

An analysis of space trajectories for the Earth-Apophis-Earth mission

Vyacheslav V. Ivashkin^{1,2}, Anqi Lang²

¹ Keldysh Institute of Applied Mathematics, RAS, Russian Federation,

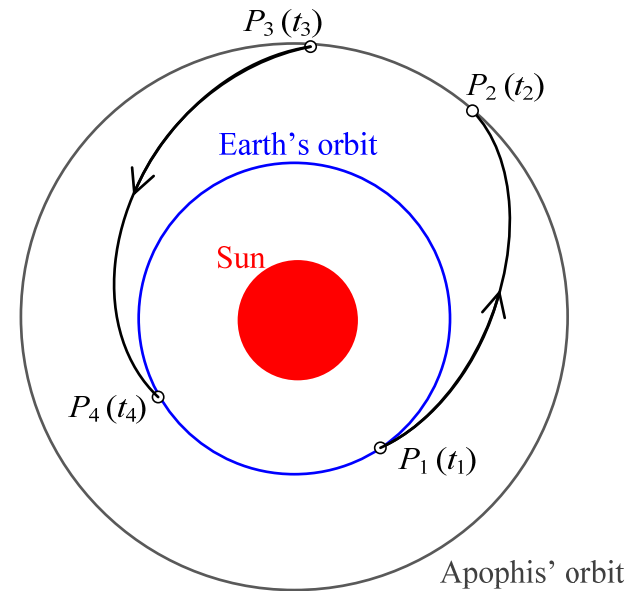
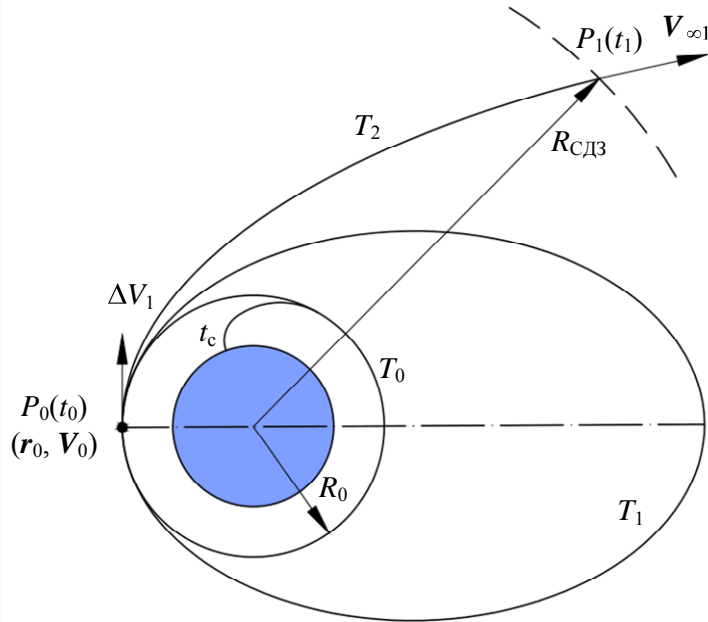
² N.E. Bauman Moscow State Technology University, Russian Federation
Ivashkin@keldysh.ru; seatu_angel@126.com

2017-3-21

Introduction

- The trajectories of spacecraft flight from Earth to Apophis with return to Earth, including the flight of orbiting around Apophis for sometime (for the main spacecraft and a special mini-satellite with a beacon (to refine the orbit of Apophis by processing the signals from it during a long-term movement around the asteroid)), are investigated.
- The optimal trajectories with maximum spacecraft payload mass are determined for the flight Earth-Apophis-Earth.
- The characteristics of the movement of the main spacecraft and the mini-satellite (probe) around the asteroid are studied, taking into account three types of perturbations: the gravitational effects of far celestial bodies (Sun, Earth, Moon, Venus, and Jupiter), non-spherical structure of Apophis and solar radiation pressure (SRP).

The flight scheme



$$\Delta t_{\Sigma} = t_4 - t_1 = \{390, 420, 450, 510, 540, 570, 600, 630, 660, 690, 730\} \text{ days.}$$

$$t_1 \in [2019.05.01; 2022.12.31]; \Delta t_A = t_3 - t_2 = 7 \text{ days}$$

More precise calculation of the trajectories

1) The correction of the trajectories, taking into account the real gravity field and SPR.

$$\frac{d^2 \mathbf{r}}{dt^2} = -\frac{\mu_E}{|\mathbf{r}|^3} \mathbf{r} - \sum_i \mu_i \left(\frac{\mathbf{r}_i}{|\mathbf{r}_i|^3} + \frac{\mathbf{r} - \mathbf{r}_i}{|\mathbf{r} - \mathbf{r}_i|^3} \right) + \Delta;$$

2) The correction of the trajectories' energy characteristics (Velocity, mass)

- Non-pulsed accelerations are considered by taking into account the gravity losses - $\Delta V_1' = \Delta V_1 - \sum \delta V_{grk}$ (Using 2 or 3 burns for the transfer from a LEO to an interplanetary orbit);
- The characteristics of the upper stage Fregat have been specified (the mass of the structure, propellants and so on);
- Taken into account the cost of the spacecraft trajectory correction and of the spacecraft motion control around asteroid $V_{K1} = 50$ m/s, $\Delta V_{K2} = 10$ m/s, $\Delta V_{K2} = 25$ m/s ;
- Given the separated mass of the mini-satellite of Apophis and the landing device – 10 kg и 20 kg (according to the Lavochkin Association recommendation).

Optimal trajectories

Table.1 Optimal trajectories for the case $\Delta u < 2\pi$

	Trajectory 1
Δu	$\Delta u < 2\pi$
Δt_{Σ} (days)	450
t_1 (data)	2021,01,23
Δt_{12} (days)	120
Δt_{34} (days)	323
t_4 (data)	2022,04,28
ΔV_1 (km/s)	3.888
ΔV_2 (km/s)	2.329
ΔV_3 (km/s)	0.912
$V_{\text{character}}$ (km/s)	7.129
m_f (kg)	379
m_p (kg)	149

Figure 1 (a)

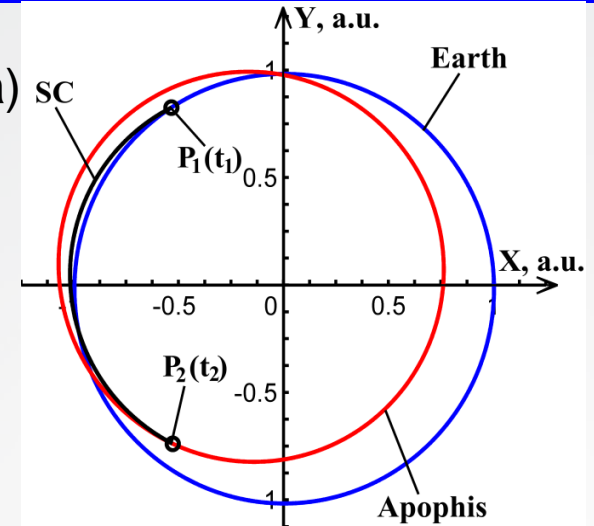
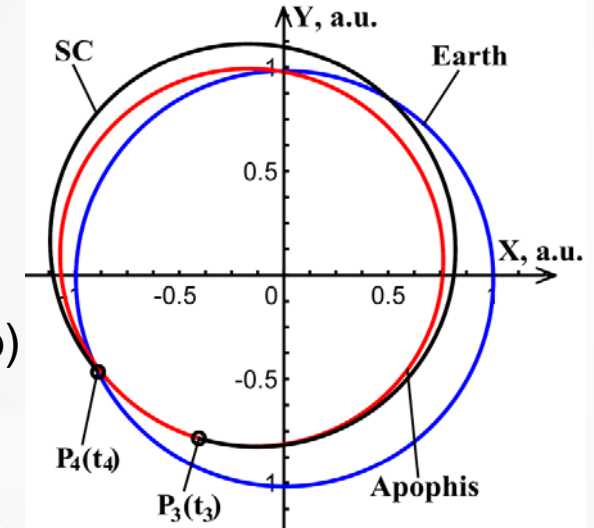


Figure 1 (b)



Optimal trajectories

Table.2 Optimal trajectories for the case $\Delta u_i > 2\pi$ ($i=1,2$)

	Trajectory 2
Δu	$\Delta u_i > 2\pi$ ($i=1,2$)
Δt_{Σ} (days)	690
t_1 (data)	2019,05,24
Δt_{12} (days)	335
Δt_{34} (days)	348
t_4 (data)	2021,04,13
ΔV_1 (km/s)	3.422
ΔV_2 (km/s)	2.851
ΔV_3 (km/s)	0.370
V_{charact} (km/s)	6.643
m_f (kg)	488
m_p (kg)	222

Figure 2 (a)

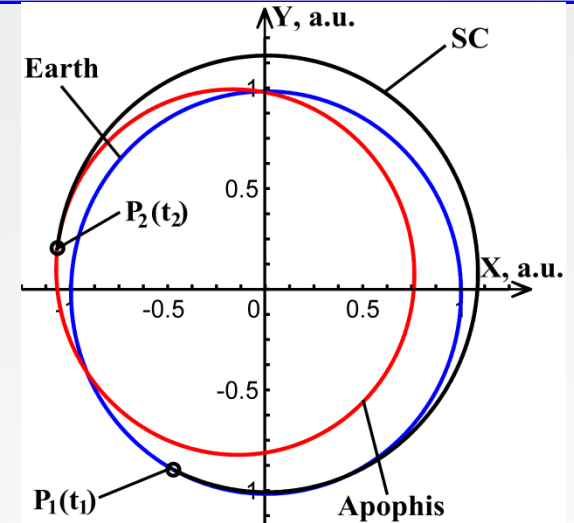
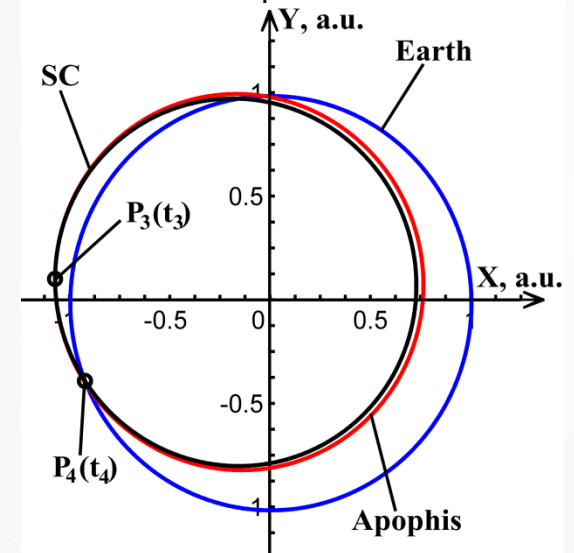


Figure 2 (b)



Payload mass m_p

Table.3 The value of payload mass m_p when using different rockets

	Trajectory 1			Trajectory 2		
	Soyuz-FG	Soyuz-2	Zenith	Soyuz-FG	Soyuz-2	Zenith
Initial mass m_0 (kg)	7130	8250	14000	7130	8250	14000
Final mass m_f (kg)	379	449	971	488	599	1183
Payload mass m_p (kg)	149	196	548	222	296	691

The equations of spacecraft motion around the asteroid (in a none-rotating Cartesian coordinate system centered at the asteroid)

$$\begin{cases} \frac{d\mathbf{r}}{dt} = \mathbf{v}; \\ \frac{d\mathbf{v}}{dt} = \mathbf{a}_0 + k_1 \cdot \mathbf{a}_1 + k_2 \cdot \mathbf{a}_2 + k_3 \cdot \mathbf{a}_3; \\ k_1 = (0;1), k_2 = (0;1), k_3 = (0;1). \\ \mathbf{a}_0 = -\frac{\mu_A}{r^3} \mathbf{r} \quad \mu_A = 1.8 \sim 2.86 \text{ m}^3/\text{s}^2 \end{cases}$$

\mathbf{a}_0 - central acceleration of gravity

\mathbf{a}_1 - gravitational perturbation of Solar system major bodies

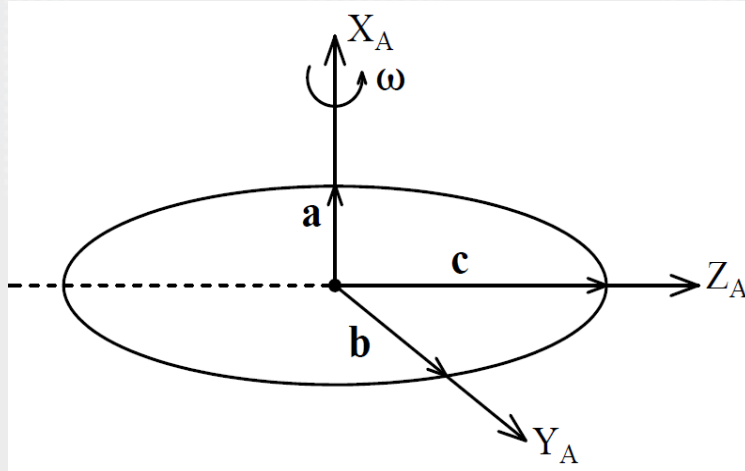
\mathbf{a}_2 - perturbation acceleration from asteroid nonsphericity

\mathbf{a}_3 - SRP acceleration

Numerical analysis (taking into account Sun, Earth, Moon, Venus, and Jupiter) are performed based on **trajectory 2**. The initial date is set on **23 April 2020**, when SC approaches to the asteroid. The initial spacecraft orbit is circular, with the radius r_0 within the range of **0.5-2 km**.

Model of Apophis

Figure 3. Model of Apophis



We assume:

1. $a_A = b_A < c_A$, elongation $\alpha = c_A/a_A = (1.3; 1.5; 1.7)$.

2. $\omega \sim a \sim L$, $\omega_{Z_A} = \omega_{Y_A} = 0$, $\omega_{X_A} = \omega$.

Parameters

μ_A (m^3/s^2)	1.8-2.86
R_A (m)	160
b_A/a_A	1.06 (± 0.02)
c_A/a_A (α)	1.5 (± 0.2)
λ_L (deg)	250
β_L (deg)	-75
P_A (hours)	30.56

P. Pravec et al. / Icarus 233 (2014)

48-60

Table.4 Parameters of asteroid
Apophis

Calculation results and analysis

1. Gravitational effects of solar system major bodies.
2. Effects of asteroid's nonsphericity.
3. Effects of solar radiation pressure (β_0).

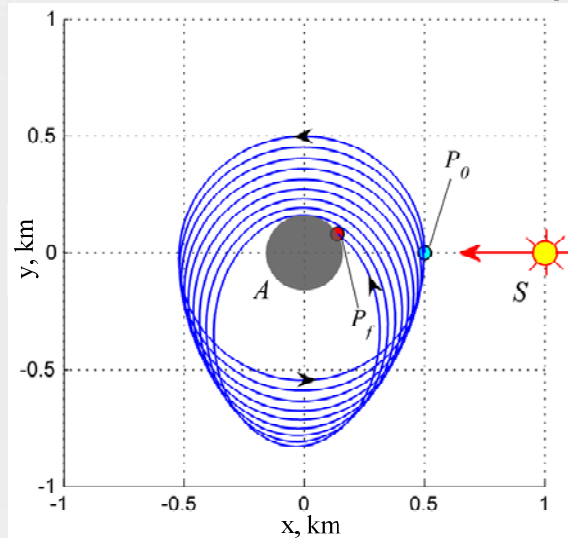


Figure 4. Main SC orbit about Apophis with only a_3 ($T \sim 5$ days) for the case $t_0 = 2020/04/23$, $\Omega_0 = 0$, $i_0 = 90^\circ$, $r_0 = 0.5$ km, $\mu_A = 2.86$ m³/s² ($\beta_0 \sim 80^\circ$)

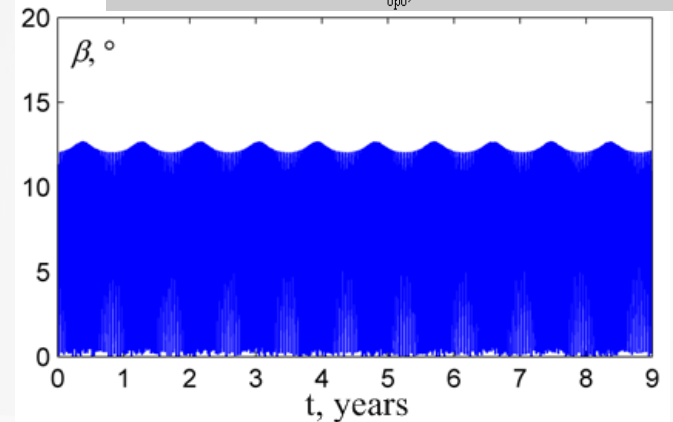
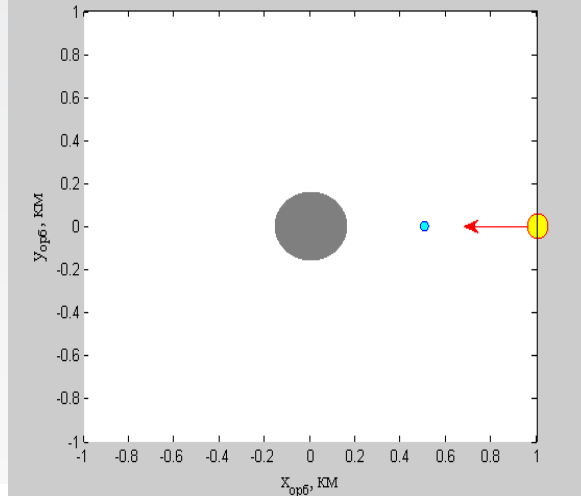


Figure 5. Evolution of angle β for the case $t_0 = 2020/04/23$, $\Omega_0 = i_0 = 90^\circ$, $r_0 = 0.5$ km, $\mu_A = 2.86$ m³/s² ($\beta_0 \sim 10^\circ$)

Joint effects of all perturbations

💡 **Attention!** The Non-linear correlation between the influence of the asteroid's nonsphericity and the influence of SRP.

Table 5: Lifetime of **the main spacecraft** motion about Apophis ($r_0 = 0.5$ km)

μ_A (m ³ /s ²)	α	T (day)
1.8	1.3	136
	1.5	88
	1.7	38 (A)
2.86	1.3	71
	1.5	310
	1.7	60

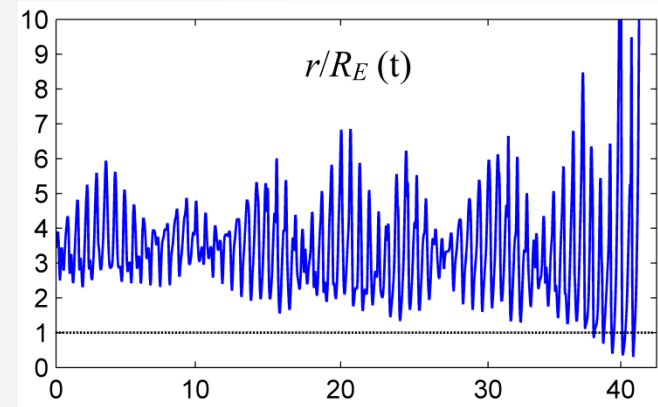


Figure 6. Evolution of the relative distance for example A : $\mu=1.8$ m³/s², $\alpha=1.7$, $r_0=0.5$ km, $e_0=0$, $\Omega_0 = i_0 = 90^\circ$

Joint effects of all perturbations

Table 6: Lifetime of **the mini-satellite** motion about

Apophis μ_A (m^3/s^2)	α	r_0 (km)	T
1.8	1.3-1.5	1.5	~ 9 years
	1.7	0.9	~ 1.2 years
		1.5	~ 9 years (B, 2020-2029)
		2.0	~ 80 days
2.86	1.7	1.0	~ 2 years
	1.3-1.7	1.5	~ 9 years
		2.0	~ 9 years
		2.5	~ 90 days

Due to Apophis' close encounter with Earth on **April 13th, 2029**, an increase of the Earth's gravitational effects results in the mini-satellite escape from the vicinity of the Apophis.

Joint effects of all perturbations

Example B: $\mu_A=1.8$
 m^3/s^2 , $\alpha=1.7$, $r_0=1.5$
 km , $\Omega_0=i_0=90^\circ$.

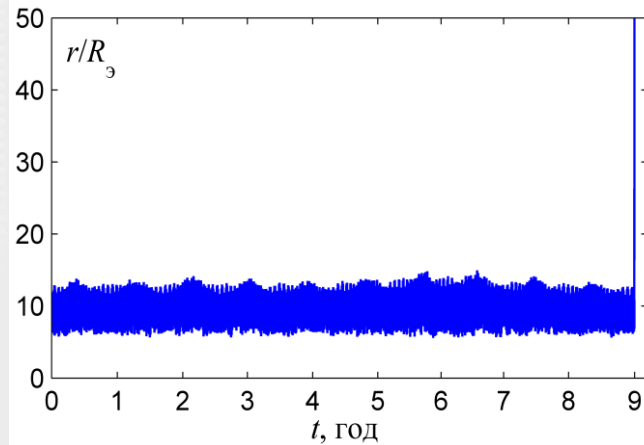


Figure 7. Evolution of the relative distance for example B.

P_0 - position of mini-satellite on **April 13th 2029**, Min(distance) ~ 38000 km.

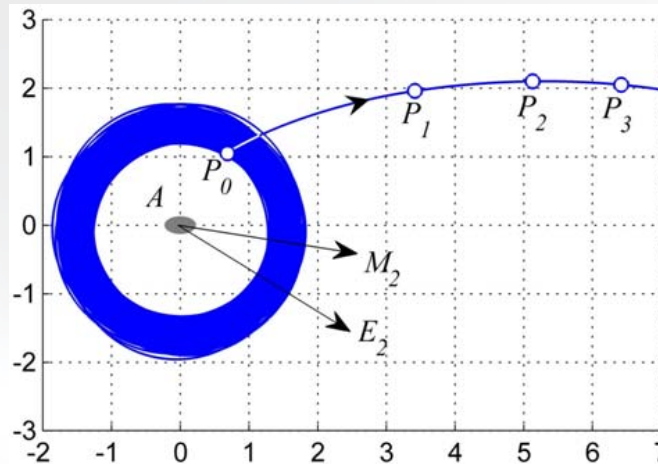


Figure 8. Mini-satellite trajectory during about 9 years (2020-2029) for example B with all perturbations. Units are in kilometers.

Joint effects of all perturbations

A number of main parameters are changed for more comprehensive analyses, in particular:

- The radius of the initial orbit r_0
- The orbital eccentricity e
- Gravitational parameter of the asteroid μ_A
- The asteroid's elongation α
- The rotation parameters of the asteroid,
- The mass and orbit parameters for SC (its plane, size, eccentricity),
- The start time of spacecraft motion
- The initial position of the satellite in orbit

Main results of the analysis presented in the paper are preserved in these cases.

Table 7: Some bounds on initial radius r_0 under which the mini-satellite can orbit the Apophis for about **9 years** in the presence of all perturbations

μ_A (m ³ /s ²)	α	r_0 (km)
1.8	1.5-1.7	1.3 - 1.6
2.86	1.7	1.4 - 2

Conclusions

- The optimal trajectories with maximum spacecraft payload mass are determined for the flight Earth-Apophis-Earth in the course of 2019-2022. Two cases are investigated: 1) interplanetary paths have no complete revolutions; 2) one of the interplanetary paths has 1 complete revolution. Energy characteristics are estimated using different launch vehicles (Soyuz-FG, Soyuz-2, Zenith) and the upper stage Fregat, taking into account gravity losses, the perturbations of the celestial bodies and SRP.
- The orbital motion of the main spacecraft and the mini-satellite with a beacon around the asteroid are studied, taking into account three types of perturbations: the gravitational effects of far celestial bodies (Sun, Earth, Moon, Venus, and Jupiter), the non-spherical structure of Apophis and solar radiation pressure (SRP). The asteroid is modeled as an ellipsoid of revolution rotating around its own minor axis.
- In the case of joint influence of all perturbations special attention should be paid to the interaction between the influence of the asteroid's nonsphericity and SRP.

Conclusions

- To improve the stability of SC motion and SC lifetime near asteroid Apophis, the optimal initial orientation of the SC orbit plane has to be normal to the asteroid-Sun direction.
- There is an optimal (in terms of "lifetime") of the initial orbital radius for the mini-satellite, $\sim 1.5\text{km}$.
- It is shown that the timing requirements for the satellite orbit of Apophis can be satisfied, "lifetime" of the main satellite (with an initial radius $r_0 \sim 0.5\text{ km}$) is about a few weeks, "lifetime" of the mini-satellite (with $r_0 \sim 1.5\text{ km}$) can be a few years, from 2020 to 2029.

Thank You for Your Attention !