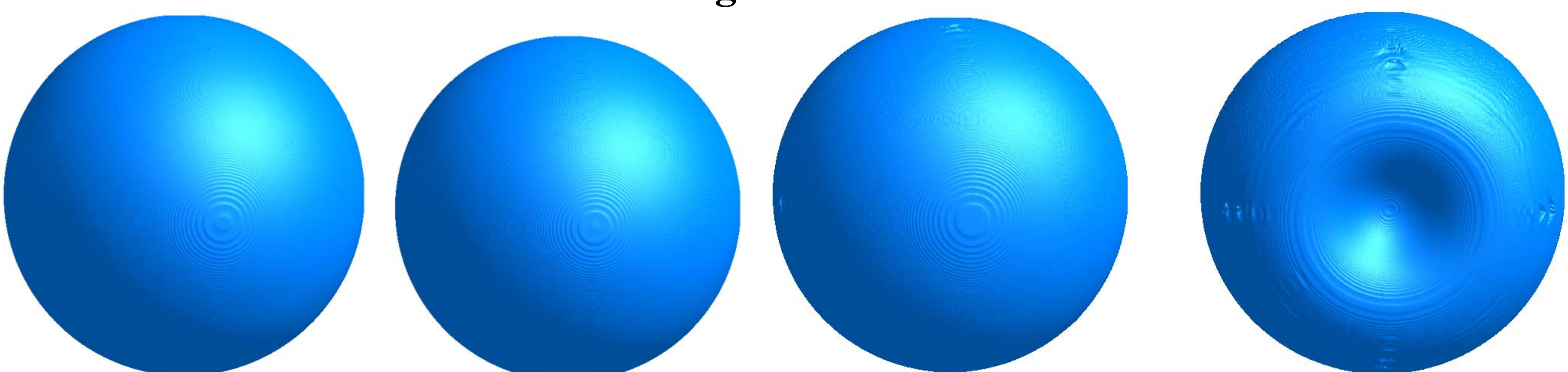
τ =0 τ =0.313 τ =0.626 τ =0.939

Isosurfaces of volume fraction $\beta=0.5$ of thermal plume material, bottom view

coarse grid

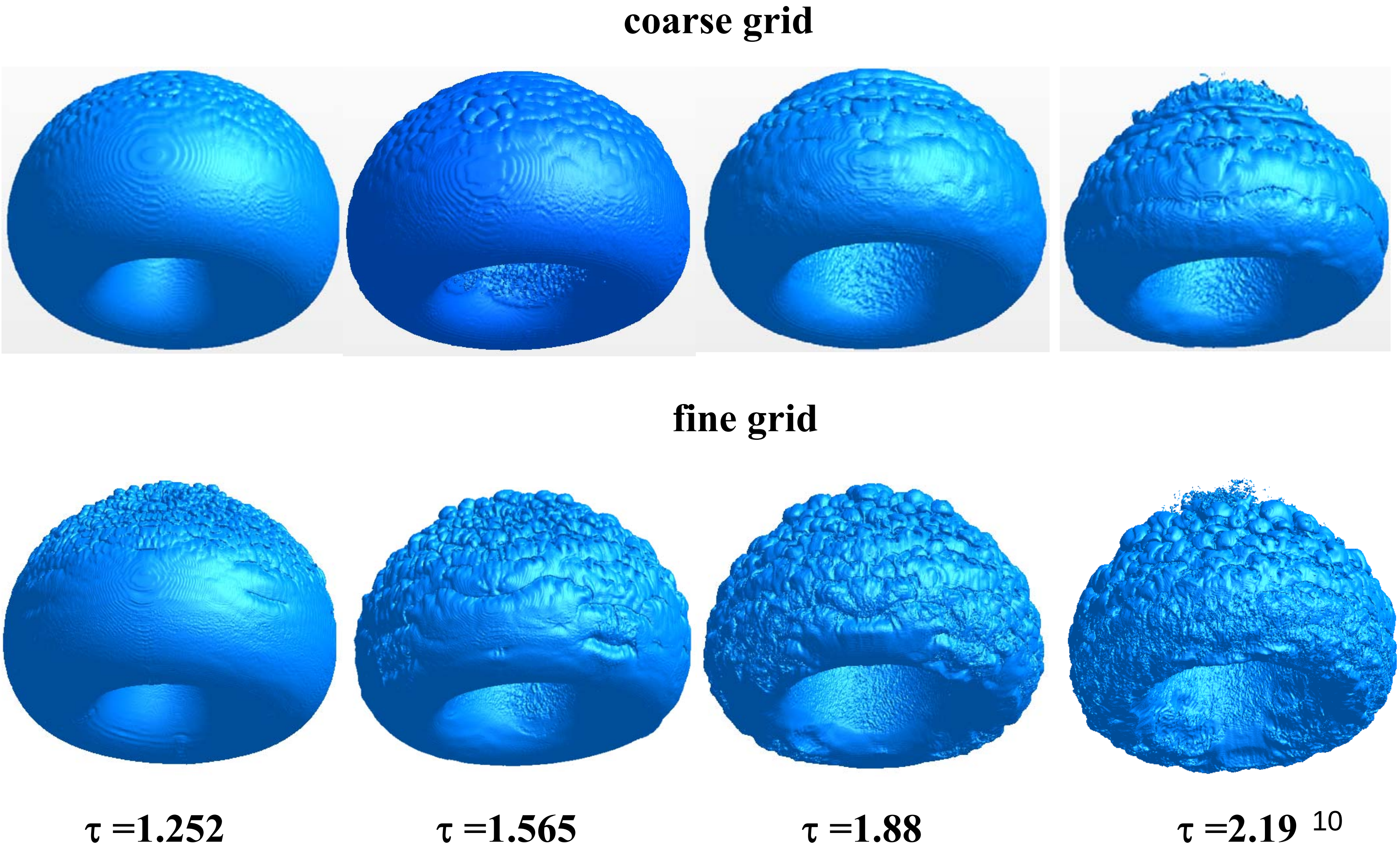


fine grid

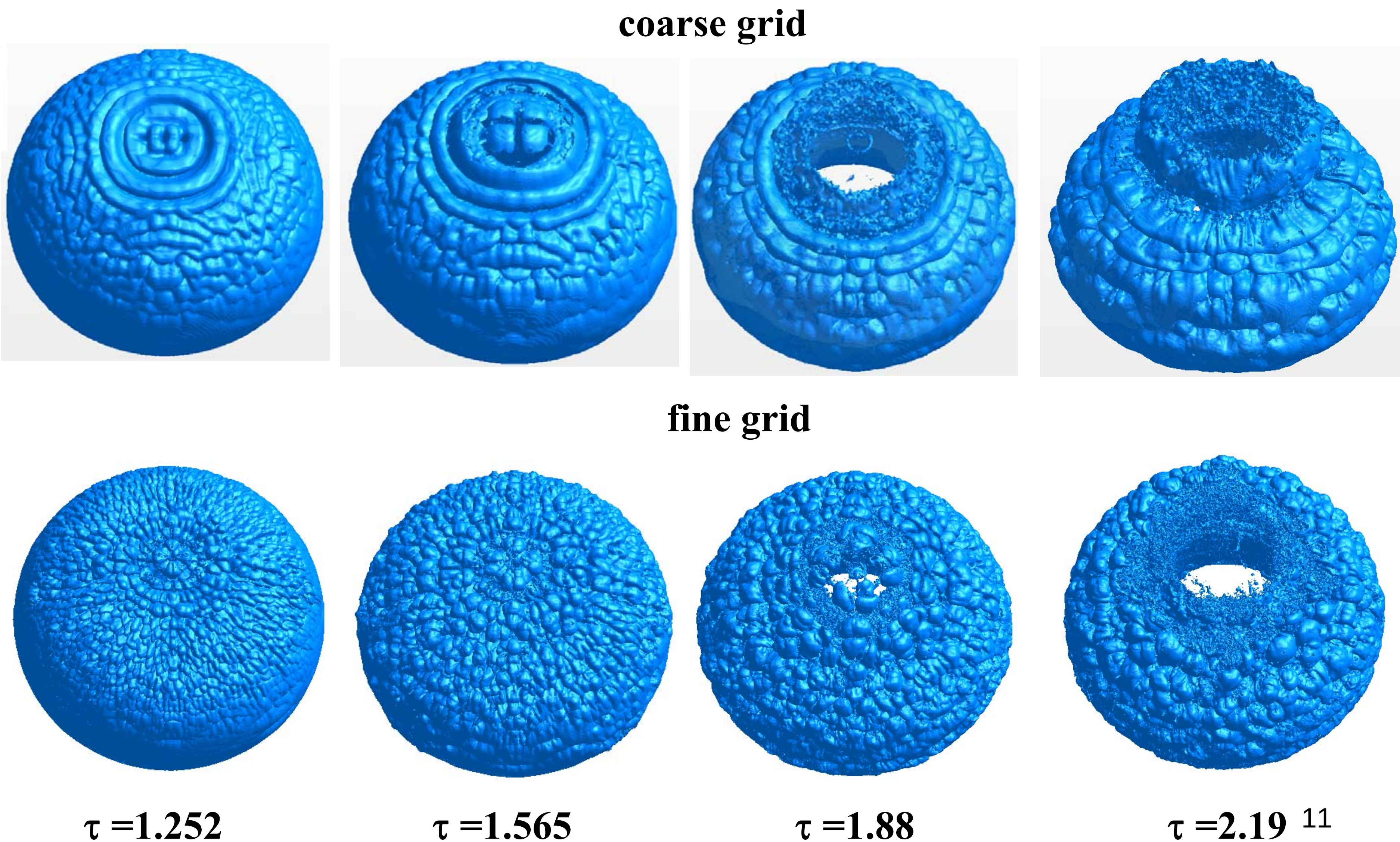


$\tau=0$ $\tau=0.313$ $\tau=0.626$ $\tau=0.939$

Isosurfaces of volume fraction β=0.5 of thermal plume material, side view

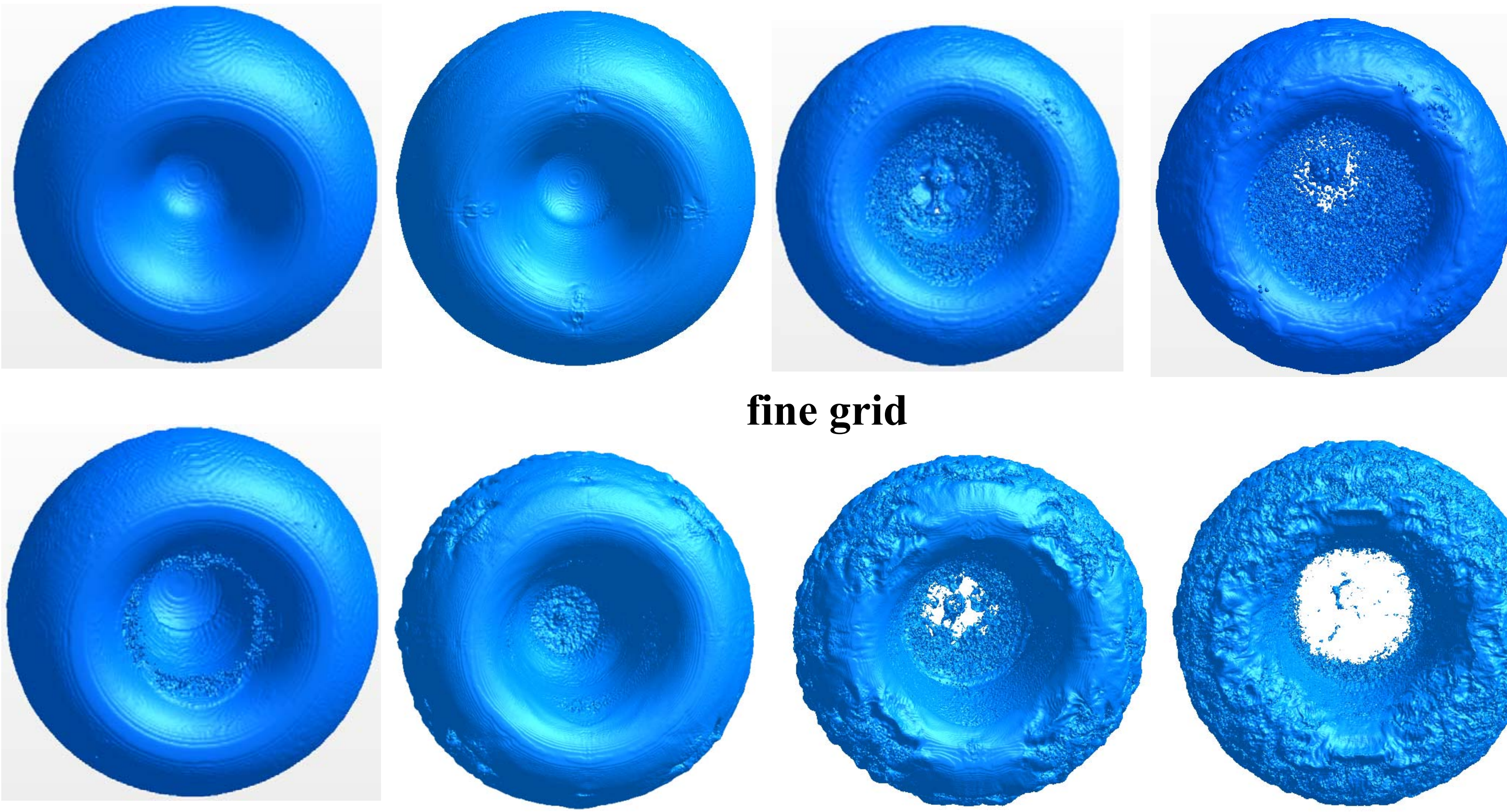


Isosurfaces of volume fraction β=0.5 of thermal plume material, top view



Isosurfaces of volume fraction β=0.5 of thermal plume material, bottom view

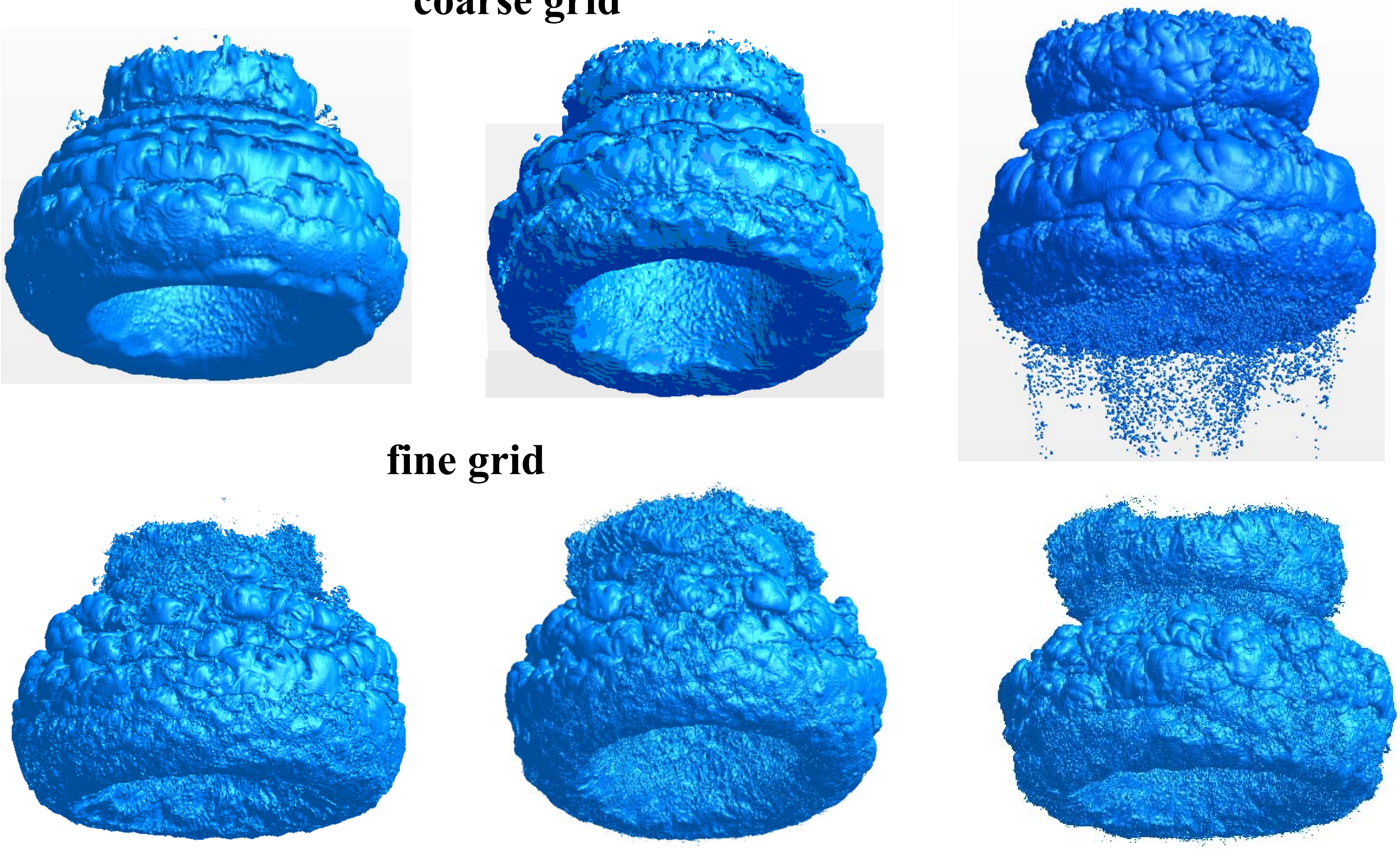
coarse grid



τ =1.252 **τ =1.565** **τ =1.88** **τ =2.19**

Isosurfaces of volume fraction β=0.5 of thermal plume material, side view

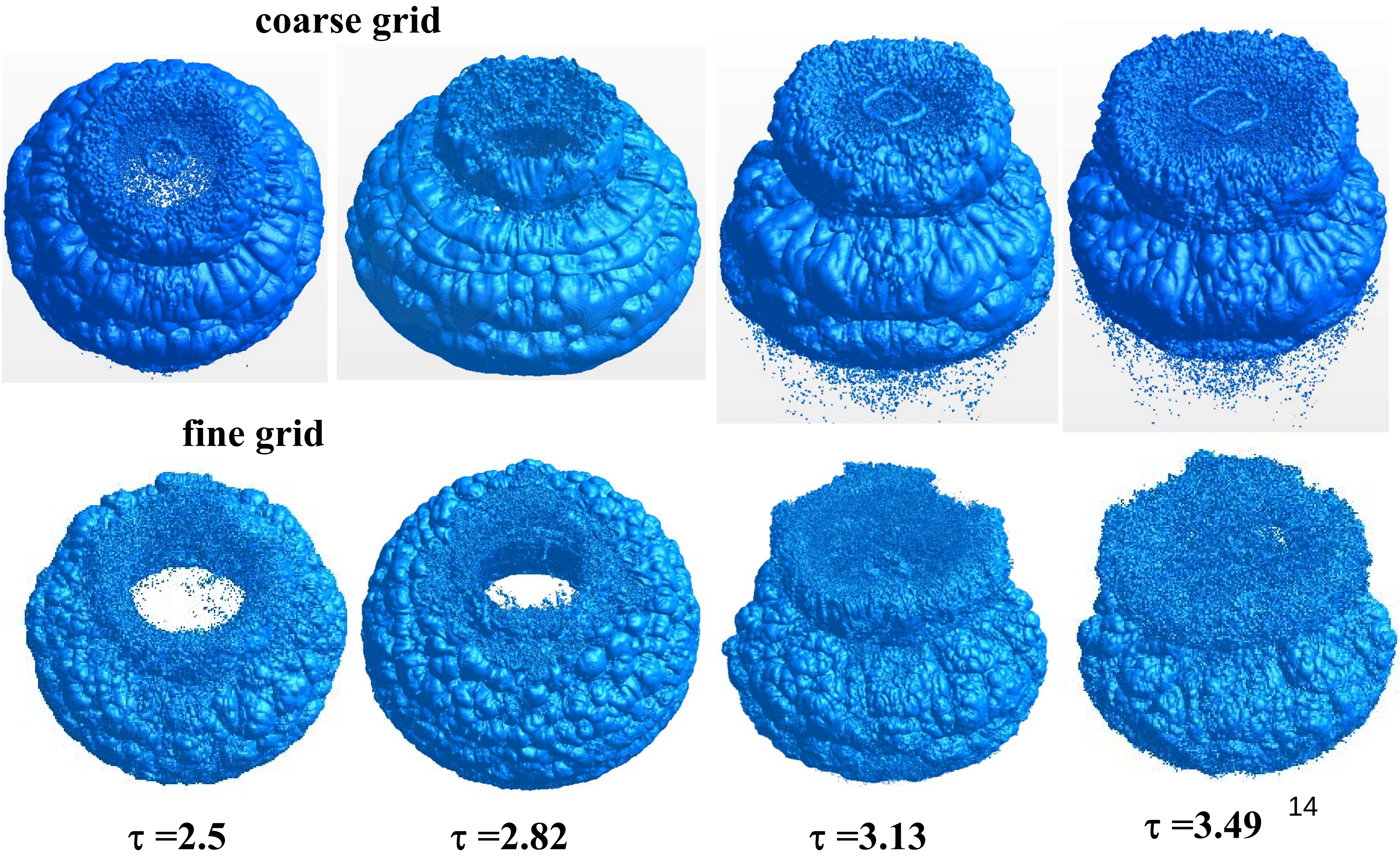
coarse grid



fine grid

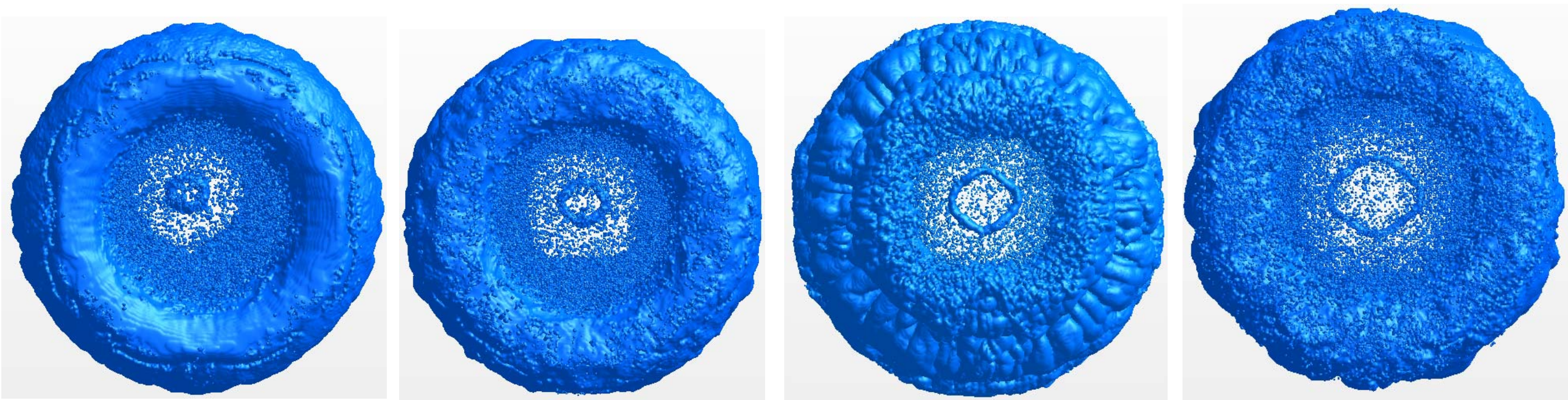
τ =2.5 **τ =2.82** **τ =3.13**

Isosurfaces of volume fraction β=0.5 of thermal plume material, top view

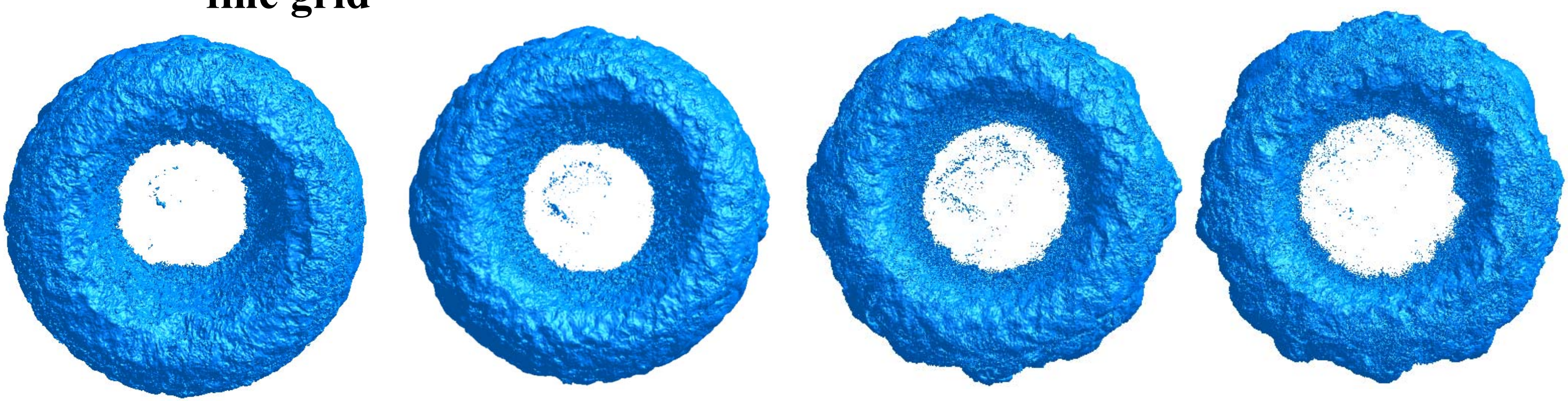


Isosurfaces of volume fraction β=0.5 of thermal plume material, bottom view

coarse grid

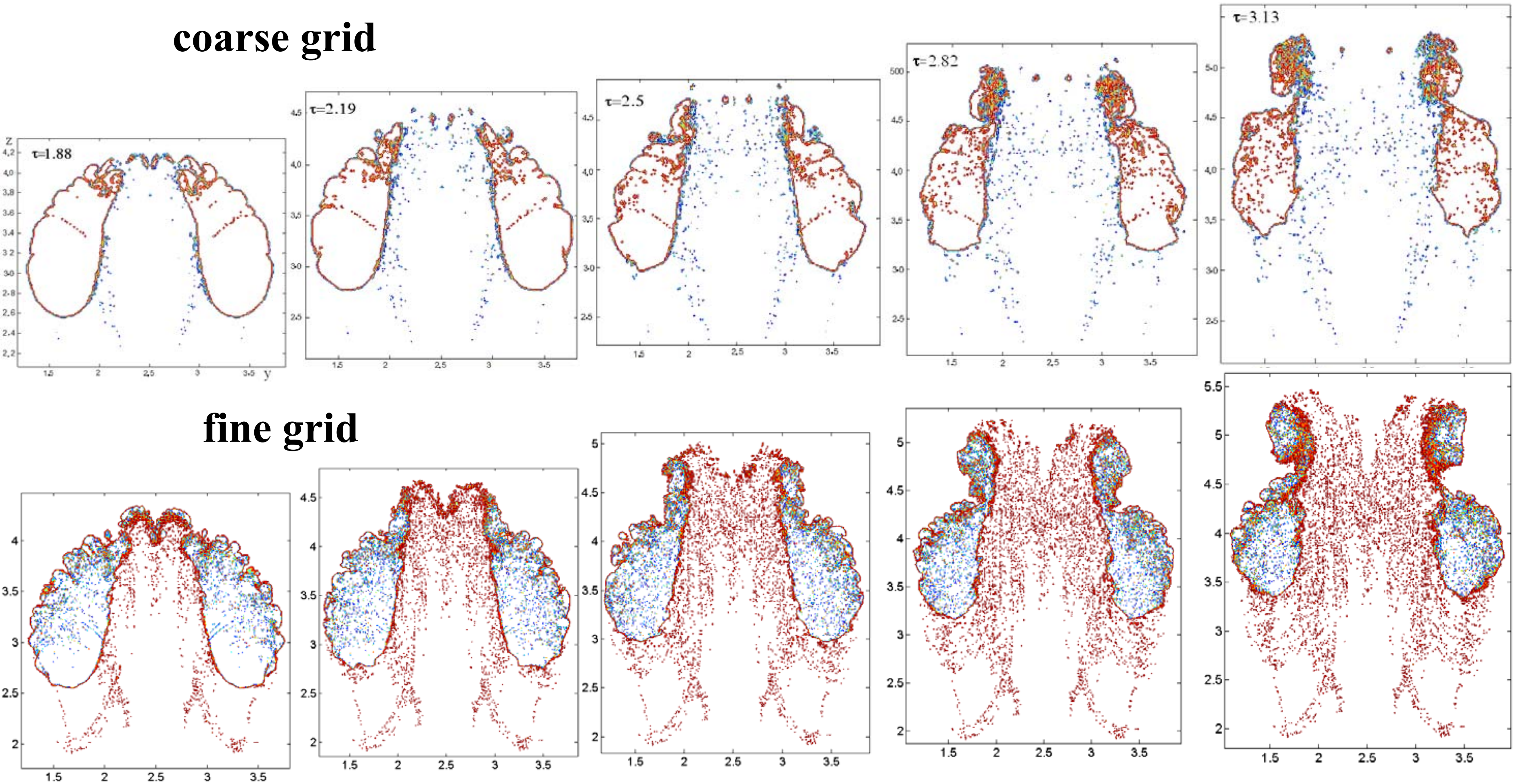


fine grid



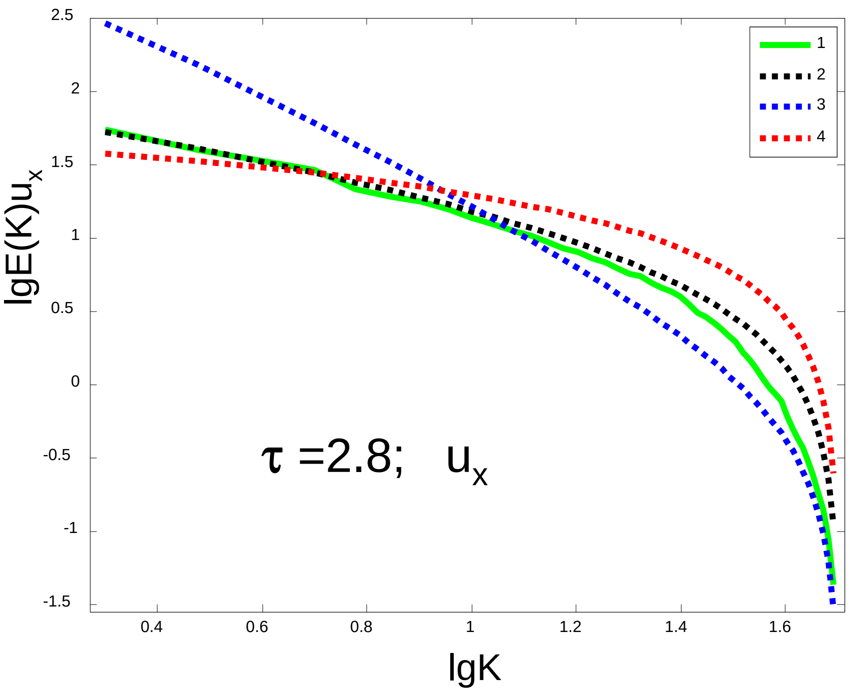
τ =2.5 **τ =2.82** **τ =3.13** **τ =3.44**

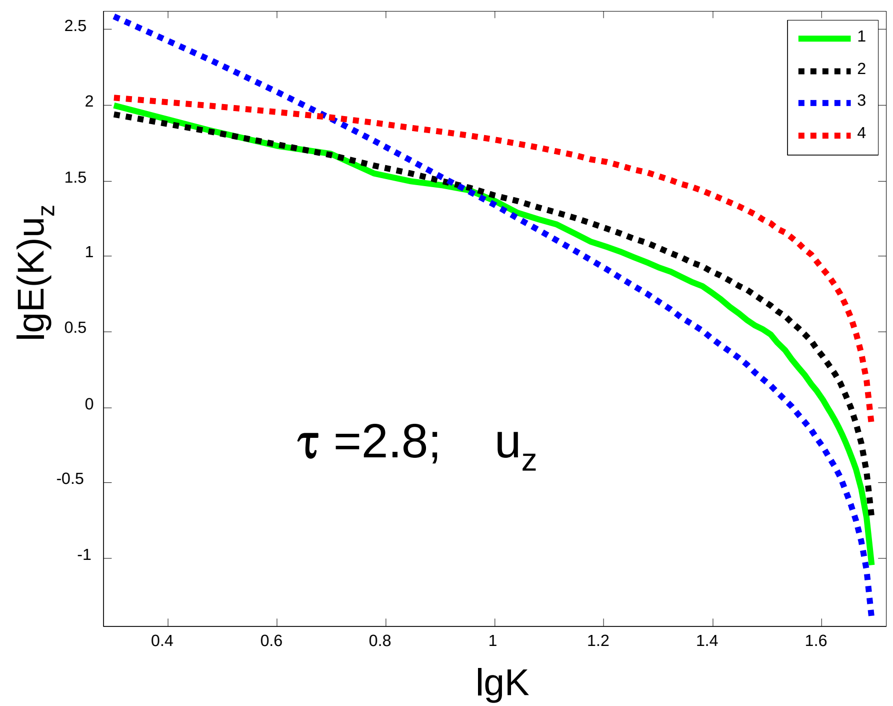
2D sections, x=0



Energy spectrum of fluctuations of quantities u_x and u_z

(fine grid)





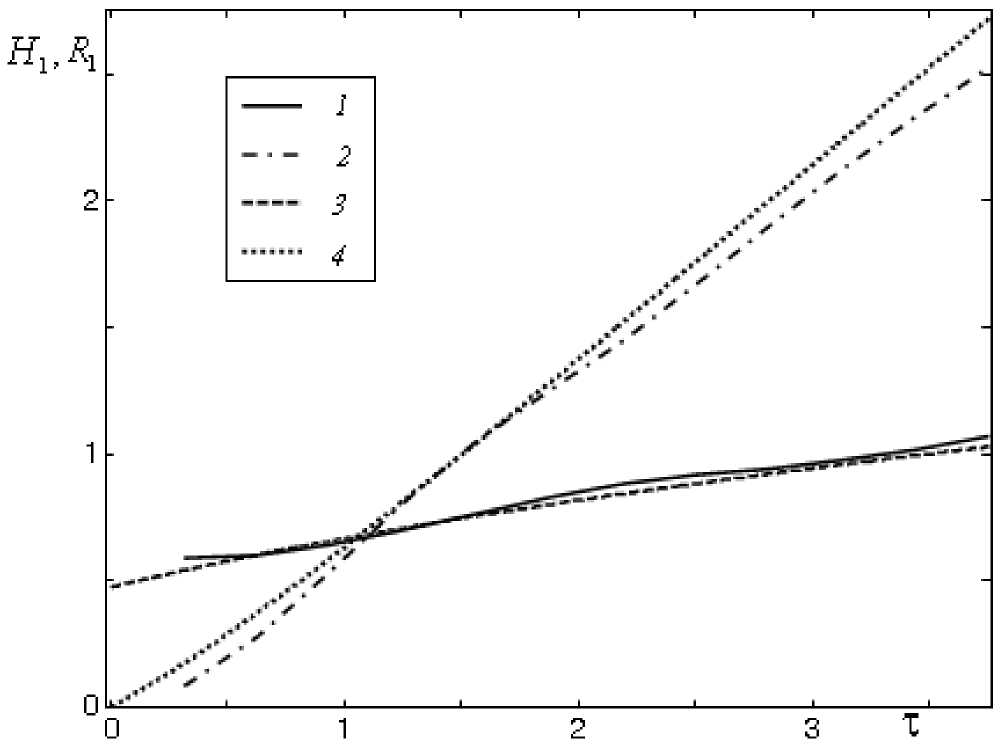
3D simulation:

3D Kolmogorov: $E(K)=const*K^{-5/3}$

2D Kolmogorov: $E(K)=const*K^{-3}$

low-frequency: $E(K)=const*K^{-1}$

Simulation vs Analytical Solution



thermal plume rising height H_1 : simulation 2N (2) -·-·-·-
analytical (4) ·········
diagram of thermal plume radius R_1: simulation 2N (1) ———
analytical (3) ------

Conclusions

- 3D simulations of vortex ring formation and motion have been conducted.

- The time of vortex ring formation is close to the time predicted theoretically and calculated earlier in 2D simulations.

- 3D simulations have been run up to a fairly late stage. At this stage, integral quantities – rising height and radius of the vortex ring– are described consistently by the analytical model, which, in turn, agrees with experiment.

- Analysis of turbulence in the vortex ring has shown that the energy spectrum of fluctuations of turbulent velocity corresponds most closely to a 3D Kolmogorov spectrum.

Thank you for your attention!

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