РФЯЦ

ВНИИТФ

V.F. Kuropatenko

Energy cumulation in cavity, shell and shock convergence

XIII Zababakhin Scientific Talks

International Conference

20.03-24.03.2017

Snezhinsk, Russia

1. Cumulation

In 2017 we celebrate the centenary of

Evegeny Ivanovich Zababakhin.

In 2017 the problem of energy cumulation celebrates its centenary, too.

The study of energy cumulation started in 1917 when Rayleigh published his paper on the collapse of spherical bubbles.

Cumulation is most pronounced in convergent shocks and shells, and in bubble collapse.

The problem of bubble collapse in liquid came from cavitation corrosion of ship propellers.

A second breath to the problem was given by the development of nuclear weapons.

A third wave of interest to the problem raised in search for new sources of energy:

- hydrodynamic fusion (shock wave)
- magnetic fusion
- laser fusion

2. Energy cumulation

Cumulation is such a redistribution of energy in a thermodynamic system that makes it grow locally.

Energy cumulates if the ratio of the maximal specific internal energy maxE to its average value E_m increases with time. Energy cumulation is characterized by a quantity

$$K = \frac{\max E(t)}{E_m(t)}.$$

If $K \to \infty$ with time t , energy cumulation is unbounded. If K<N=const, energy cumulation is bounded.
E.I. Zababakhin proposed that energy cumulation should be characterized by K in the form

$$K = \frac{\max P(t)}{\max P(t_0)}$$

since P has the dimensions of J/cm^3 (energy per unit volume).

E.I. Zababakhin, Unbounded cumulation phenomena. Mechanics in the USSR over 50 years.
V. 2. Fluid mechanics. Moscow, NAUKA Publishers, 1970, P. 313-342.

3. Expectations and disappointments

Unbounded energy cumulation was theoretically proved for many problems.

But it has not been achieved in experiment.

Only bounded energy cumulation is observed in experiment.

E.I. Zababakhin: - “Consideration to the maximum possible extent of actual conditions (including energy dissipation due to viscosity and heat conduction) does not eliminate theoretically unbounded cumulation, and the question of what after all bounds it remains undetermined". “Probably, cumulation is bounded by instability". (1988)

4. The history of cumulation study

Evgeny Zababakhin made a fundamental contribution to cumulation study.

Many authors drew brilliant theoretical results from cumulation modeling.

Analytical solutions for bubble collapse were derived by:

1917 – Rayleigh (ideal incompressible liquid)
1960 – Hunter (ideal gas)
1963 – Kazhdan and Brushlinsky (ideal gas)
1970 – Zababakhin (viscous incompressible liquid)
1996 – Krayko (ideal gas)

1965 – Zababakhin solved spherical incompressible shell convergence.
1975 – 2017 Nigmatulin's results for bubbles in real liquids
1960-2017 – studies with numerical methods

5. The history of cumulation in shock waves

Self-similar solutions to spherical shock convergence in ideal gas:

1942 – Guderley, 1945 – Sedov, 1945 – Stanyukovich,

1955 – Landau, Stanyukovich, 1996-2014 – Krayko

The effect of material properties on cumulation:

1957 – Zababakhin, Nechayev (shock waves in EM field)

1960 – Zababakhin (energy cumulation in multi-layer spherical systems)

1965 – Zababakhin, Simonenko (heat conduction effects of shock parameters)

Our work aims to derive reference solutions which are used to verify accuracy of numerical methods for cumulation modeling.
It differs from other types of work in problem statement, interpretation of results, approach to the construction and analysis of equations and methods for their solution.

6. Shock in a cold gas sphere

Problem statement: at $t = t_0$,

- a gas sphere of radius r_0 and mass $m_0 = \frac{4}{3}\pi r_0^3 \rho_0$,
- gas parameters: $P_0 = 0$, $\rho_0 = \text{const}$, $U_0 = 0$, $E_0 = 0$,
- $U_{g0} < 0$ on the boundary.

EOS: $P=f(s)\rho^{\gamma}$ or $P=(\gamma-1)\rho E$

At $t > t_0$, a shock wave will move through gas. Its parameters are:

In Eulerian coordinates r, t

$$\rho_w = \frac{\gamma+1}{\gamma-1}\rho_0, \quad U_w = \frac{2}{\gamma+1}D,$$

$$P_w = \rho_0 D U_w.$$

Shock velocity

$$D = \frac{dr}{dt}.$$

In Lagrangian coordinates m, t

$$\rho_w = \frac{\gamma+1}{\gamma-1}\rho_0,$$

$$U_w = \frac{2}{(\gamma+1)(4\pi\rho_0)^{1/3}(3m_w)^{2/3}}W,$$

$$P_w = \left(\frac{\rho_0}{3m_w}\right)^{2/3}(4\pi)^{-1/3}U_w W.$$

Shock velocity

$$W = \frac{dm}{dt}.$$

7. Shock trajectory

The shock trajectory is defined by:

In Eulerian coordinates

$$r_w = r_0 \left(\frac{t_f - t}{t_f - t_0} \right)^n .$$

Shock velocity

$$D = D_0 \left(\frac{t_f - t}{t_f - t_0} \right)^{n-1} ,$$

$$D_0 = \frac{\gamma + 1}{2} U_{g0} .$$

Convergence time

$$t_f = t_0 - \frac{r_0 n}{D_0} .$$

The self-similarity index n is yet undefined.

In Lagrangian coordinates

$$m_w = m_0 \left(\frac{t_f - t}{t_f - t_0} \right)^k .$$

Shock velocity

$$W = W_0 \left(\frac{t_f - t}{t_f - t_0} \right)^{k-1} ,$$

$$W_0 = \frac{\gamma + 1}{2} U_{go} \left(4\pi\rho_0 \right)^{1/3} \left(3m_0 \right)^{2/3} .$$

Convergence time

$$t_f = t_0 - \frac{m_0 k}{W_0} .$$

The self-similarity index k is yet undefined.

8. Gas motion between shock and boundary

The amplitude of a shock depends on distance to the symmetry center and overriding compression and rarefaction waves. Gas behavior between the shock and the boundary is defined by the following system of equations.

In Eulerian coordinates, these are the mass conservation equation, the equation of motion, the equation for pressure, and EOS.

$$\left(\frac{\partial \rho}{\partial t}\right)_r + U\frac{\partial \rho}{\partial r} + \rho\frac{\partial U}{\partial r} + \frac{2\rho U}{r} = 0,$$

$$\left(\frac{\partial U}{\partial t}\right)_r + U\frac{\partial U}{\partial r} + \frac{1}{\rho}\frac{\partial P}{\partial r} = 0, \qquad (8.1)$$

$$\left(\frac{\partial P}{\partial t}\right)_r + U\frac{\partial P}{\partial r} + \gamma P\left(\frac{\partial U}{\partial r} + \frac{2U}{r}\right) = 0,$$

$$P = F(s)\rho^{\gamma} \quad \text{or} \quad P = (\gamma - 1)\rho E.$$

Sought are ρ, U, and P.

In Lagrangian coordinates, these are the equation of trajectory, the mass conservation equation, the equation of motion, and EOS. After conversion from r and U to R=r and $C=r^2U$ they read as

$$\left(\frac{\partial R}{\partial t}\right)_m - 3C = 0,$$

$$\left(\frac{\partial \rho}{\partial t}\right)_m + 4\pi\rho^2\frac{\partial C}{\partial m} = 0, \qquad (8.2)$$

$$\left(\frac{\partial C}{\partial t}\right)_m + 4\pi R^{4/3}\frac{\partial\left(F\rho^{\gamma}\right)}{\partial m} - \frac{2C^2}{R} = 0,$$

$$P = F(s)\rho^{\gamma} \quad \text{or} \quad P = (\gamma - 1)\rho E.$$

Sought are R, C, and ρ.

9. Separation of variables

Convert to new independent variables and functions.

In Eulerian coordinates,

from t, r tp t, ξ(t, r), where

$$\xi = \frac{r}{r_0}\left(\frac{t_f - t_0}{t_f - t}\right)^n .$$

In Lagrangian coordinates,

from t, m to t, η(t, m), where

$$\eta = \frac{m}{m_0}\left(\frac{t_f - t_0}{t_f - t}\right)^k .$$

On the shock front

ξ=1

η=1

we will seek solution in the form

$$P = \alpha_P(t)\Pi(\xi),\ \ \rho = \alpha_\rho(t)\delta(\xi),$$

$$U = \alpha_u(t)M(\xi).$$

$$R = \beta_R(t)T(\eta),\ \ \rho = \beta_\rho(t)\delta(\eta),$$

$$C = \beta_c(t)Z(\eta).$$

Each of systems (8.1) and (8.2) divides into two systems of equations: one for the functions of t and the other for the functions of ξ (η, respectively).

10. Systems of equations

For the functions of t:

In Eulerian coordinates

$$\alpha_\rho = \frac{\rho_0}{\delta_1}\left(\frac{\gamma+1}{\gamma-1}\right), \quad \alpha_u = \frac{D_0}{M_1}\left(\frac{t_f - t}{t_f - t_0}\right)^{n-1},$$

$$\alpha_p = \frac{\rho_0 D_0^2}{\Pi_1}\left(\frac{t_f - t}{t_f - t_0}\right)^{2(n-1)}.$$

$\delta=\delta_1$, $\Pi_1=1$, $M_1=1$ on the shock front.

In Lagrangian coordinates

$$\beta_R = \frac{R_0}{T_1}\left(\frac{t_f - t}{t_f - t_0}\right)^{k}, \quad \beta_\rho = \frac{\rho_0}{\delta_1}\left(\frac{\gamma+1}{\gamma-1}\right),$$

$$\beta_c = \frac{C_0}{Z_1}\left(\frac{t_f - t}{t_f - t_0}\right)^{k-1}.$$

T_1, δ_1, Z_1 are the values of T, δ, Z on the shock front.

For the functions of ξ or η:

$$\begin{aligned}
&(M-\xi)\delta' + \delta M' = -\frac{2M\delta}{\xi},\\
&\delta(M-\xi)M' + \Pi' = -\frac{n-1}{n}\delta M,\\
&\gamma\Pi M' + (M-\xi)\Pi' = -\frac{2\gamma M\Pi}{\xi} - \frac{n-1}{n}\Pi.
\end{aligned} \tag{10.1}$$

$$\begin{aligned}
&\eta T' = A_1,\\
&\delta B_1 Z' - \eta Z_1 \delta' = 0,\\
&-\frac{\eta}{Z_1}Z' + \frac{C_1\gamma\eta}{\delta_1}\delta' = C_2.
\end{aligned} \tag{10.2}$$

Here

$A_1(T,Z), B_1(\delta), C_1(T,\delta,\eta), C_2(T,\delta,Z,\eta).$

11. Solution

Equations (10.1) are linear with respect to δ', M', Π'; equations (10.2) are linear with respect to T', δ', Z'. They are inhomogeneous. Their solutions exist if the appropriate determinants are not zero.

In Eulerian coordinates

$$\Delta = (M-\xi)\left(\delta(M-\xi)^2 - \gamma\Pi\right) \neq 0. \quad (11.1)$$

Solution:

$$M' = \frac{R_1(M-\xi)}{\xi n \Delta}, \quad \delta' = \frac{\delta R_2}{\xi n \Delta},$$

$$\Pi' = -\frac{\xi R_3 (M-\xi)^2}{\xi n \Delta}.$$

In Lagrangian coordinates

$$\Delta = (B_1 C_1 \gamma - \xi)\xi \neq 0. \quad (11.2)$$

Solution:

$$T' = \frac{A_1}{\eta}, \quad \delta' = \frac{B_1 C_2 \delta_1}{\Delta}, \quad Z' = \frac{\eta C_2 Z_1}{\Delta}.$$

A solution also exists if simultaneously $\Delta=0$ and $C_2=0$. This is possible at $K=K_*$

A solution also exists if simultaneously $\Delta=0$ and all minors in the augmented matrix of order three are zero. This is possible at $n=n_*$.

γ	1,1	1,2	4/3	1,4	5/3
n_*	0,7959	0,7571	0,7293	0,7172	0,6884
k_*	2,3879	2,2714	2,1831	2,1515	2,0651

12. Reference solution

Initial data: $P_0 = 0$, $\rho_0 = 1$, $U_0 = 0$, $U_{g0} = -1$.

$$\gamma = \frac{5}{3}, \qquad n = 0{,}688377.$$

Boundary velocity $U_{гр}(t)$.

Figures 1,2,3:

—— analytical solution;

–o– VOLNA calculations with shock captured;

\- - - with shock smearing.

Profiles $t_1=0{,}4$; $t_2=0{,}45$; $t_3=0{,}5$; $t_ф=0{,}51628$;

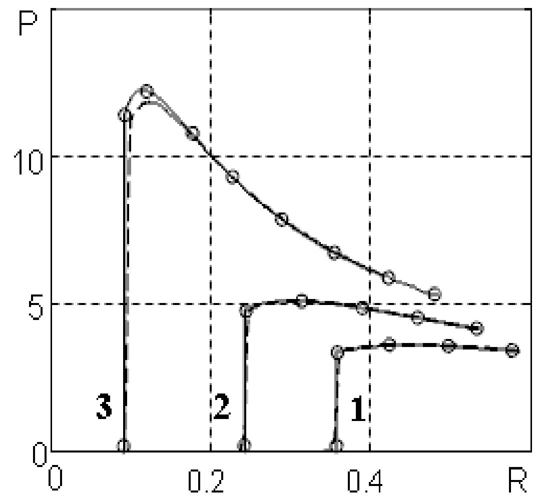


Fig. 1

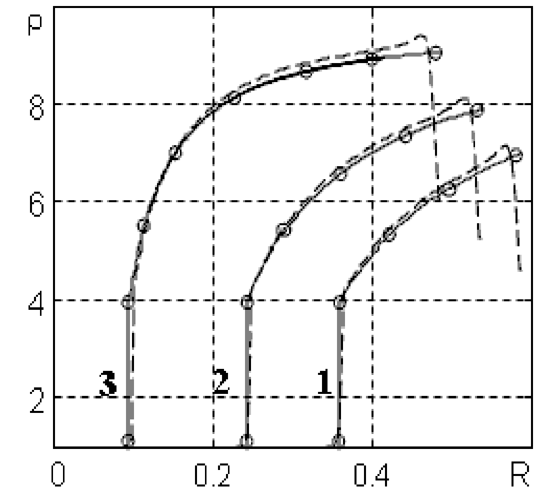


Fig. 2

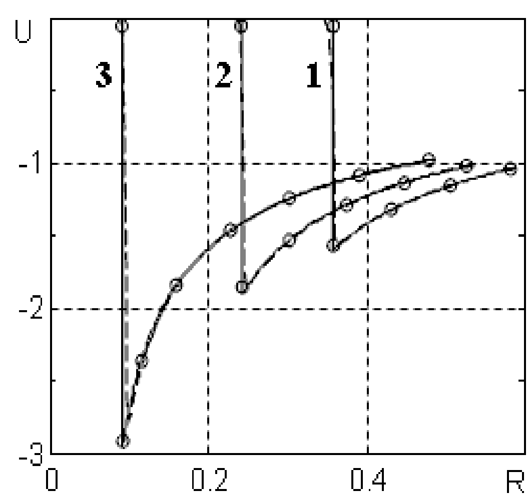
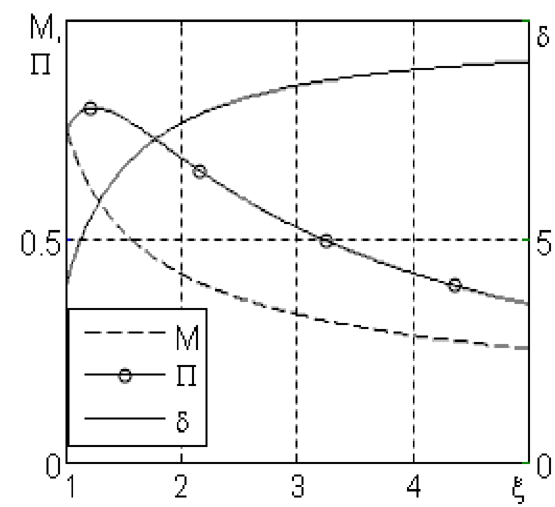


Fig. 3



13. Compressible incompressible continuum

In Lagrangian coordinates, the equations of mass conservation, the equation of motion, and the equation of internal energy take the form

$$\frac{\partial V}{\partial t} - 4\pi \frac{\partial r^2 U}{\partial M} = 0, \qquad \frac{\partial U}{\partial t} + 4\pi r^2 \frac{\partial P}{\partial M} = 0,$$

$$\frac{\partial E}{\partial t} + P \frac{\partial V}{\partial t} = 0.$$

Compressibility β_s and sound velocity C are related by

$$\beta_s = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_s, \qquad C^2 = -V^2\left(\frac{\partial P}{\partial V}\right)_s, \qquad C^2 = \frac{V}{\beta_s}.$$

There is no incompressible matter in nature.

There is a class of flows where V=const.

Many understand constancy of V as incompressibility, i.e., $\beta_s = 0$ and $C^2 = \infty$. It is an error.

The property of flow is not the property of matter.

14. Flows with constant density

For the flows where V = const and $\frac{\partial V}{\partial t} = 0$, the system reduces to two equations:

$$\frac{\partial r^2 U}{\partial M} = 0, \quad \frac{\partial U}{\partial t} + 4\pi r^2 \frac{\partial P}{\partial M} = 0. \qquad (14.1)$$

With the equation of particle trajectory

$$\left(\frac{\partial r}{\partial t}\right)_M = U$$

the system of 3 equations contains 3 functions:

$$r(t,M),\ U(t,M),\ P(t,M).$$

Rayleigh, Hunter and Zababakhin derived their solutions from these equations.

15. Fundamental contradiction

The Navier-Stokes model does not consider energy conservation and EOS. This is a source of the following contradiction:

1. From equation of motion (14.1), it follows that pressure
   P changes at V=const.

2. From equation $\frac{\partial E}{\partial t} + P\frac{\partial V}{\partial t} = 0$ at $\frac{\partial V}{\partial t} = 0$, it follows that $\frac{\partial E}{\partial t} = 0.$

3. From EOS P = P(V,E) at V = const and E =const, it follows that
   P does not change.

The contradiction dissolves if assume that there is a source of energy in the medium. In the EOS at V=const

$$P_x\left(V\right) = \text{const},\ \ E_x\left(V\right) = \text{const} \qquad \text{and} \qquad \frac{\partial E_T}{\partial t} = \frac{\partial g}{\partial t}.$$

A wide class of EOS have the form $P_T V = (V)E_T$ and thus pressure at V = const depends on the dissipative function g.

Viscosity and heat conduction are not enough to obtain a flow with V = const.

16. General solution

The first equation

$$\frac{\partial r^2 U}{\partial M} = 0$$

has a solution

$$r^2 U = f(t). \qquad (1)$$

This is true for arbitrary M. On the bubble boundary at M=0,

$$r_B^2 U_B = f(t).$$

This gives the dependence of velocity on coordinate

$$U = U_B r_B^2 r^{-2}.$$

The equation of boundary motion $\frac{dr_B}{dt} = U_B$ is integrated with respect to t

$$r_B = \left(r_{B0}^3 + \int_{t_0}^{t} 3f(t)\,dt \right)^{1/3}. \qquad (2)$$

The time t_f of cavity convergence is determined from (2) at $r_B = 0$

$$r_{B0}^3 + \int_{t_0}^{t_f} 3f(t)\,dt = 0.$$

17. Determination of pressure

Substitute $r^2U = f(t)$ in the equation of motion

$$\frac{\partial U}{\partial t} + 4\pi r^2 \frac{\partial P}{\partial M} = 0$$

and find $\frac{\partial P}{\partial M}$

$$\frac{\partial P}{\partial M} = -\frac{1}{4\pi r^2}\left(\frac{1}{r^2}\frac{df}{dt} - \frac{2f^2}{r^5}\right). \qquad (1)$$

Multiply by $dM = 4\pi r^2 V_0^{-1} dr$ and integrate between 0 and M, i.e., between r_B and r

$$P = P_B(t) + \frac{1}{V_0}\left(\frac{df}{dt}\left(\frac{1}{r} - \frac{1}{r_B}\right) - \frac{f^2}{2}\left(\frac{1}{r^4} - \frac{1}{r_B^4}\right)\right). \qquad (2)$$

P_B is defined either by gas in the cavity, or by surface tension. If these are absent, $P_B = 0$.

At $r = \infty$

$$P_\infty = P_B(t) - \frac{1}{V_0}\left(\frac{df}{dt}\frac{1}{r_B} - \frac{f^2}{2}\frac{1}{r_B^4}\right). \qquad (3)$$

Any two of the functions $P_\infty(t)$, $P_B(t)$, $r_B(t)$, and $f(t)$ are independent. That is, this is a flow with two-function arbitrariness.

18. Dissipation

Write P(t, r) in the form

$$P = P_\infty(t) + \frac{1}{V_0}\left(\frac{df}{dt}\frac{1}{r} - \frac{f^2}{2r^4}\right). \qquad (1)$$

Change from P to E with $PV_0 = \Gamma E$. Differentiate E with respect to time to obtain the dissipation rate which gives “incompressibility” V = const.

$$\left(\frac{\partial g}{dt}\right)_M = \frac{1}{\Gamma}\left(V_0\frac{dP_\infty(t)}{dt} + \frac{1}{r}\frac{d^2f}{dt^2} - \frac{2f}{r^4}\cdot\frac{df}{dt} + \frac{2f^3}{r^7}\right).$$

19. Determination of mean energy

Since V=const, P_x and E_x do not vary. Let $P_x=0$ and $E_x=0$. Dissipation only changes the thermal energy

$$E_T = P \cdot \frac{V_0}{\Gamma} \tag{1}$$

The expression for E(r, t) is obtained from (17.2)

$$E = \frac{1}{\Gamma}\left(P_B(t)V_0 + \frac{df}{dt}\left(\frac{1}{r} - \frac{1}{r_B}\right) - \frac{f^2}{2}\left(\frac{1}{r^4} - \frac{1}{r_B^4}\right)\right).$$

Multiply E by $dM=4\pi r^2\rho_0 dr$ and integrate between r_B and r_A, where r_A is a boundary of some mass

$$M_0 = \frac{4}{3}\pi\rho_0\left(r_A^3 - r_B^3\right).$$

Dividing by M_0 gives the mean specific energy

$$E_m = \frac{1}{\Gamma}\left(P_B(t)V_0 + \frac{df}{dt}\left(\frac{3\left(r_A^2 - r_B^2\right)}{2\left(r_A^3 - r_B^3\right)} - \frac{1}{r_B}\right) - \frac{f^2}{2}\left(\frac{3\left(r_A - r_B\right)}{\left(r_A^3 - r_B^3\right)r_A r_B} - \frac{1}{r_B^4}\right)\right). \tag{2}$$

20. Determination of maxP and maxE

At any time, P is maximal at point r_M, where

$$\left(\frac{\partial P}{\partial M}\right)_t = 0. \qquad (1)$$

From (17.1) and (20.1) we obtain the coordinate of the point where P=maxP

$$r_M = \left(2f^2 \Big/ \frac{df}{dt}\right)^{1/3}. \qquad (2)$$

From (17.2) and (20.2) we obtain

$$\max P = P_B(t) + \frac{1}{V_0}\left(\frac{df}{dt}\left(\frac{1}{r_M} - \frac{1}{r_B}\right) - \frac{f^2}{2}\left(\frac{1}{r_M^4} - \frac{1}{r_B^4}\right)\right). \qquad (3)$$

From (19.1) and (20.3) we obtain

$$\max E = \frac{V_0 P_B(t)}{\Gamma} + \frac{1}{\Gamma}\left(\frac{df}{dt}\left(\frac{1}{r_M} - \frac{1}{r_B}\right) - \frac{f^2}{2}\left(\frac{1}{r_M^4} - \frac{1}{r_B^4}\right)\right). \qquad (4)$$

There are three functions of t in these equations. Two of them are independent.
Let them be $P_B = (t)$ and $f(t)$.

21. The class of elementary solutions

Consider closed cavities in infinite liquid ($r_A = \infty$). Choose the functions $P_B(t)$ and $f(t)$ from the condition that the cavity contains vacuum ($P_B = 0$) and the trajectory of its surface is

$$r_B = r_{B0}\varphi^n \quad \text{where} \quad \varphi = \frac{t_f - t}{t_f - t_0}. \qquad (1)$$

Differentiate r_B and substitute in the general solution to obtain

$$U_B = U_{B0}\varphi^{n-1}, \qquad U_{B0} = -\frac{nr_{B0}}{t_f - t_0}. \qquad (2)$$

From the condition that r_B=0 and U_B= - ∞ at $t=t_f$, it follows that

$$0 < n < 1.$$

The functions f(t) and $\dfrac{df}{dt}$ are determined from (1), (2) and (16.2):

$$f(t) = r_{B0}^2 U_{B0}\varphi^{3n-1}, \qquad \frac{df}{dt} = \frac{3n-1}{n} \cdot r_{B0}\, U_{B0}^2\, \varphi^{3n-2}.$$

The conditions U_{B0}< 0, f(t)<0, and $\dfrac{df}{dt} > 0$ are satisfied if $n>1/3$. From P_∞ (t)>0, it follows that n<0,4. Hence a solution exists if

$$1/3 < n < 0,4.$$

22. Cumulation coefficients

Max P at $t=t_0$ follows from (20.3) at $\varphi=1$. So,

$$K_P = \frac{\max P(t)}{\max P(t_0)} = \left(\frac{r_{B0}}{r_B}\right)^{2(1-n)/n}.$$

As $r_{B0}/r_B \to \infty$, unbounded cumulation is reached at 1/3<n<1.

At n=0.4 , the result coincides with the Rayleigh-Zababakhin solution.

For $r_B \ll r_{B0}$,

$$\max E = \frac{U_{B0}^2}{\Gamma}\left(\frac{r_{B0}}{r_B}\right)^{2(1-n)/n}\left(\frac{3}{2}\left(\frac{3n-1}{2n}\right)^{4/3} - \frac{5}{2} + \frac{1}{n}\right), \quad E_m = \frac{U_{B0}^2}{2\Gamma}\left(\frac{r_{B0}}{r_B}\right)^{2(1-n)/n}\frac{(2-5n)}{n}.$$

Then

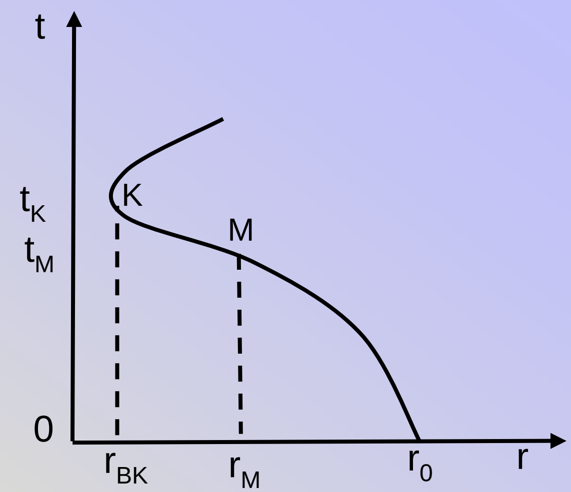
$$K_E = 1 + \frac{3n}{2-5n}\left(\frac{3n-1}{2n}\right)^{4/3}.$$

K_E is independent of time. E_m increases with maxE, so their ratio remains constant. For 1/3<n<0.4, K_E takes values within $1 < K_E < \infty$.

23. Gas-filled bubble collapse

The model is simple. The density of gas in the bubble only depends on time and gas compression is isentropic. So,

$$P_B = P_{B0}\left(\frac{r_{B0}}{r_B}\right)^{3\gamma}, \quad f = U_{B0} r_{B0}^2 \sqrt{\frac{r_B - r_{BK}}{r_{B0} - r_{BK}}},$$



where r_{BK} is minimal bubble radius.
Differentiate f(t) to obtain

$$\frac{df}{dt} = \frac{f^2}{2r_B^2\left(r_{B0} - r_{BK}\right)}.$$

At the time when the boundary stops,

$$t_K = t_0 - \frac{2\left(r_{B0} - r_{BK}\right)}{15U_{B0}\cdot r_{B0}^2}\left(3r_{B0}^2 + 4r_{B0}r_{BK} + 8r_{BK}^2\right), \quad f_K = 0. \quad \left(\frac{df}{dt}\right)_K = 0, \quad U_{BK} = 0.$$

P is maximal at the point

$$r_M = \left(2f^2 \Big/ \frac{df}{dt}\right)^{1/3} = \left(4r_B^2\left(r_B - r_{BK}\right)\right)^{1/3}.$$

r_M decreases as r_B decreases and $r_B = r_M = \frac{4}{3}r_{BK}$ at time t_M.

From this time the boundary decelerates: $\left(\frac{\partial P}{\partial M}\right)_B < 0$ and $\max P = P_B\left(t\right)$.

24. Cumulation coefficients

At time t_K,

$$\max P_K = P_{B0}\left(\frac{r_{B0}}{r_{BK}}\right)^{3\gamma}, \qquad \max E_K = \frac{V_0}{\Gamma} P_{B0}\left(\frac{r_{B0}}{r_{BK}}\right)^{3\gamma}.$$

At time t_K, $f_K=0$ and $\left(\frac{df}{dt}\right)_K = 0$, then $E_{mK} = \frac{V_0}{r} P_{B0}\left(\frac{r_{B0}}{r_{BK}}\right)^{3\gamma}$.

From the equality $\max E_K = E_{mK}$, it follows that cumulation is absent.

$$K_E = 1.$$

Determine $\max P_0$ at t_0 from (19.3)

$$\max P_0 = P_{B0} + \frac{U_{B0}^2}{2V_0}\left(\frac{3}{2^{4/3}\left(1 - r_{BK}/r_{B0}\right)^{4/3}} - \frac{r_{BK}}{r_{B0}}\left(1 - \frac{r_{BK}}{r_{B0}}\right)^{-1}\right).$$

So, according to the cumulation coefficient

$$K_P = \frac{\max P_K}{\max P_0},$$

energy cumulation is bounded.

25. Shell convergence

On the inner $r = r_B$ and outer $r = r_A$ shell surfaces, $P_B = 0$, $P_A = 0$. All processes depend on initial energy and energy release g. The function f depends on $r_B(t)$ and $r_A(t)$:

$$f = U_{B0} r_{B0}^2 \left(\frac{r_A r_B (r_{A0} - r_{B0})}{r_{A0} r_{B0} (r_A - r_B)} \right)^{1/2}, \qquad \frac{df}{dt} = \frac{f^2 (r_A + r_B)(r_A^2 + r_B^2)}{2 r_A^3 \cdot r_B^3}.$$

maxP is achieved in the shell at the point

$$r_M = r_A r_B \left(0{,}25 (r_A + r_B)(r_A^2 + r_B^2) \right)^{-1/3}.$$

So, $r_M \to 0$ as $r_B \to 0$. The average energy of the shell increases in convergence, but maxE increases greater. Compare the cumulation coefficients

$$K_p \approx G_4 \left(\frac{r_{B0}}{r_B} \right)^3 \text{ and } \quad K_E \approx G_5 \left(\frac{r_{B0}}{r_B} \right),$$

where G_4= const, G_5= const at $r_B = 0$. The value of K_p was obtained by Zababakhin in 1965.

26. Conclusion

1. Interest to energy cumulation does not fade. Material models get more and more sophisticated. Mathematical modeling plays an important role in energy cumulation studies.

2. Analytical solutions grow in importance as references for verification of numerical accuracy.

3. Solutions with arbitrary self-similarity indises are derived for shock convergence in ideal gas. Entropy there is essentially dependent on the Lagrangian coordinate. Comparison with VOLNA calculations is provided.

4. Solutions to bubble collapse and converging shell in flows with constant density are derived. It is shown that energy needs to be applied to the system to ensure ρ = const in compressible liquid flows. With this energy input the cumulation coefficient reduces.


Thank you
for your
time.