Academician Evgeny Zababakhin Centenary
International Conference

«13th Zababakhin scientific talks»

Numerical solution of one-dimensional ideal gas shock-free ultrahigh compression problem subject to the conditions on characteristic

Bautin S.P., Novakovskiy N.S.

n.s.novakovskiy@yandex.ru
sbautin@usurt.ru

Plan

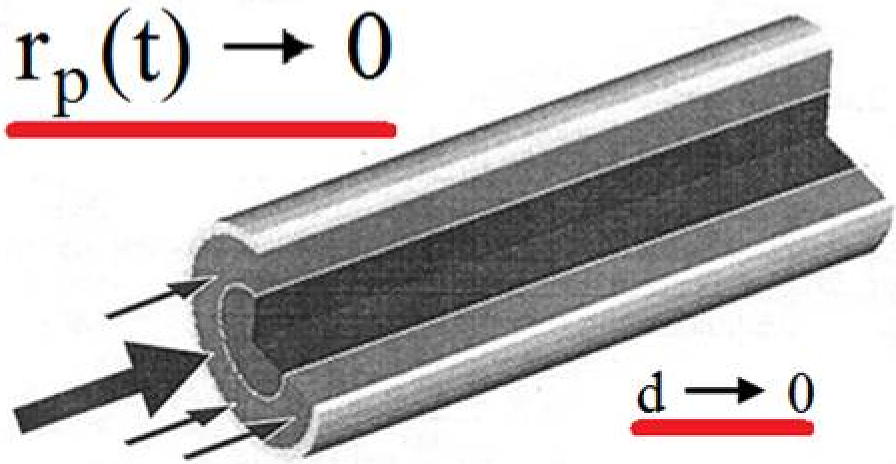
1. Problem’s general information

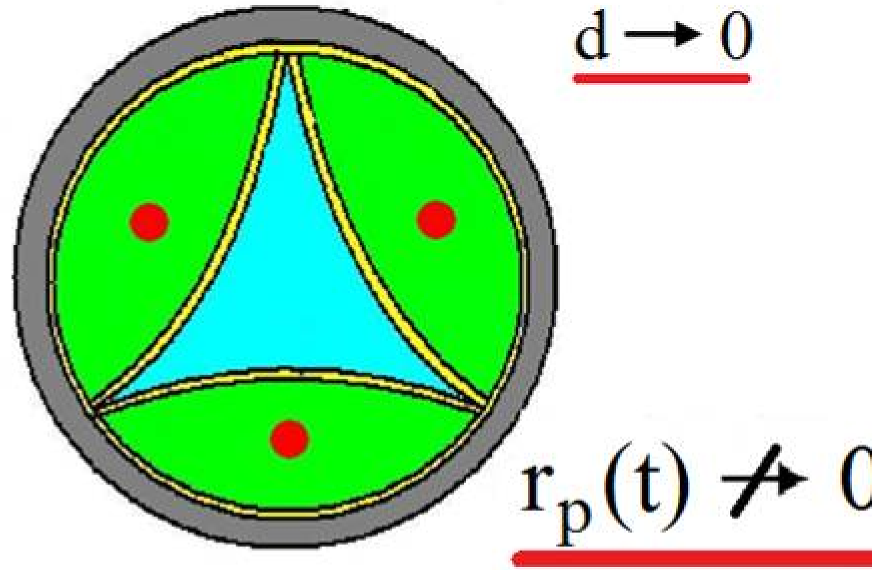
2. Application of characteristic’s method for solving the problem in time decreasing case.

3. The numerical method with bound condition set on characteristic for solving the problem in time increasing case. Calculations results.

4. Conclusions.

ICF target examples





All in Russian:

[1] **Zababakhin E. I., Zababakhin I. E. Phenomena of unlimited cumulation. M.: Nauka, 1988.**

[2] Dolgoleva G.V., Zabrodin A.V. Energy cumulation in layer systems and shockless comression realisation, M, FIZMATLIT, 2004.

[3] Bautin S.P. Gas strong compression mathematical modeling. Novosibirsk: Nauka. 2007.

[4] Suchkov V.A. The outflow of gas into the vacuum on the oblique wall//AMM 1963. V. 27. r.4. P. 739-740.

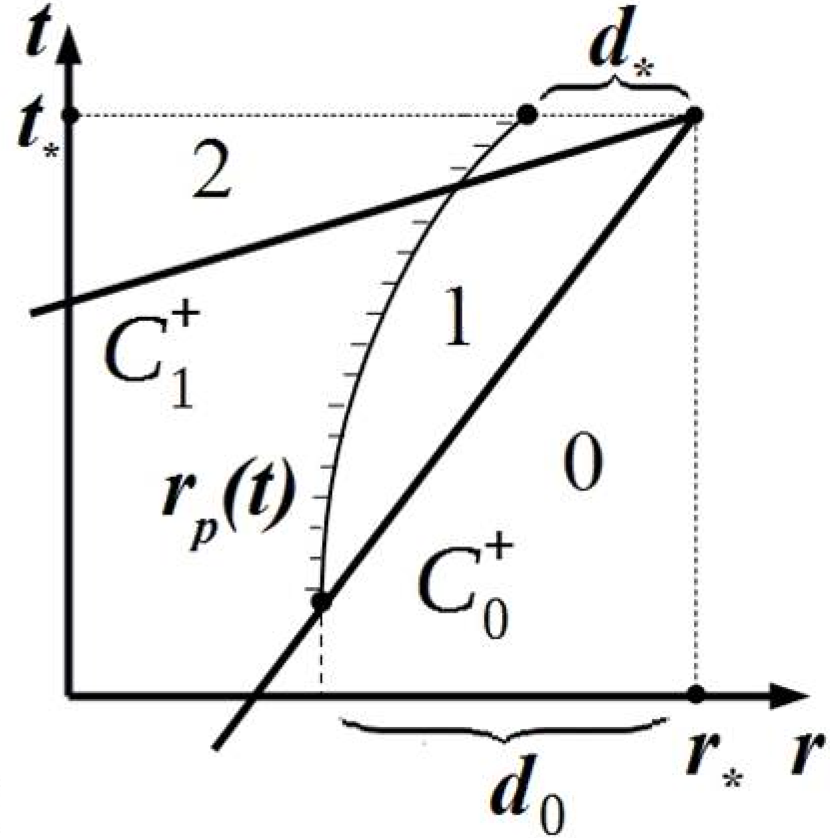
[5] Sidorov A.F. New regimes of unlimited shockless compression of a gas.//Dokl. Phys. 44, #.1, 36-40(1999)

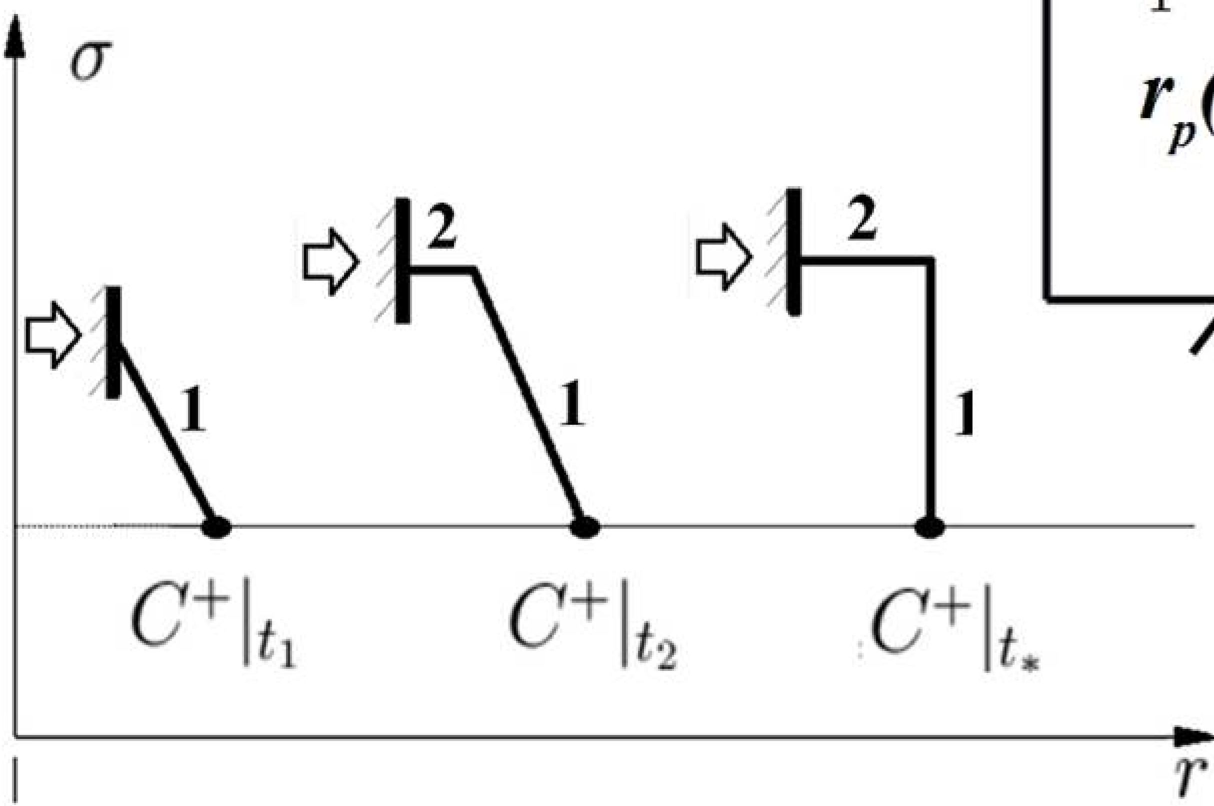
[6] Nikolaev Yu.V. About numerical solution of shockless powerful compression of unidimensional gas layers problem.//Computational technologies.. 2001. V. 6. N. 2. P. 104-108.

[7] Anuchin M. G. Influence of thermal conductivity on unlimited shockless compression of a flat gas layer. // AMTF. 1998. V 39, N. 4. P. 25–32.

[8] Artem'ev A.Yu., Delov V. I., Dmitriyeva L. V. and others. Numerical simulation of shockless unconfined gas compression in Lagrange variables by the method of D.// VANT. Ser.: Mat. Mod. Fiz. Proc 1995 No.4. p. 42-47.

(σ-r) – compression diagram





The 1D gas dynamics equations (GDE)

$$\begin{cases} \sigma_t + u\sigma_r + \dfrac{(\gamma-1)}{2}\sigma\left(u_r + \nu\dfrac{u}{r}\right) = 0, \\ u_t + \dfrac{2}{(\gamma-1)}\sigma\sigma_r + uu_r = 0, \\ s = 1. \end{cases}$$

$$\sigma = \rho^{\frac{\gamma-1}{2}} \qquad \nu = 0, 1, 2$$

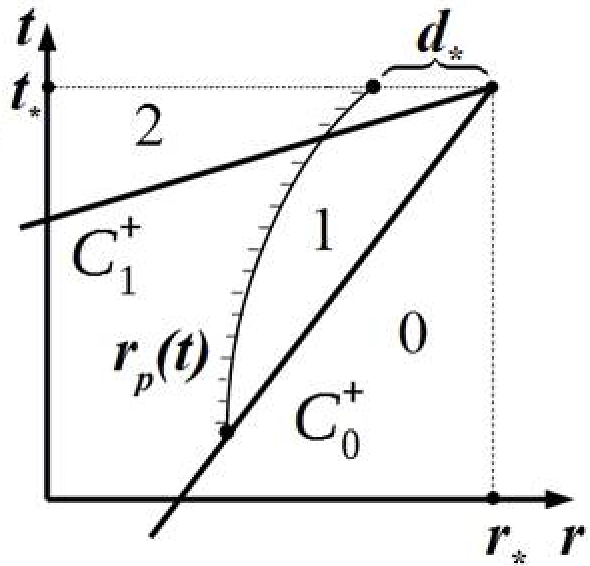
$$r = \begin{cases} x_1, \nu = 0, \\ \sqrt{\sum_{i=1}^{\nu+1} x_i^2} \geq 0, \nu = 1, 2\,. \end{cases}$$

Exact solution in region 1

$\nu = 0$ Rieman's centered wave (CW)

$$u(t,r) = \frac{2}{\gamma-1}\sigma - \frac{2}{\gamma-1}\text{const},$$

$$\frac{r-r_*}{t-t_*} = \frac{\gamma+1}{\gamma-1} - \frac{2}{\gamma-1}\text{const}$$



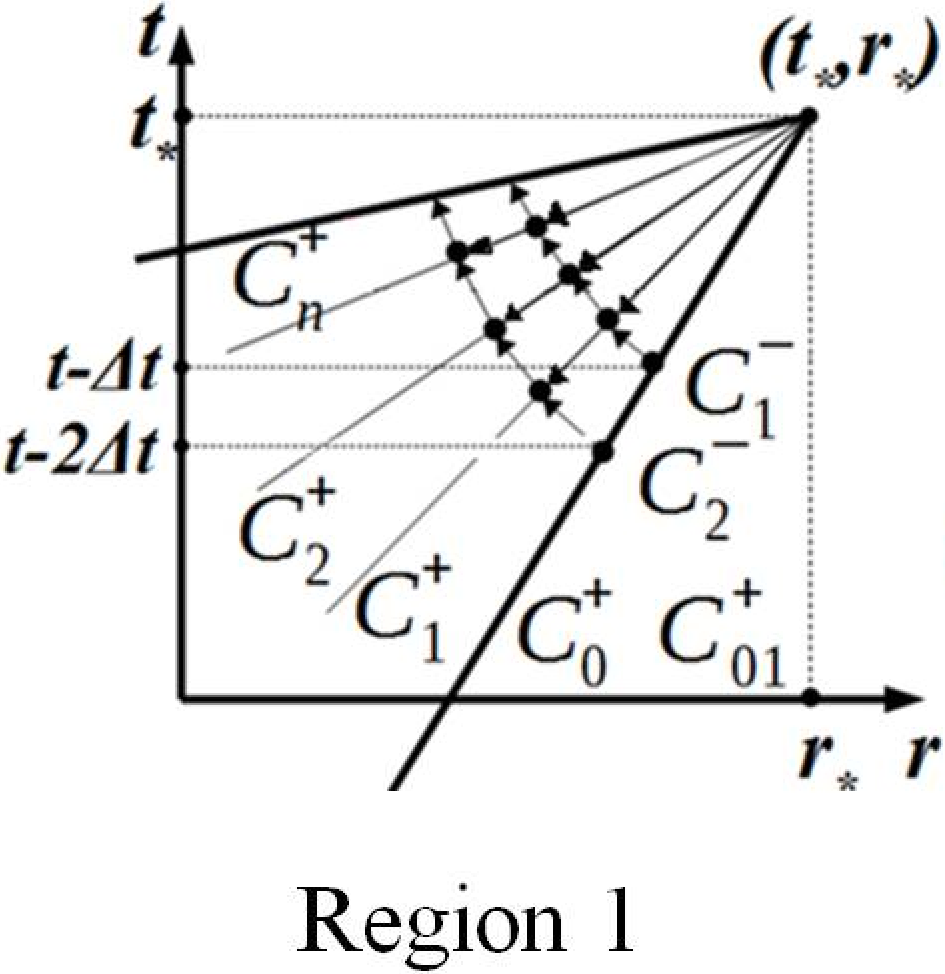
$\nu = 1, 2$ generalizated Rieman's CW

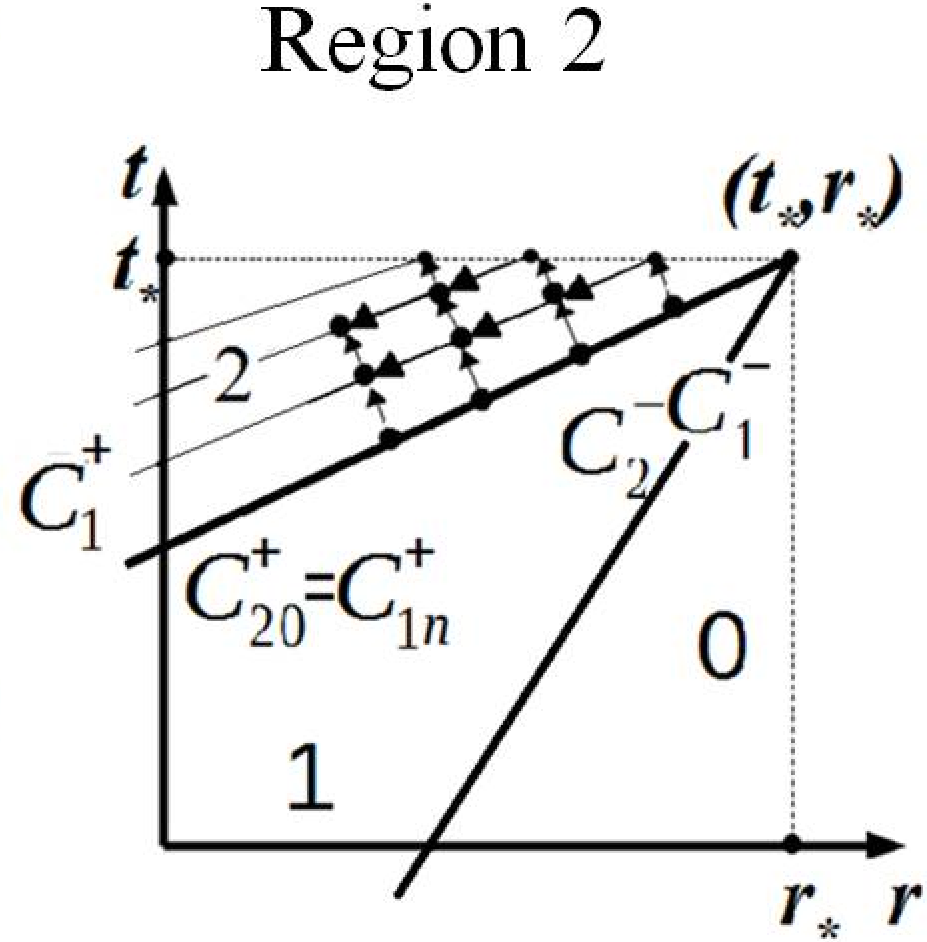
$$u(t,\sigma) = \left[\frac{2}{\gamma-1}\sigma + u_*\right] + (t-t_*)\widetilde{u}(t,\sigma)$$

$$r(t,\sigma) = r_* + \left[\frac{\gamma+1}{\gamma-1}\sigma + u_*\right](t-t_*) + (t-t_*)^2\widetilde{r}(t,\sigma)$$

$$\frac{r-r_*}{t-t_*} = \left[\frac{\gamma+1}{\gamma-1}\sigma + u_*\right] + (t-t_*)\widetilde{r}(t,\sigma)$$

Characteristic's method for solving the problem in time decreasing case



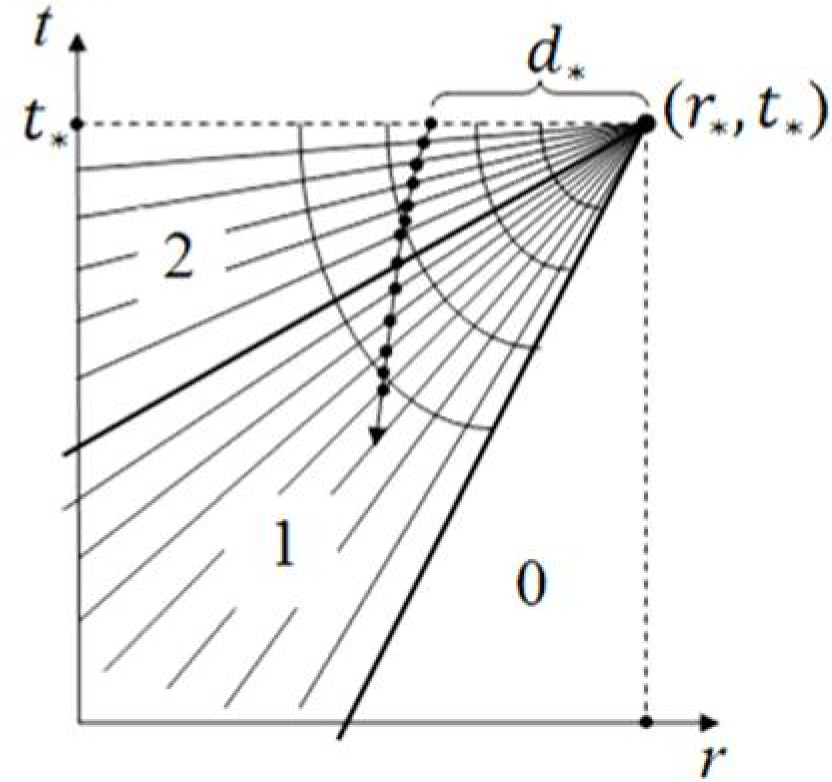


Solution has constructed.

How the recommendation for experiment can be formulated ?

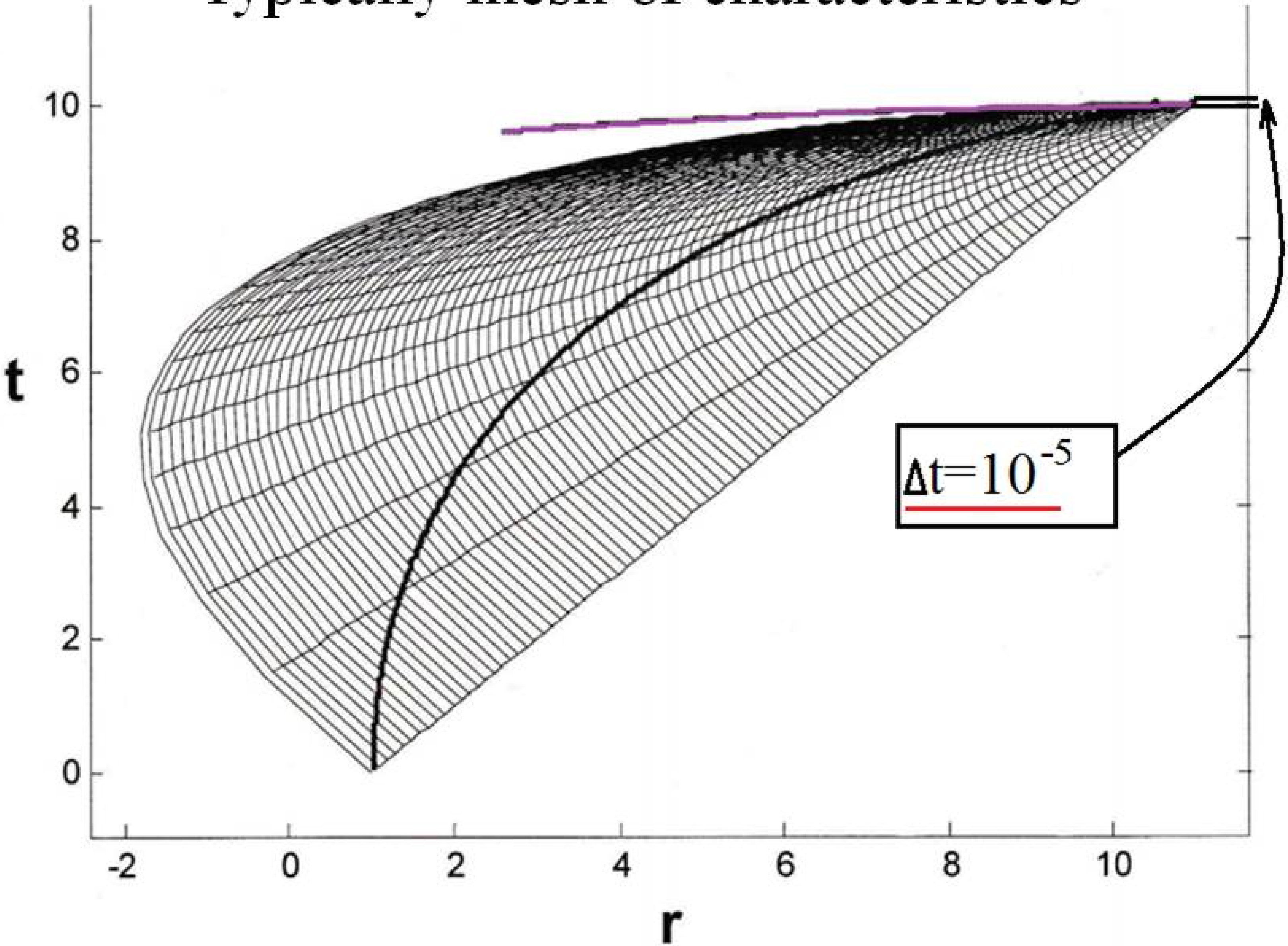
Compressing piston's moving law restoring algorithm

$$\frac{dr}{dt} = u(t,r), r|_{t=t_*} = r_* - d_*$$



1. Find the characteristic mesh's cell which contain point $(r_* - d_*, t_*)$
2. Interpolate velocity to $(r_* - d_*, t_*)$
3. Solve finite-difference equation of piston's trajectory. Calculate new cross point
4. Finish the algorithm if new point is on the C_0^+-characteristic of region **1**. In other cases repeat steps 2-4.

Typically mesh of characteristics



Results obtained in case of time decrease

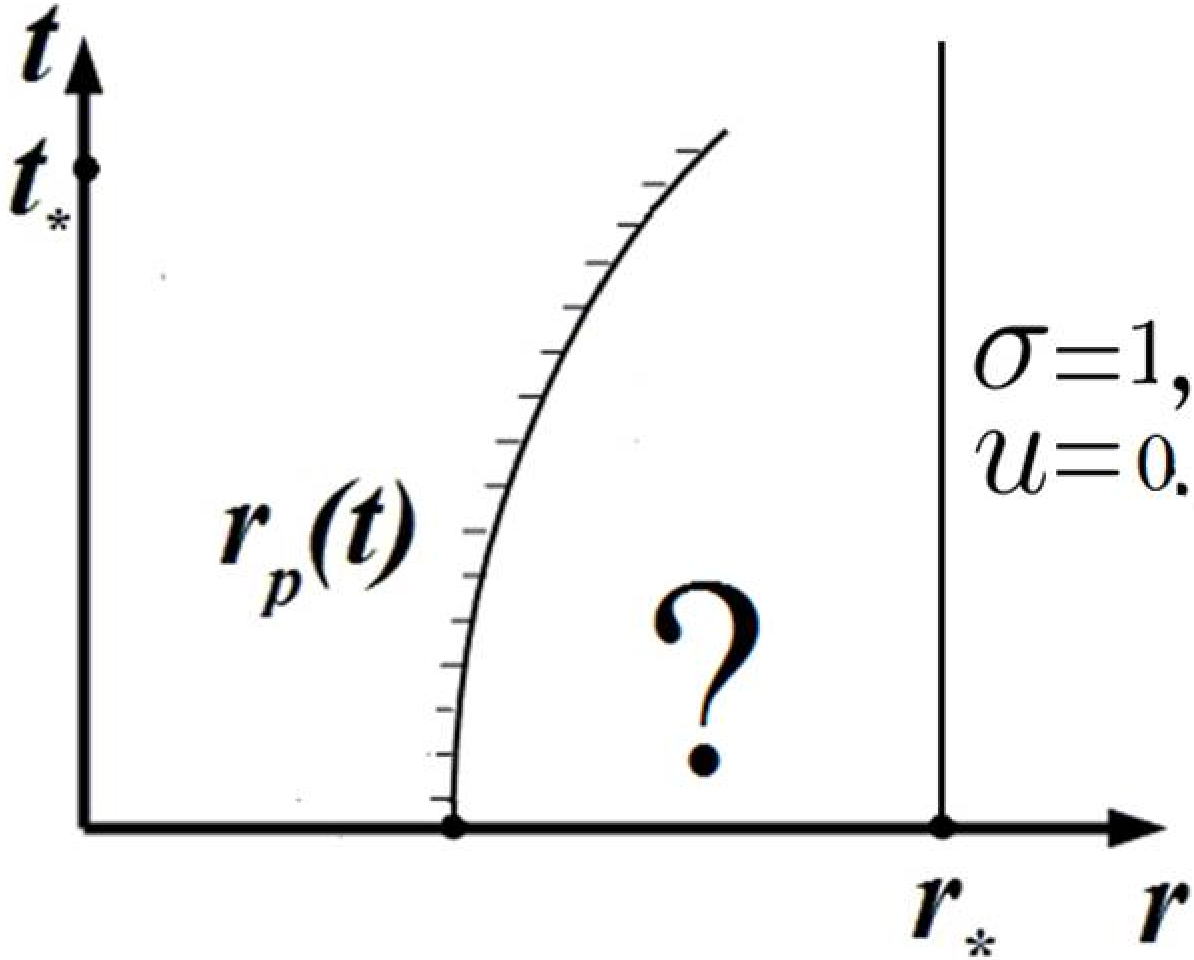
1. Test compression in plane symmetry ($\nu = 0$) has calculated. Calculated values of velocity and sound velocity fully match with exact solution – Rieman's CW.

2. The compressing piston's trajectory definition computational error is decreased in proportion to the increase in mesh points.

3. The calculation results of some other problems in cylindrical and spherical symmetry are in good agreement with previously obtained results.

N.S. Novakovskiy. One-dimensional math modeling of ideal gas strong compression in R. Mises configurtion.//Mathematical Structures and Modeling. 2016 #3 (39).

The time increasing case calculations.



$r_p(t)$ law is given

1D GDE in Lagrangian mass coordinates. The implicit approximation

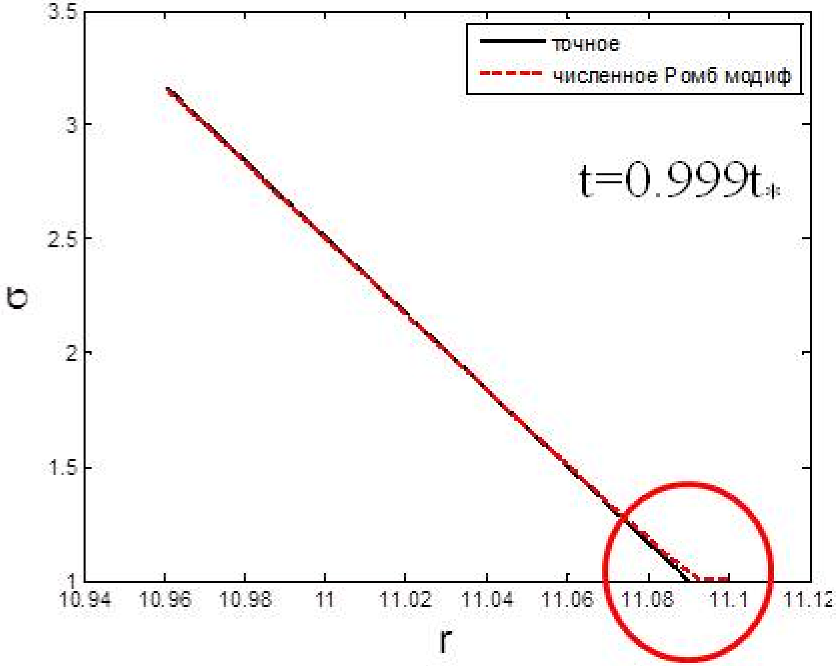
$$\frac{\partial v}{\partial t} = \frac{\partial u}{\partial m},$$
$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial m},$$
$$\frac{\partial E}{\partial t} + v\frac{\partial (pu)}{\partial x} = 0,$$
$$p = (\gamma - 1)\rho c_v T, \varepsilon = c_v T$$
$$u|_{x=x_l}(t) = u_p(t),$$
$$u|_{x=x_r}(t) = 0,$$
$$u(x,0) = 0.$$

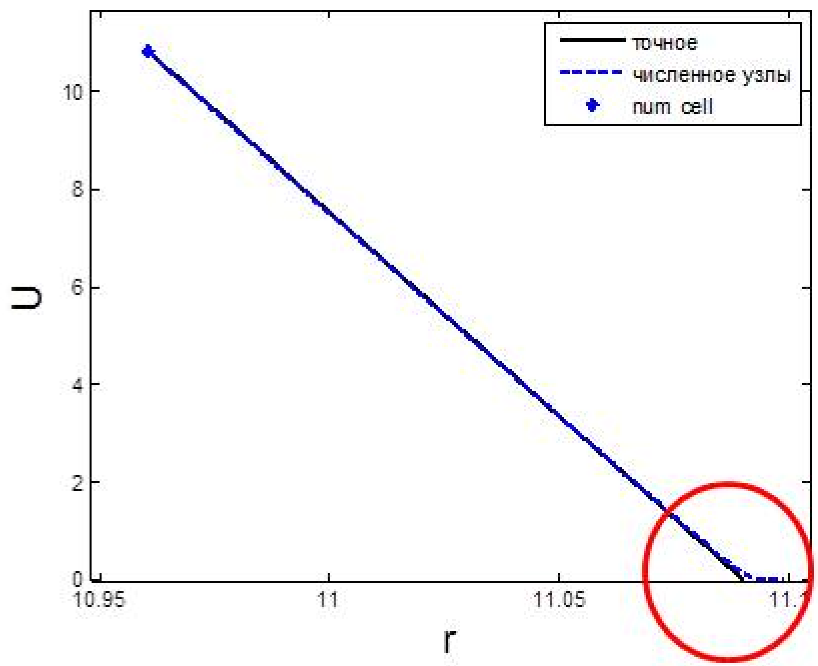
$$\frac{v^{n+1} - v^n}{\tau} = \left(\frac{\partial u}{\partial m}\right)^{n+1},$$
$$\frac{u^{n+1} - u^n}{\tau} = -\left(\frac{\partial p}{\partial m}\right)^{n+1},$$
$$\frac{E^{n+1} - E^n}{\tau} + v^{n+1}\left(\frac{\partial (pu)}{\partial x}\right)^{n+1} = 0,$$
$$u_0^n = u_p(t^n),$$
$$u_N^n = 0,$$
$$u(t^0) = 0; v(t^0) = 1;$$
$$E(t^0) = 1/(\gamma(\gamma - 1))$$

A. D. Gadzhiev, V. N. Pisarev, "“Romb”: An implicit finite-difference method for the numerical solution of the equations of gas-dynamics with heat conduction" // Zh.V.Mat.Mat. Fiz..1979. Vol. 19. № 5. P. 1288-1303.
A.D. Gadzhiev, S.Yu. Kuzmin, S.N. Lebedev, V.N. Pisarev. The implicit finite-difference method “Romb” for solving 2D gas-dymanics equations. VANT. Ser.: Mat. Mod. Fiz. Proc 2001. No.4. C. 11-21.

Results of calculations by "Romb" method

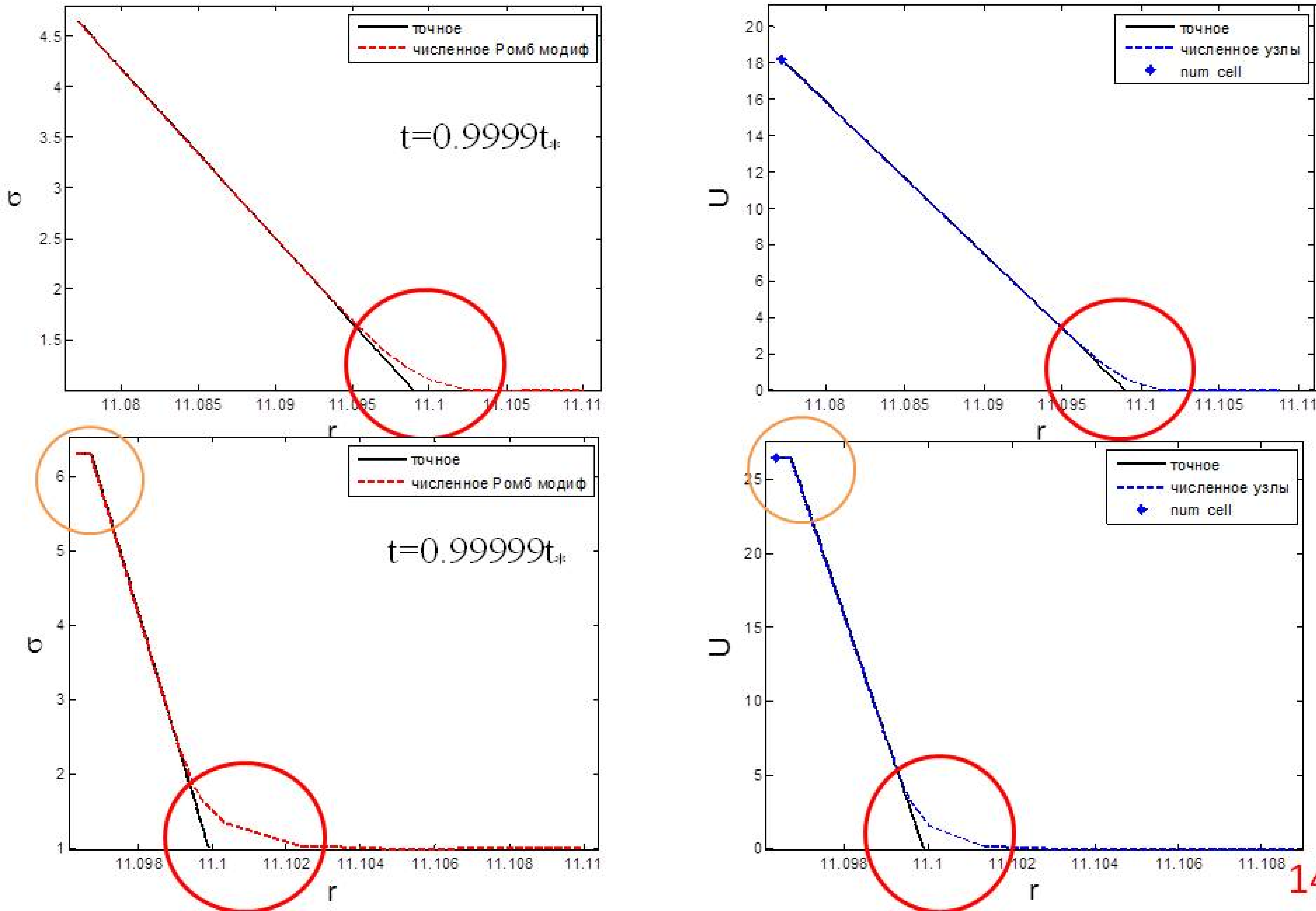
Symmetry	Time moment of mass part maximal compression($t_* = 1$)				
	10%	30%	50%	70%	90%
0	0.999980525	0.9999889	0.9999921	0.9999953	0.999998425
1	0.999983714	0.999991876	0.999994142	0.999996381	0.999998857
2	0. 9999899	0.9999921	0.9999944	0.9999966	0.9999989





symmetry	ν=0	ν=1	ν=2
dm, %	0.1	0.08	0.14

Results of calculations by “Romb” method

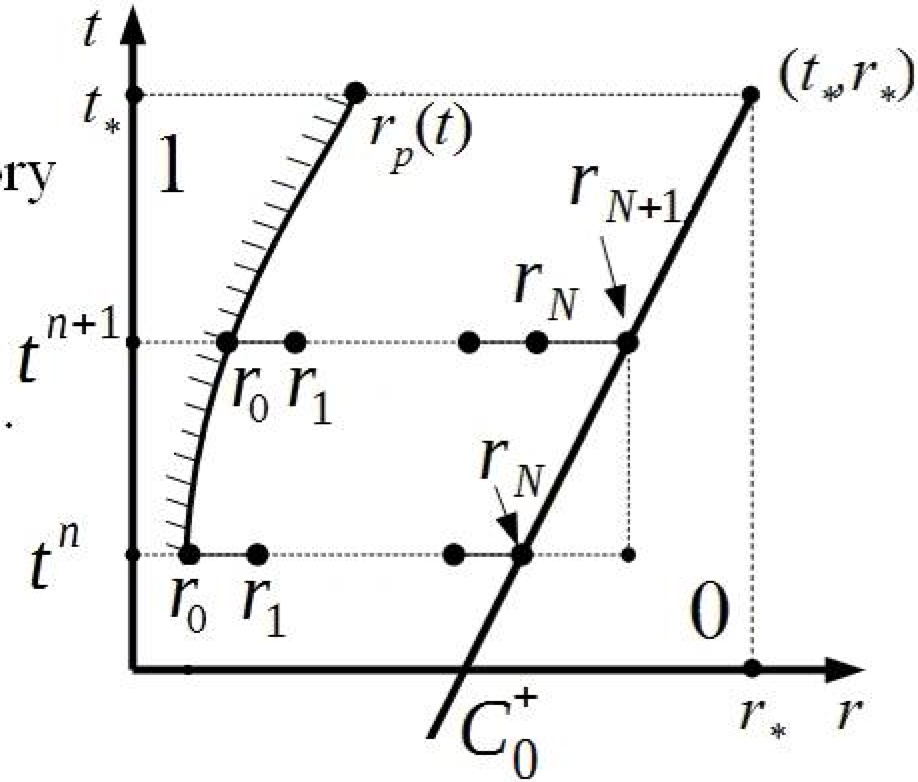


Computational method for the compression region only (from piston to characteristic C_0^+).

1. Get current time's coordinate of C_0^+ characteristic using its trajectory

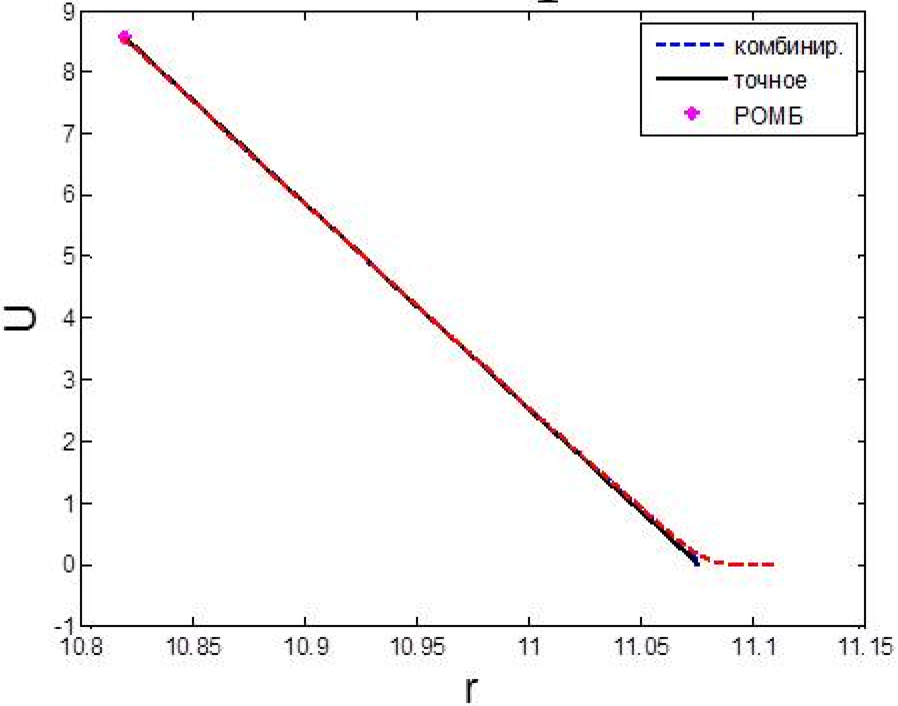
2. Move the right bound - add new interval for computational domain .

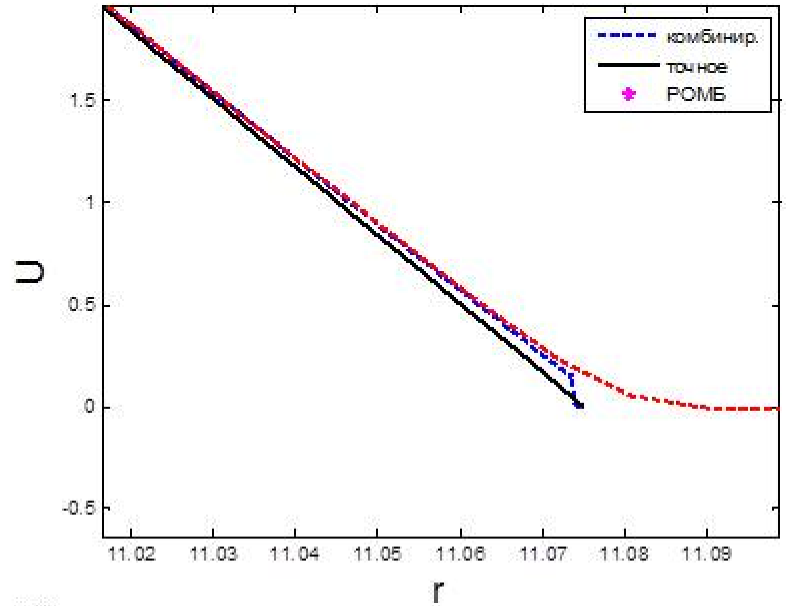
3. Using "Romb" method solve gas dynamics equations in domain between compressing piston and characteristic C_0^+

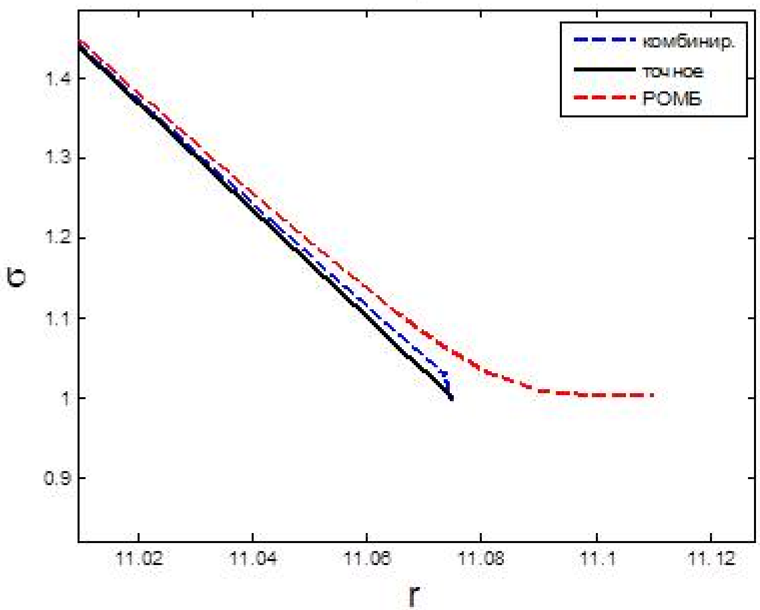


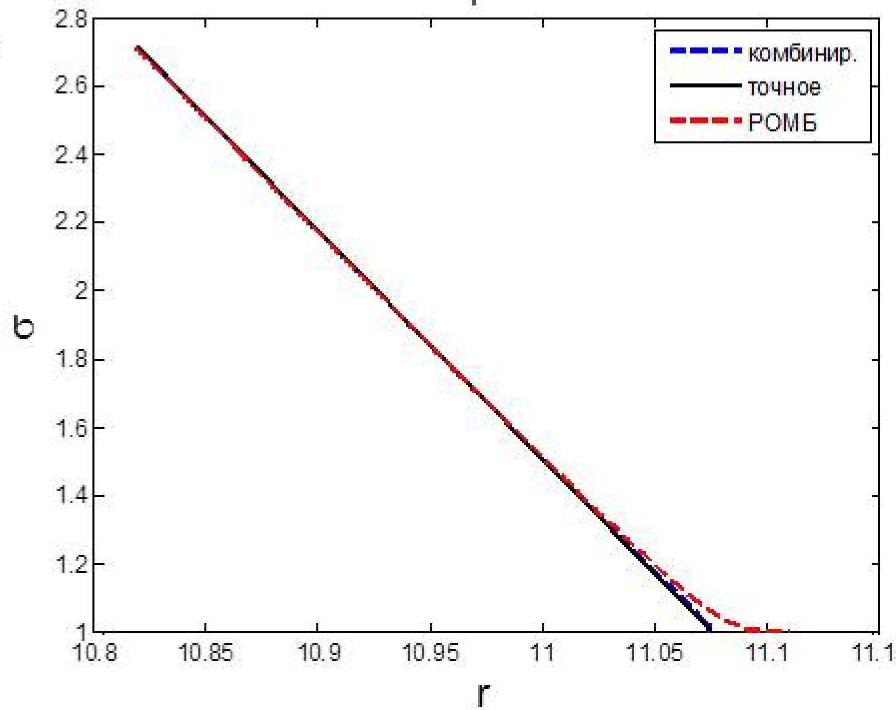
N.S. Novakovskiy. The combined numerical method for solving the one-dimensional ideal gas shock-free strong compression problem in R. Mises configuration//Mathematical Structures and Modeling. 2017 #1 (41).

Comparison with “Romb” results

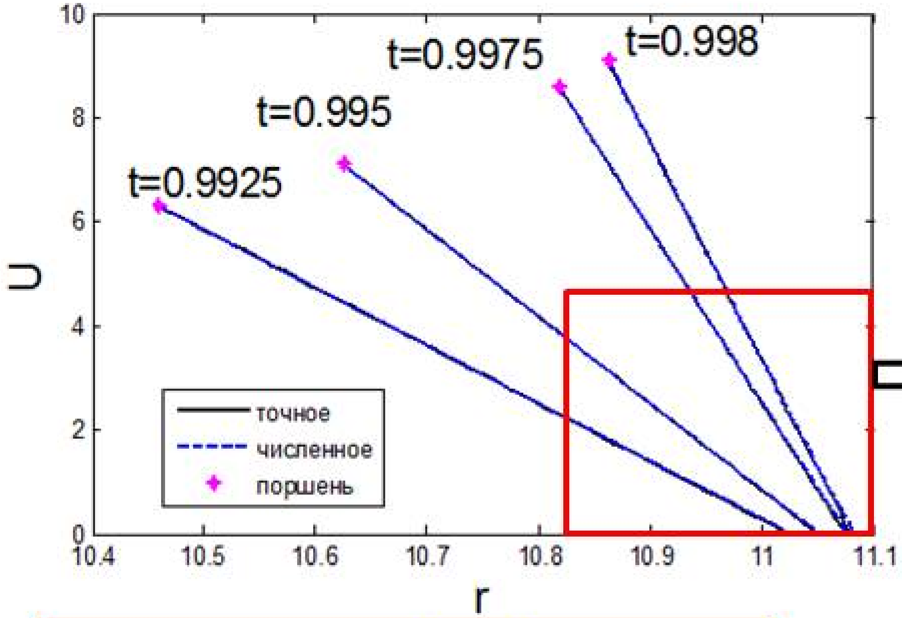


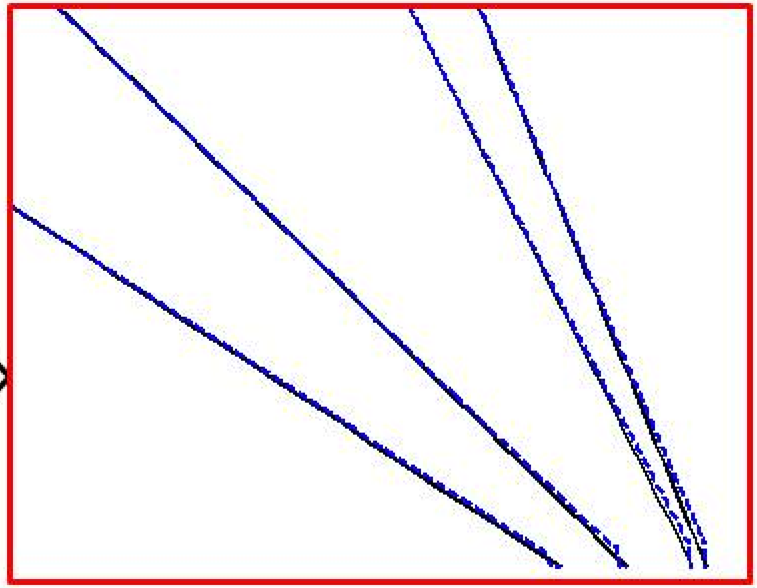


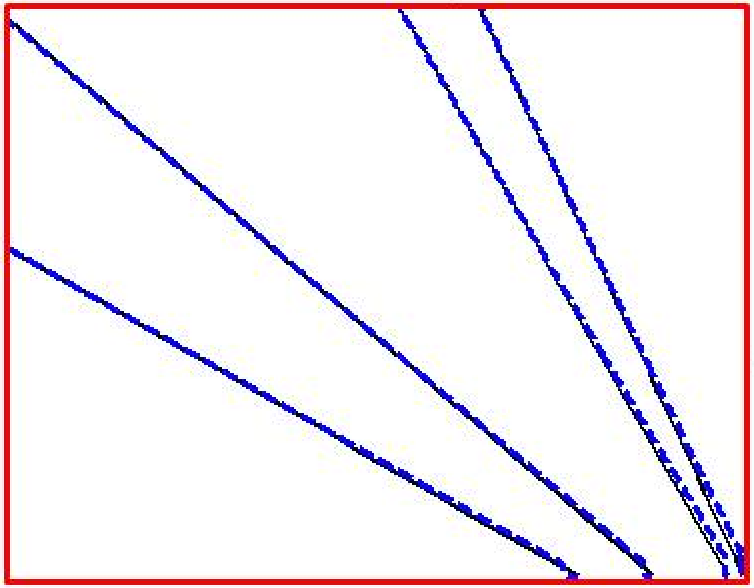


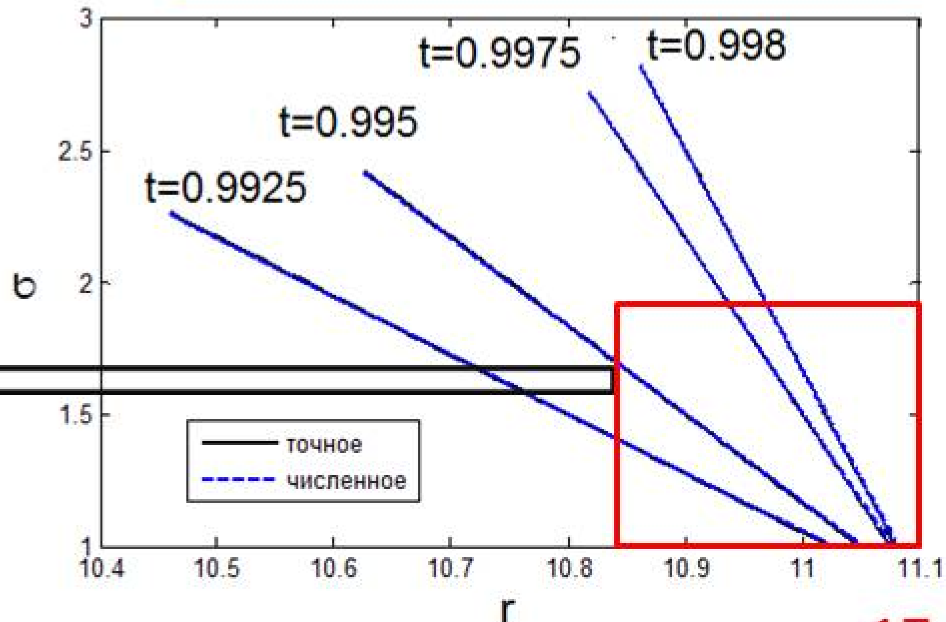


Right bound of computational domain is C_0^+ $v=0$

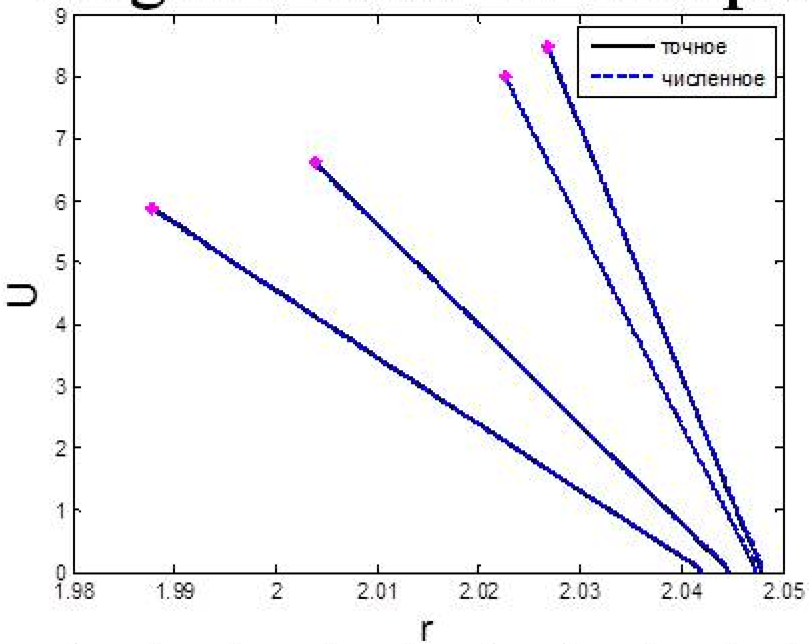


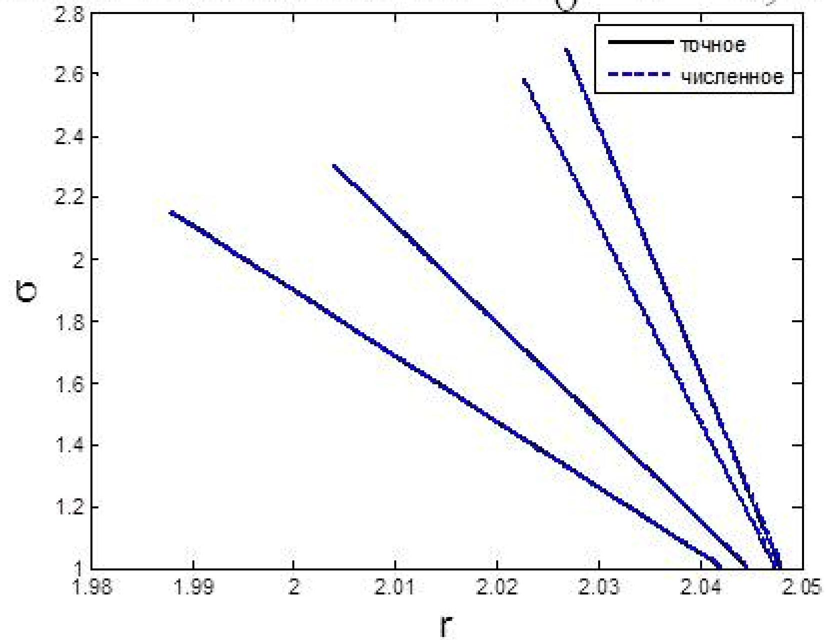


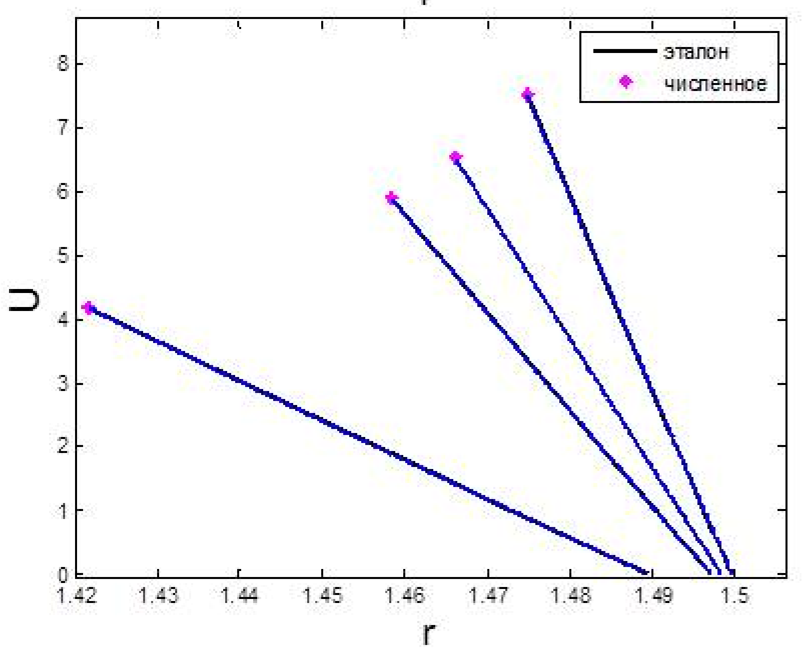


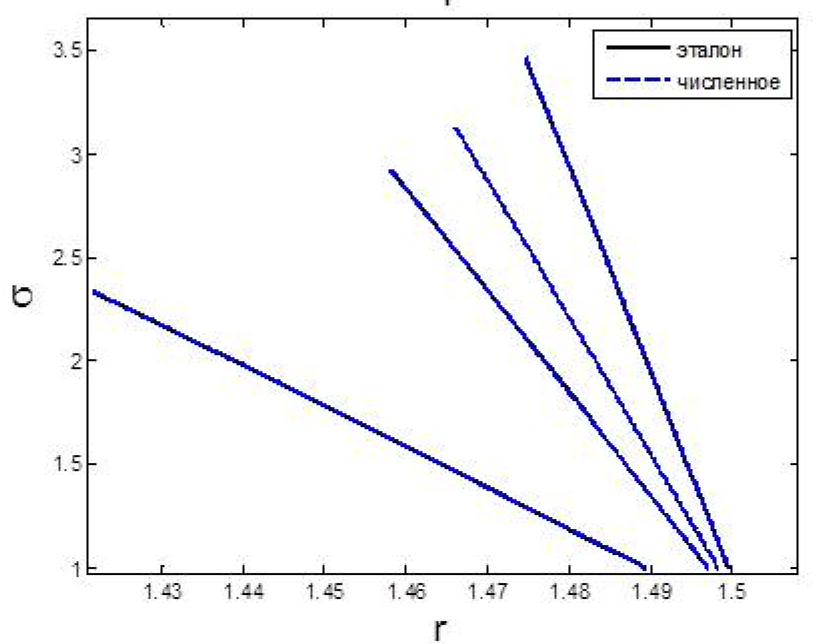


Right bound of computational domain is C_0^+ ν=1,2





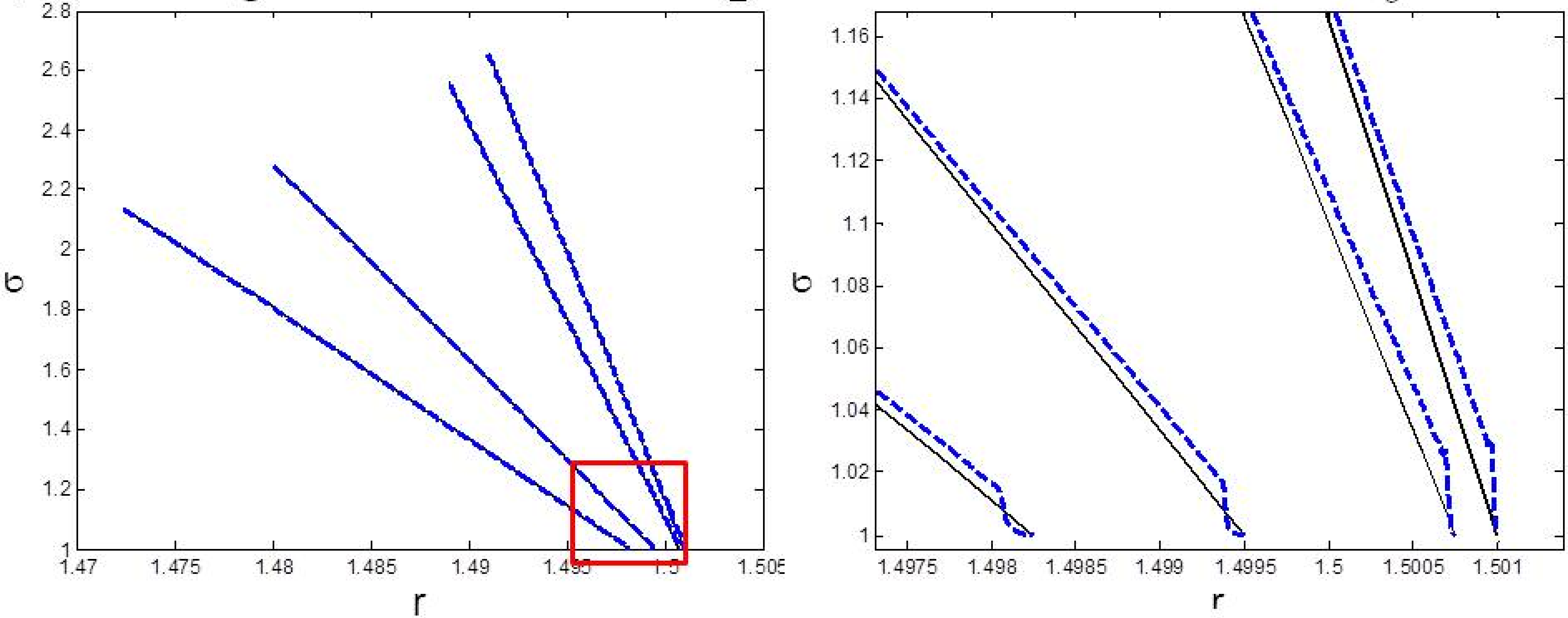




c_i^+

c_i^+

Right bound of computational domain is C_0^+

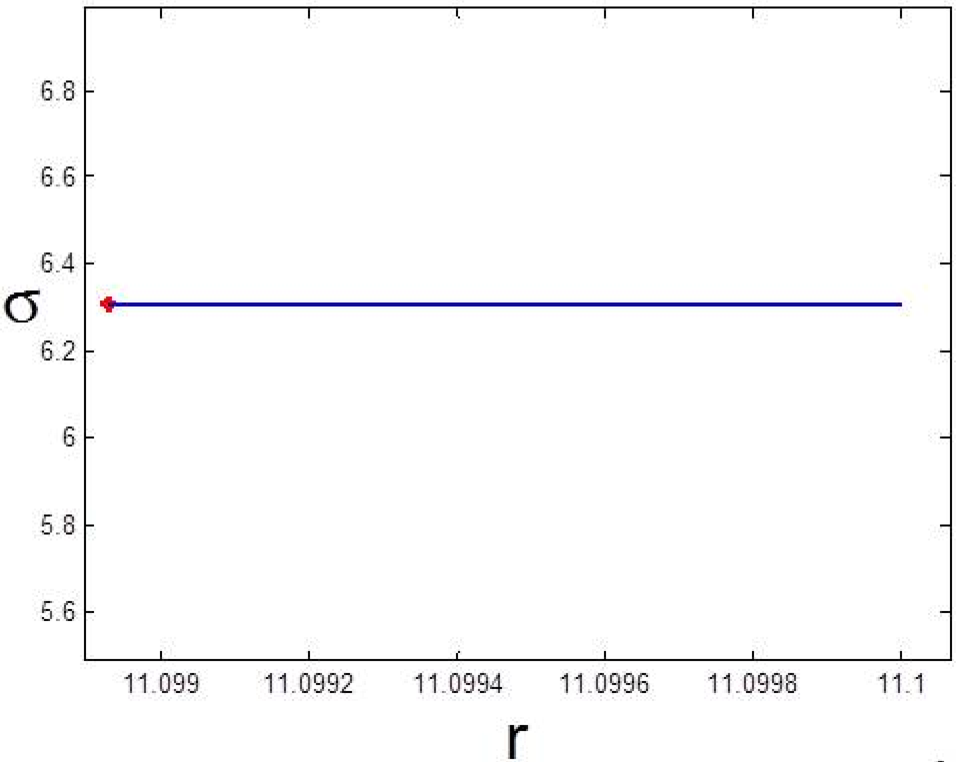


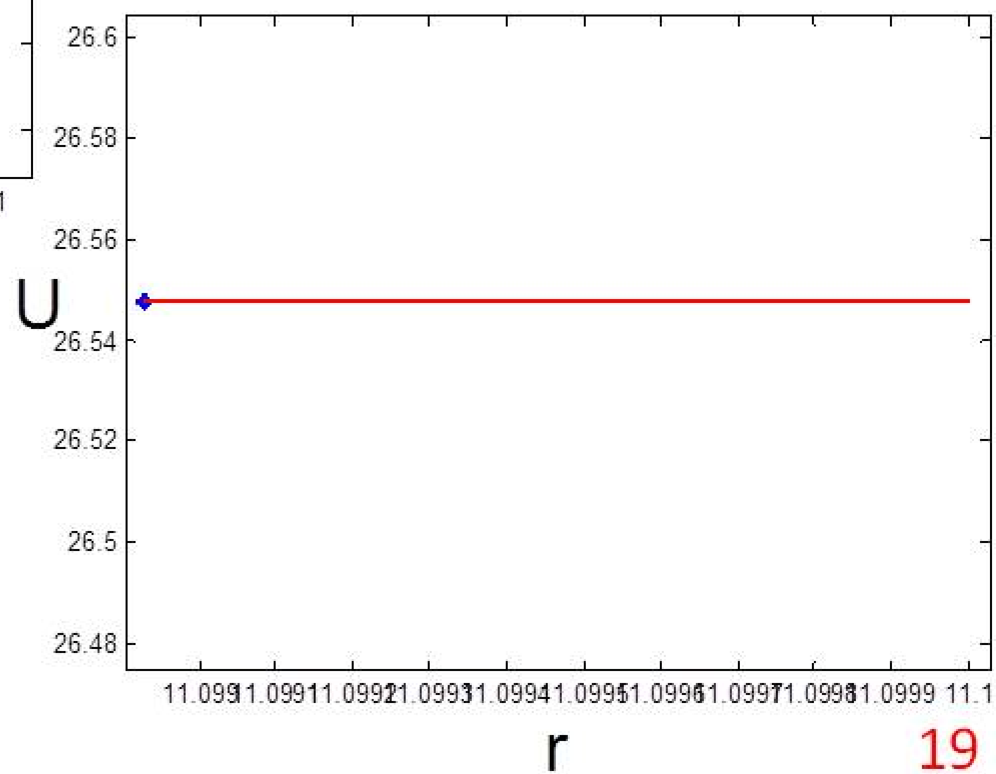
Conclusion:

Use data on other characteristics C_i^+ obtained in time decreasing case problem solution.

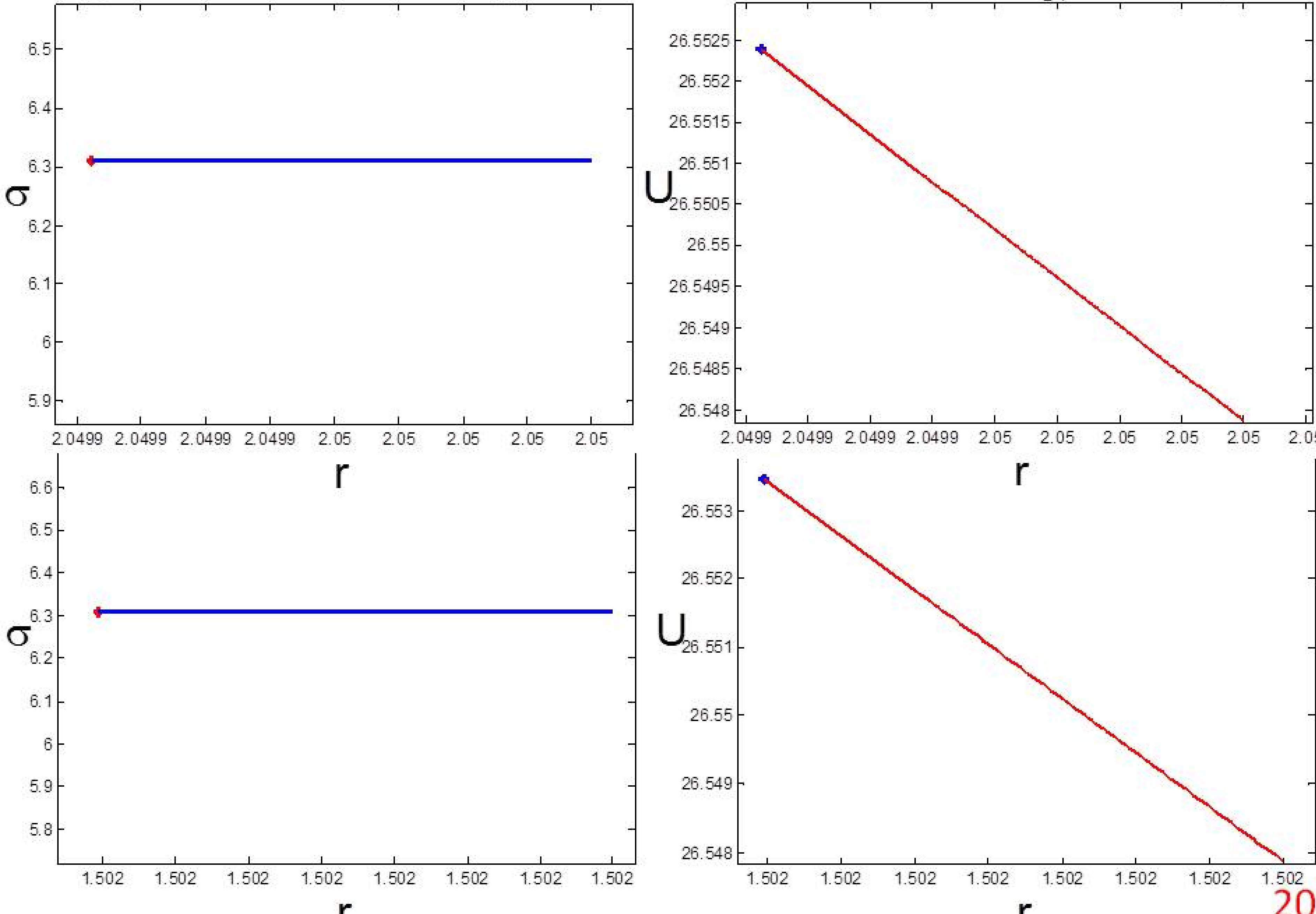
The right bound of computational domain - C_i^+

Right bound of computational domain is $C_N^+ v=0$





Right bound of computational domain is C_N^+ v=1,2



Conclusions

1. The compressing piston moving law has been computed using method of characteristics while solving gas dynamic equations in decreasing time case. Locally-analytic in a neighborhood of the point (r_*, t_*) exact solution was used.

2. Compression of a sufficiently large mass of gas by piston has been numerically modeled by finite-difference method with considering of new special bound condition on characteristic.

3. Solution has been constructed up to time t_*- all gas mass compressed to a given density.

Thank you for attention!