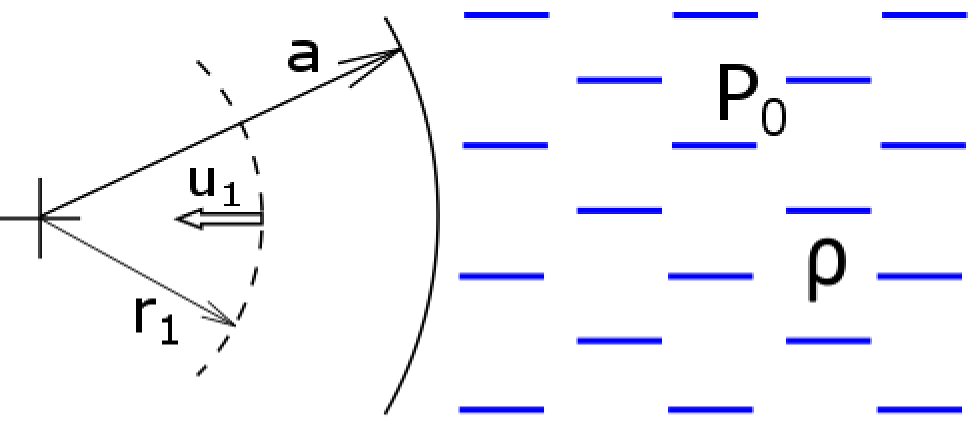
Evgeny I. Zababakhin Research into Cumulation

Based on “Phenomena of Unlimited Cumulation” monograph

Igor E. Zababakhin

Spherical bubble. Uncompressible ideal fluid

(Raley, 1917)

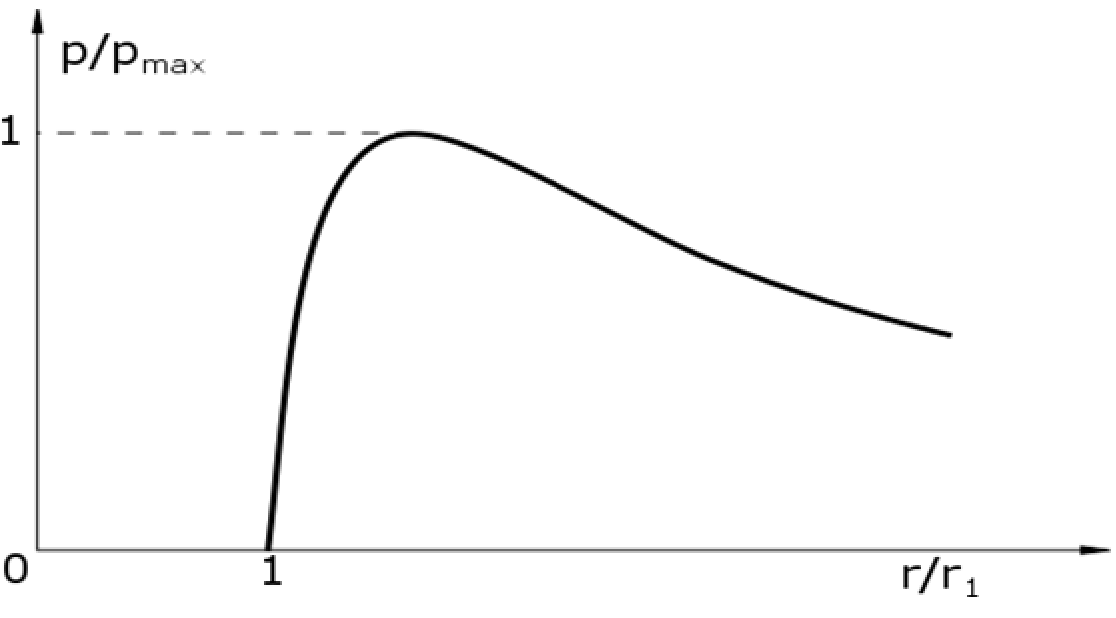


Self-similar solution (r/r_1).

At $r_1 \to 0$

$u_1 \sim 1/r_1^{3/2}$

$p_{max} \sim 1/r_1^{3/2}$

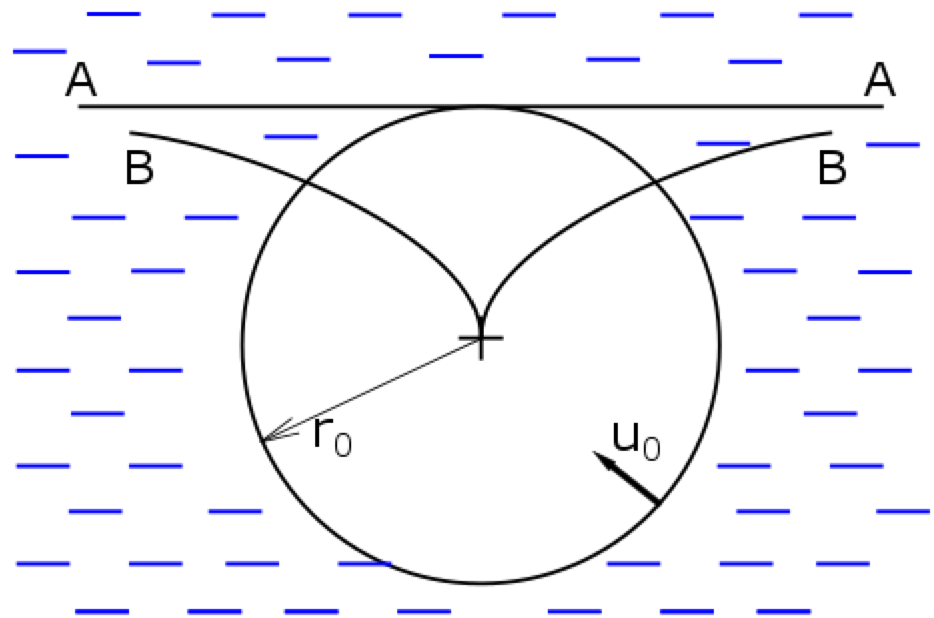


Cavitation.

Spherical bubble. Uncompressible viscous fluid.
(Evgeny I. Zababakhin, 1960)

Inertial Motion

$$\frac{|u_0| r_0}{\nu} = \mathrm{Re}$$ - Reynolds number



Re < 8 a shell stops, not reaching the center;

Re > 8 unlimited cumulation.

$u_1 \sim 1/r_1^{3/2}$. Viscosity decreases a constant in asymptotics by a factor of (1 + 8/Re).

Fluid under pressure

$$a_{\kappa p} = 8{,}4\nu\sqrt{\frac{p}{\rho}}$$

$a < a_{\kappa p}$ a bubble is slowly filled up

$a > a_{\kappa p}$ cumulation, the same as without viscosity $u_1 \sim 1/r_1^{3/2}$

Account for dissipation does not make cumulation infeasible.

Spherical bubble in compressible fluid

(Ya.B. Zeldovich, I.M. Gelfand, K.V. Brushlinsky and Ya.M. Kazhdan (1963), Hunter (1960) and many other researchers)

Cubic equation of state. $p = A\rho^3$ or $p = A(\rho^3 - \rho_0{}^3)$

$$\frac{\partial \alpha}{\partial t} + \alpha \frac{\partial \alpha}{\partial r} + \frac{\alpha^2 - \beta^2}{2r} = 0 \qquad \frac{\partial \beta}{\partial t} + \beta \frac{\partial \beta}{\partial r} + \frac{\alpha^2 - \beta^2}{2r} = 0$$

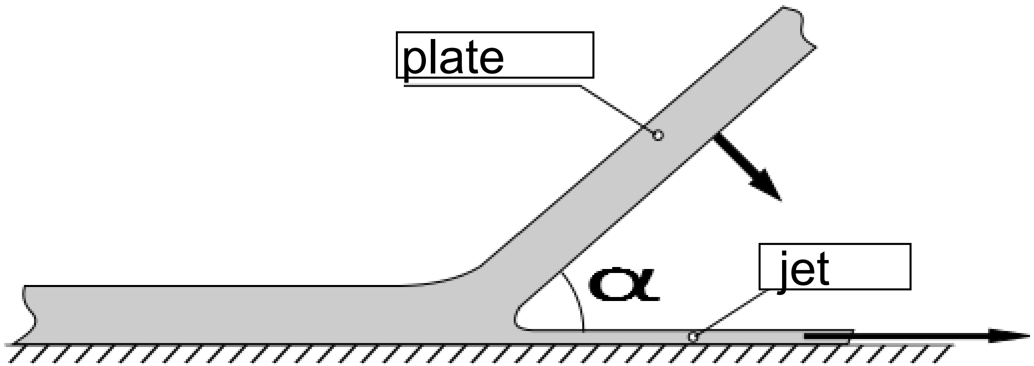
In the stage of focusing an extreme solution is derived in a self-similar form

$$\alpha(r,t) = \frac{r}{kt} a(\xi) \qquad \beta(r,t) = \frac{r}{kt} b(\xi) \qquad \xi = \frac{t}{r^k}$$

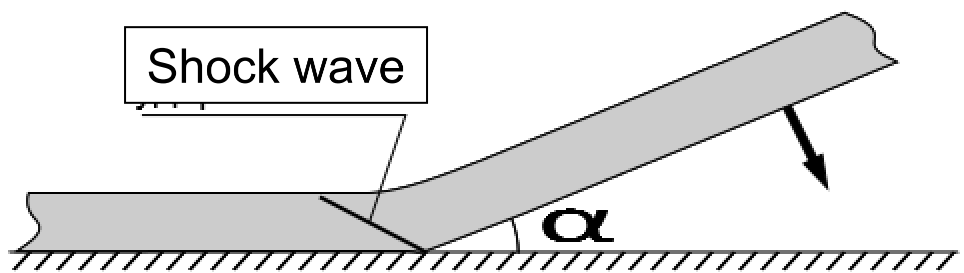
Continuum of solutions exists $1.411 < k < 1.5$ I.M. Gelfand (1952).

Near focus $u \sim \rho \sim 1/r^{0,411}$.

Planar 2D case



Incompressible fluid

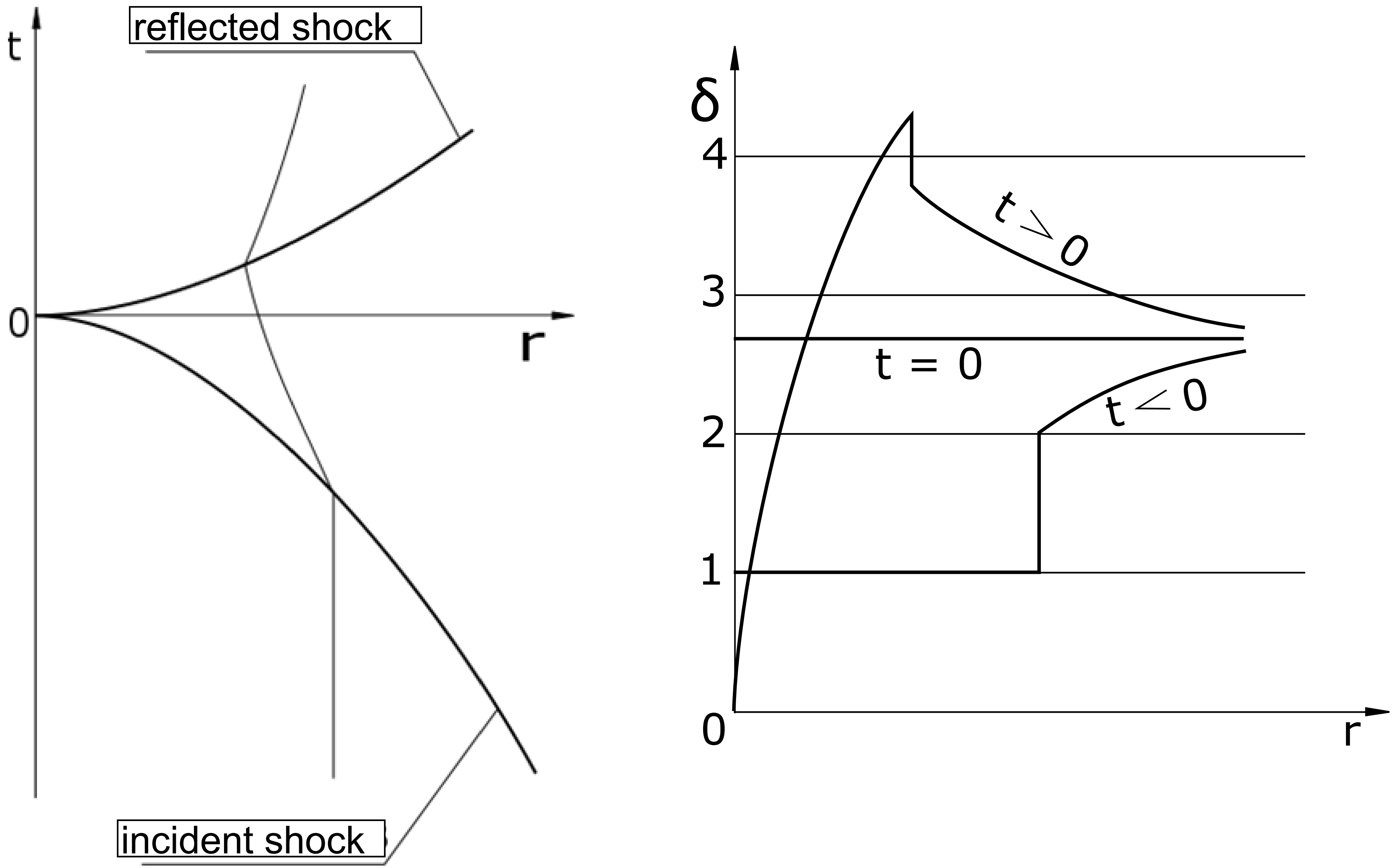


compressible fluid, small angle – no jet

Focusing of a strong shock wave.

(L. D. Landau, K. P. Stanyukovich (1945), Guderlay (1942) and others)

γ= 3



Solution: $u_1 \sim 1/R^{k-1}$, where k is a fitting parameter.

For $\gamma=3$ $k=1{,}571$. $u_1 \sim 1/R^{0,571}$; $p \sim u^2 \sim 1/R^{1,142}$. For some values of $\boldsymbol{\gamma}$ there exist several values of **k** (K.V. Brushlinsky, Ya.M. Kazhdan).

At focusing, **u** and **c** are finite everywhere except the center. $\delta = 2{,}73$

Solution beyond focusing area was proposed by G.M. Gandelman (1951)

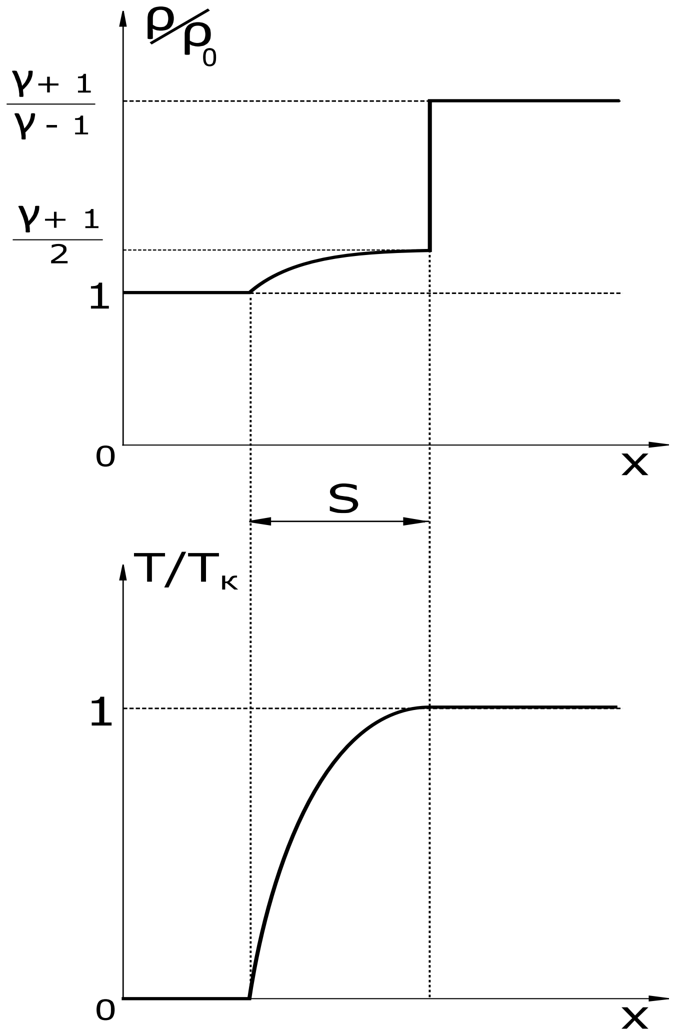
At a certain profile of $\rho_0 \sim r^n$ material beyond the shock is at rest, i.e. the reached state is conserved till rarefaction wave arrives from outward.

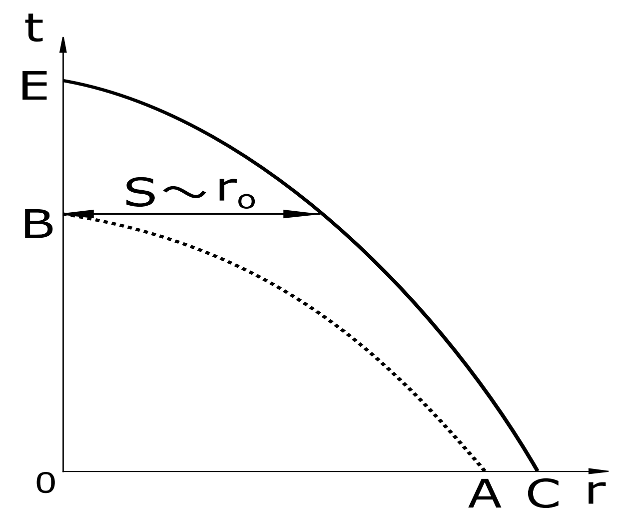
Wave focusing in heat-conducting gas

(V. A. Simonenko, T.I. Zababakhin 1965)

Radiation energy density is much smaller than that of matter. Radiation-to-matter equilibrium.

Thermal precursor.





Maximum temperature of the process is finite. Focusing of the second wave is an isothermal process, i.e. T→const and ρ→∞ (instead of T →∞ and ρ→const in the case of no heat conduction).

Energy density is infinitely large.

Converging detonation wave

(Ya.B. Zeldovich, E.I. Zababakhin, B.V. Ajvazov, et al.)

Cubic equation of state for PW.

Jouger condition is not fulfilled. Equations of spherical motion for variables α and β are as follows:

$$\frac{\partial\alpha}{\partial t}+\alpha\frac{\partial\alpha}{\partial r}+\frac{\alpha^2-\beta^2}{2r}=0 \qquad \frac{\partial\beta}{\partial t}+\beta\frac{\partial\beta}{\partial r}+\frac{\alpha^2-\beta^2}{2r}=0$$

A solution is given at initial moment, when the process starts:

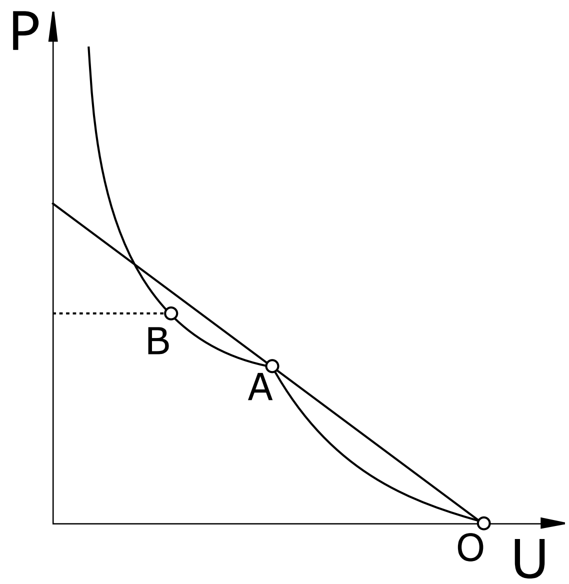
$$\frac{dp}{dr}=-\frac{3}{8}\frac{p_H}{R}\leq 0$$

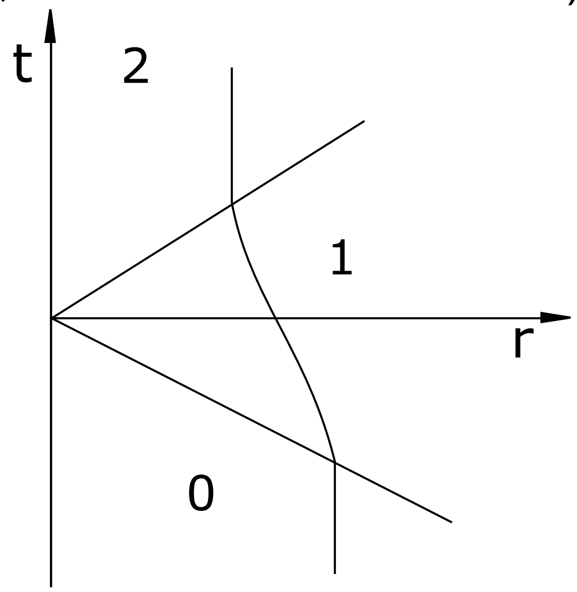
Qualitatively the same is true for a cylindrical wave (Ya.B. Zeldovich considered a 3D case).

Before focusing, HE power does not matter.

Focusing of a shock wave with phase transition

(Bancroft D., Minshell C., Peterson E. L at al. 1956)





Hugoniot with a bend caused by phase transition.
01 – phase precursor; 12 – wave reflected from the center (behind it, matter at rest).

No cumulation.

Cubic equation of state.

$$p - p_A = \frac{\rho_A c_A^{\;2}}{3}\left[\left(\frac{\rho}{\rho_A}\right)^3 - 1\right]$$

Equation of motion

$$\frac{\partial \alpha}{\partial t} + \alpha \frac{\partial \alpha}{\partial r} + \frac{\alpha^2 - \beta^2}{2r} = 0 \qquad \frac{\partial \beta}{\partial t} + \beta \frac{\partial \beta}{\partial r} + \frac{\alpha^2 - \beta^2}{2r} = 0$$

A self-similar solution is derived assuming that

$$\alpha(r,t) = a(\xi)\frac{r}{t} \qquad \beta(r,t) = b(\xi)\frac{r}{t} \qquad \xi = \frac{t}{r}$$

Solution:

$$u = u_0 \frac{1 - (c_A t / r)^2}{1 - (c_A / c_0)^2} \qquad \frac{\delta_m - 1}{\delta_A - 1} = \frac{(2c_0 - c_A)(c_0 + c_A)}{c_A(c_0 - c_A)} \qquad \frac{p_m}{p_A} = \frac{c_0 + c_A}{c_0 - c_A}$$

δ_A and p_A correspond to the beginning of phase transition; δ_m and p_m – the reflected wave.

Velocity equals zero at the reflected wave.

Focusing of elastic precursor

(E.I. Zababakhin)

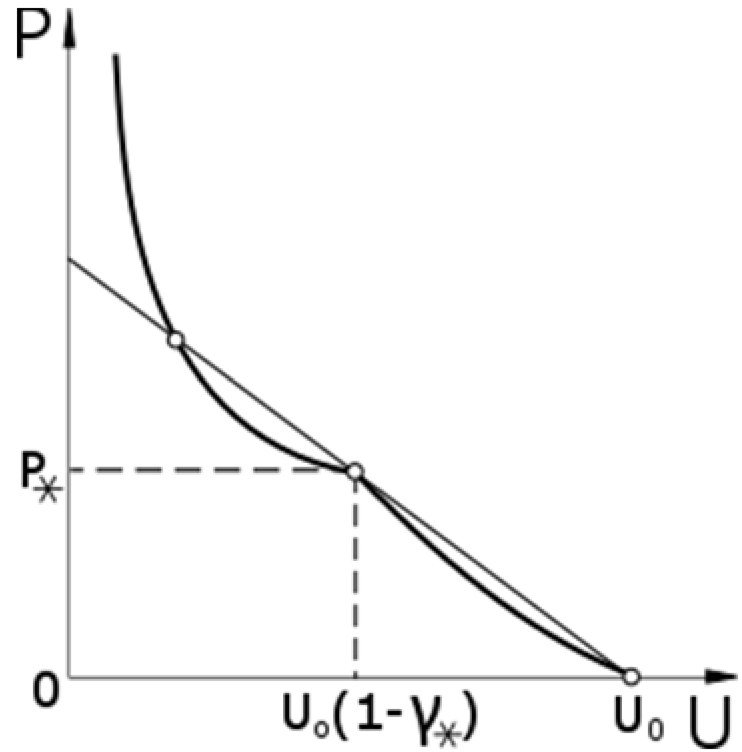
SW shows uniaxial strain. Anisotropy. Exceeding shear strength is accompanied by material softening, which gives a bend at the Hugoniot.
The Poisson ratio is 1/3.

Then in the area of elasticity (strength)

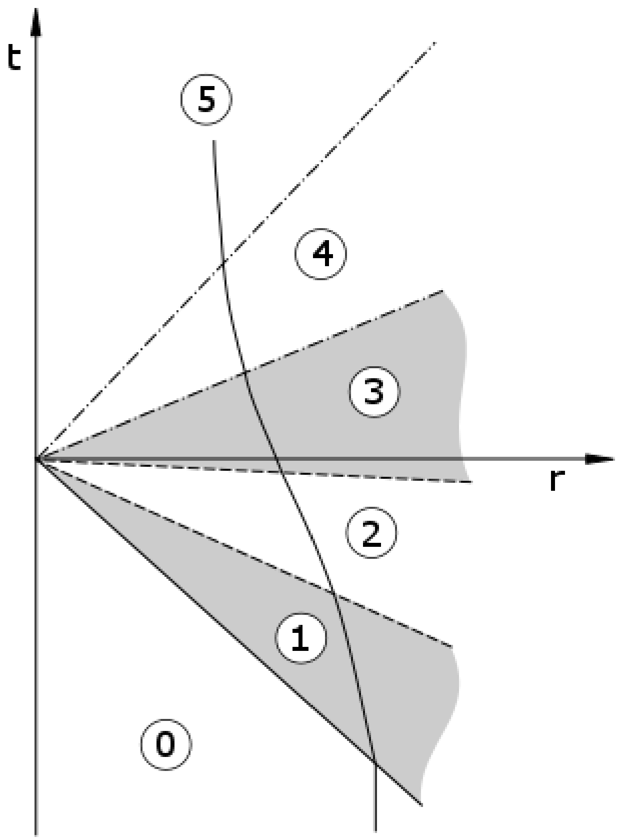
$$\frac{dp}{d\rho} = \frac{3E}{2\rho} = c_y^{\ 2}$$

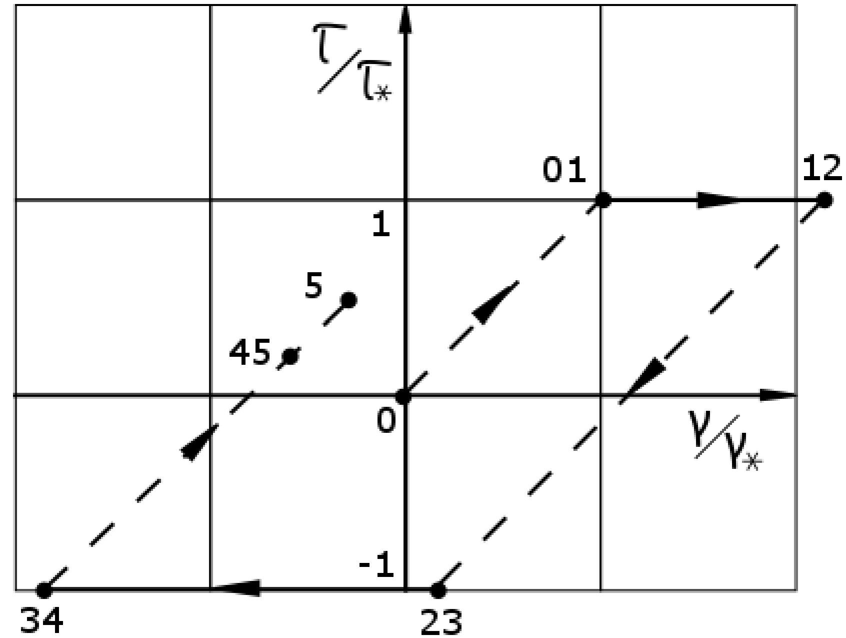
Above this area we get for elastis-plastic body

$$\frac{dp}{d\rho} = \frac{E}{\rho} = c_n^{\ 2}$$



The Hugoniot bend causes wave splitting into two. Elastic precursor, at which pressure is a constant.





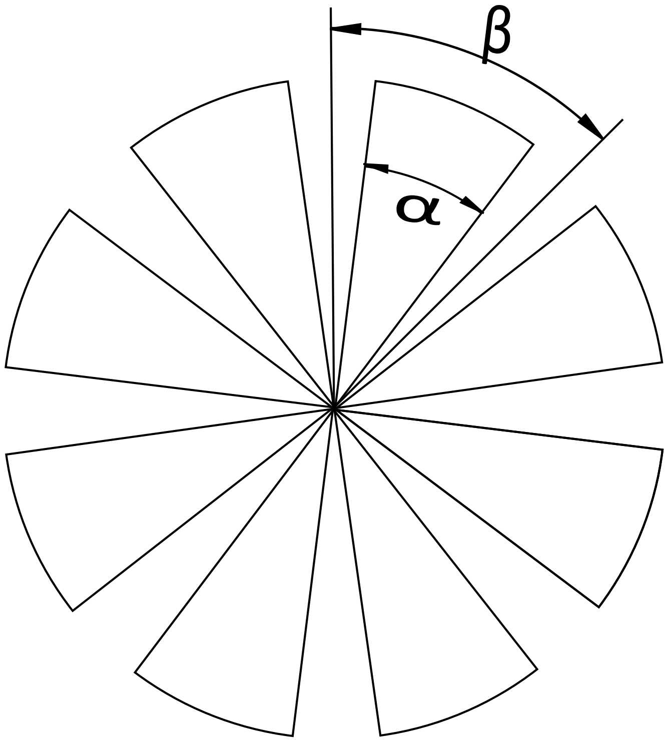
Result: material has a trace in the form of normal and shearing stress, constant within the entire volume. And

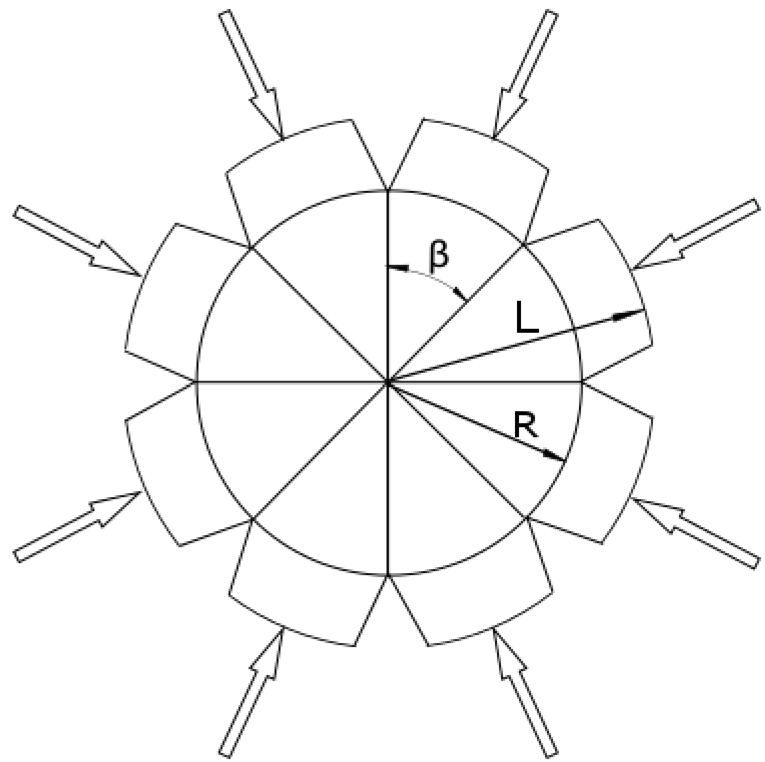
$$\frac{p}{p_*} = 0{,}467 \ln \frac{R}{r}$$

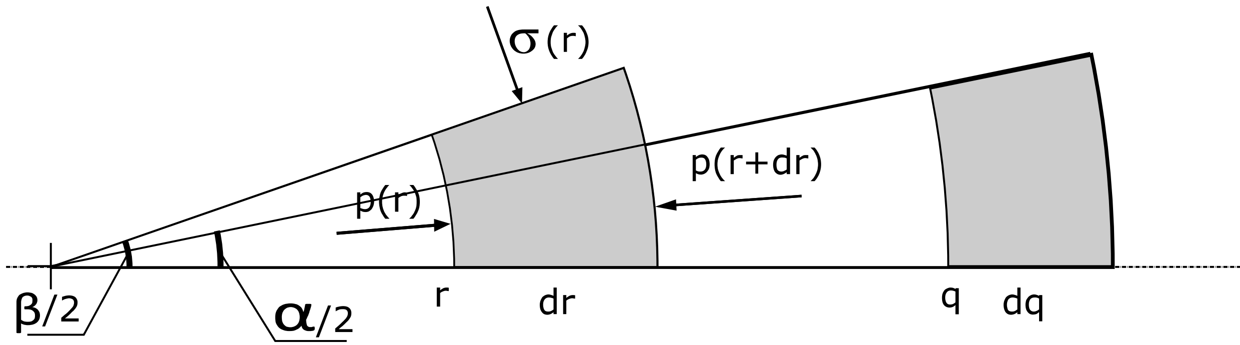
R is sphere radius after unloading

Concentric press

(E.I. Zababakhin, I.E. Zababakhin)







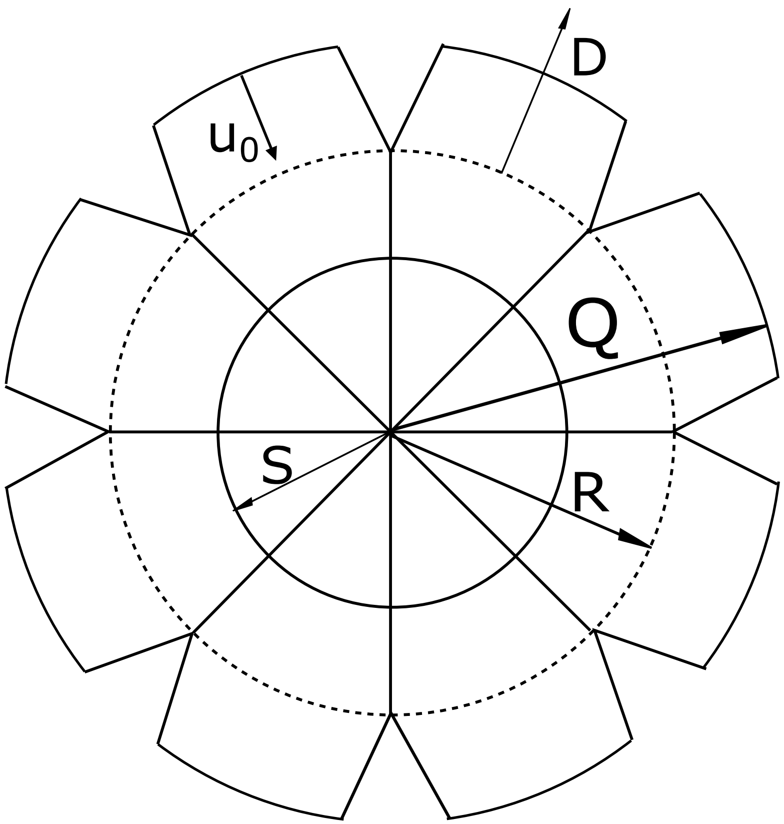
Condition of element equilibrium $$\frac{dp}{dr} = -\frac{2(p-\sigma)}{r}$$

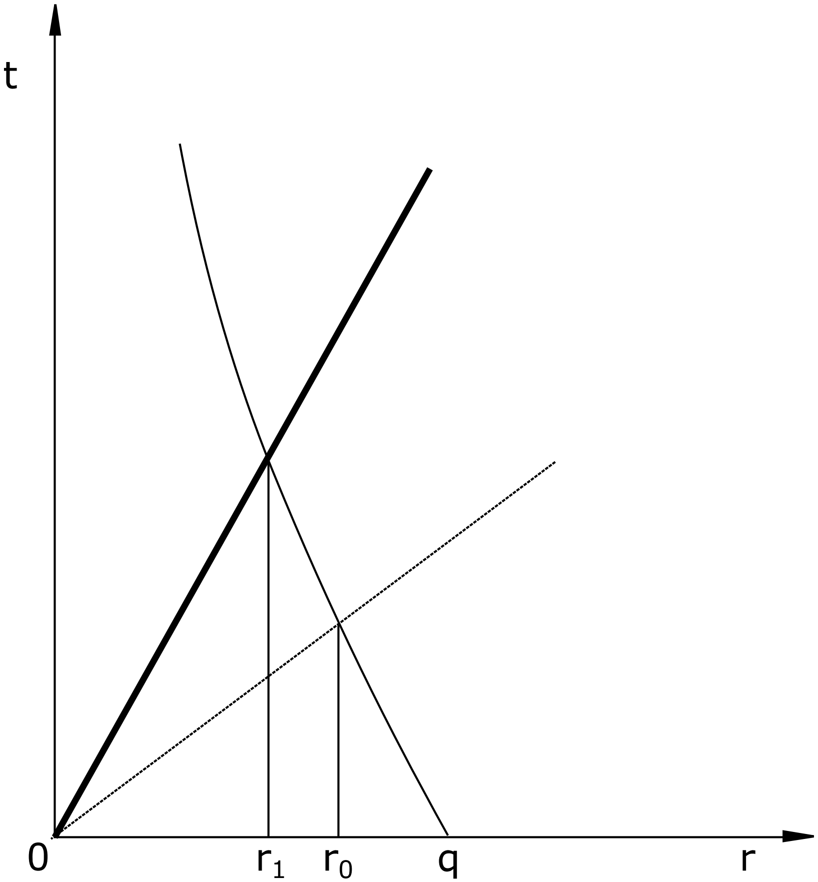
Select k so that material at the boundary of continuous compression is at strength limit.
Finally

$$\lg\frac{r}{L} = \lg\sqrt{\frac{p_0}{p_*}} - \left(\frac{1}{2} + \frac{E_0}{6p}\right)\lg\frac{1+3p/E_0}{1+3p_*/E_0}$$

Static cumulation.
For strong steel (E = 2000 Kbar, 200 kg/mm^2), pressure of 100 Kbar is reached at radius r/L = 0.15 under external pressure of 10 Kbar.

Concentric shock of sharp bodies



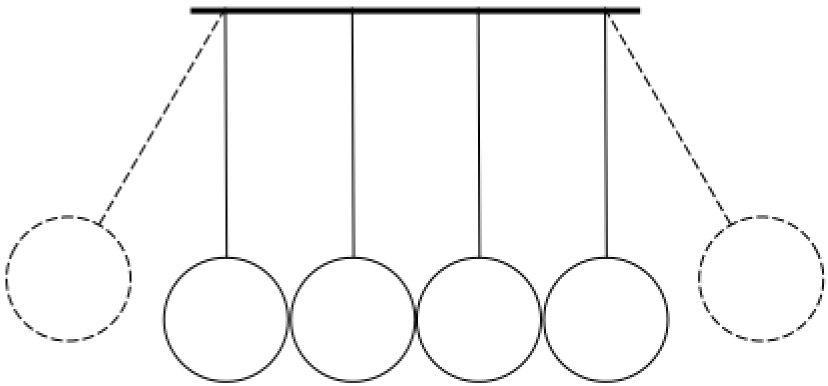


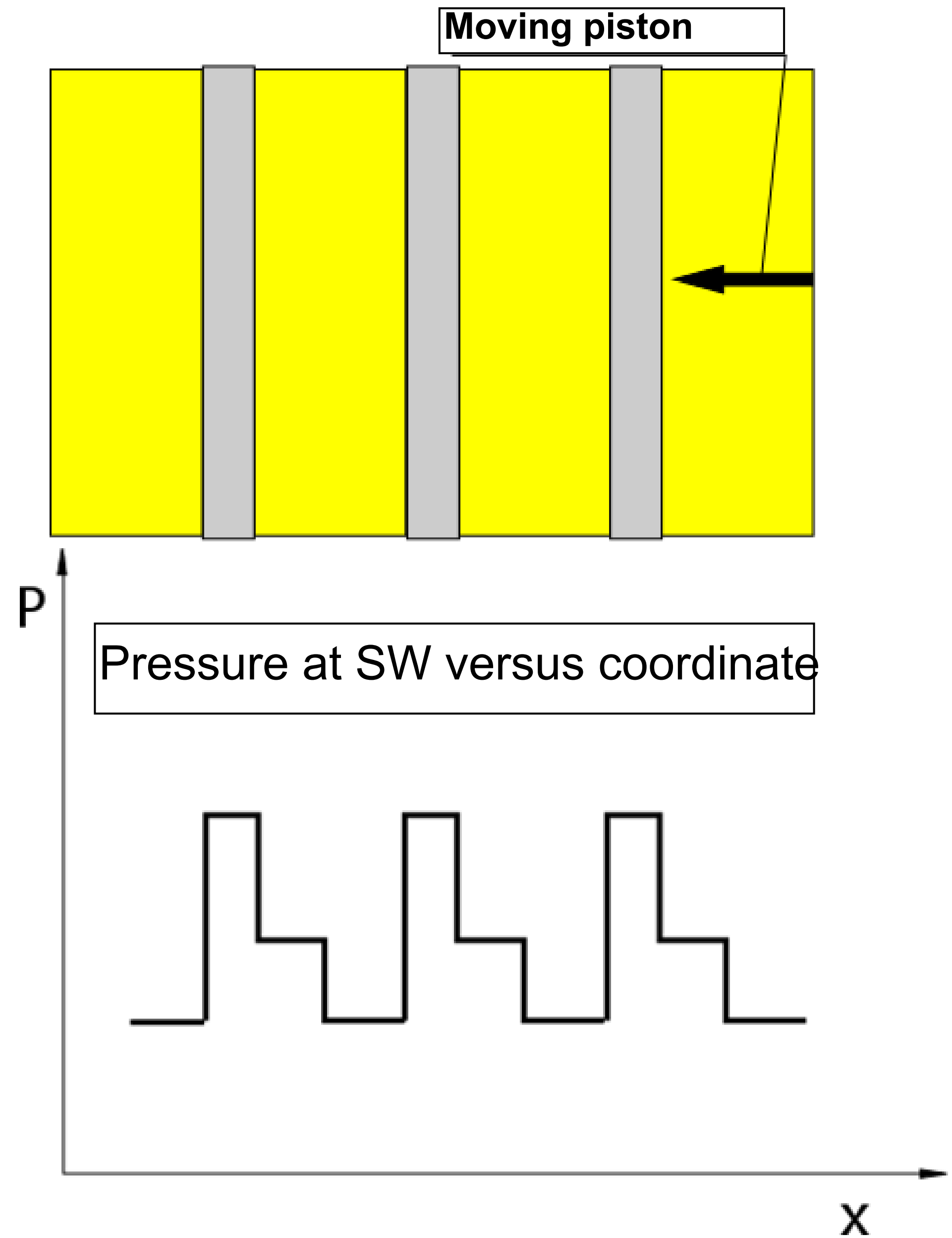
A minimal shock velocity exists.

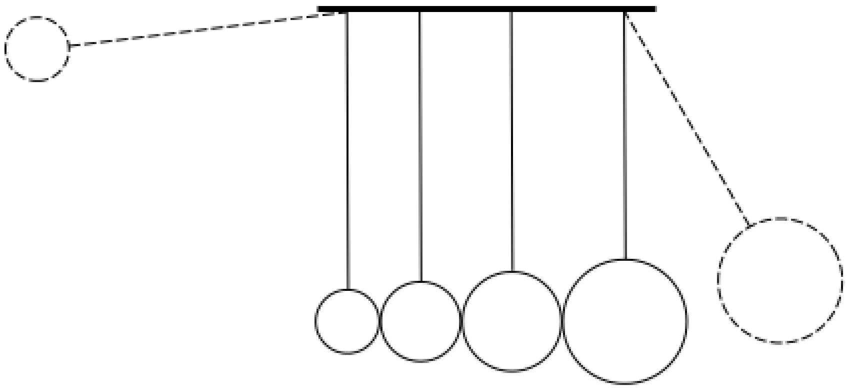
At equal small radius and equal pressures reached, radius of static and shock press is
Q/L < 0.4,
and at double shock velocity
Q/L < 0.06

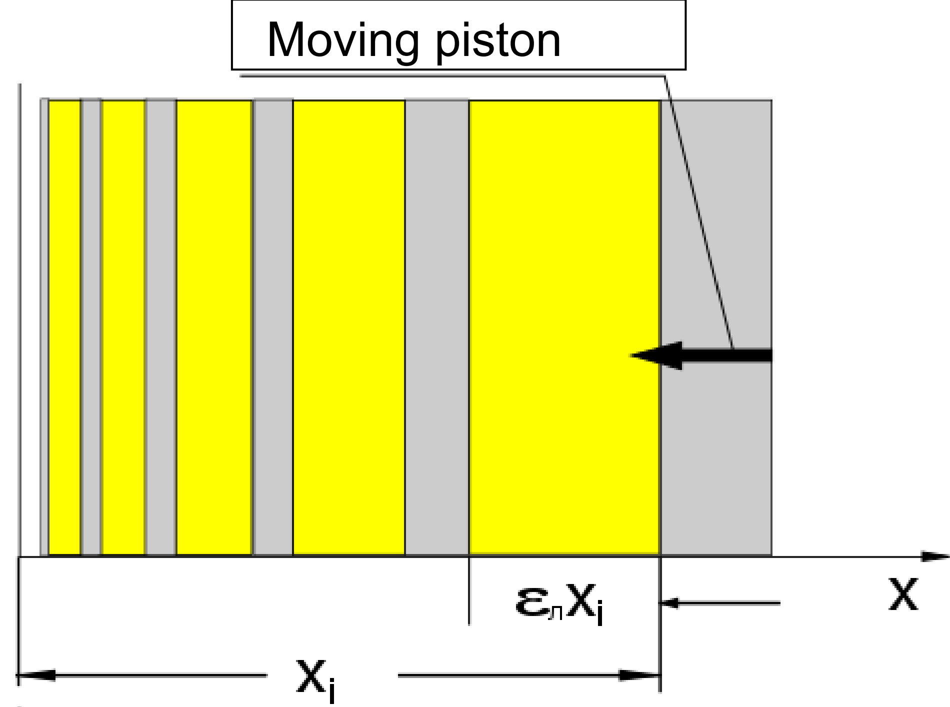
Cumulation with periodic self-similarity

(E.I. Zababakhin, A.A. Bunatyan, V.F. Kuropatenko, K.K. Krupnikov)

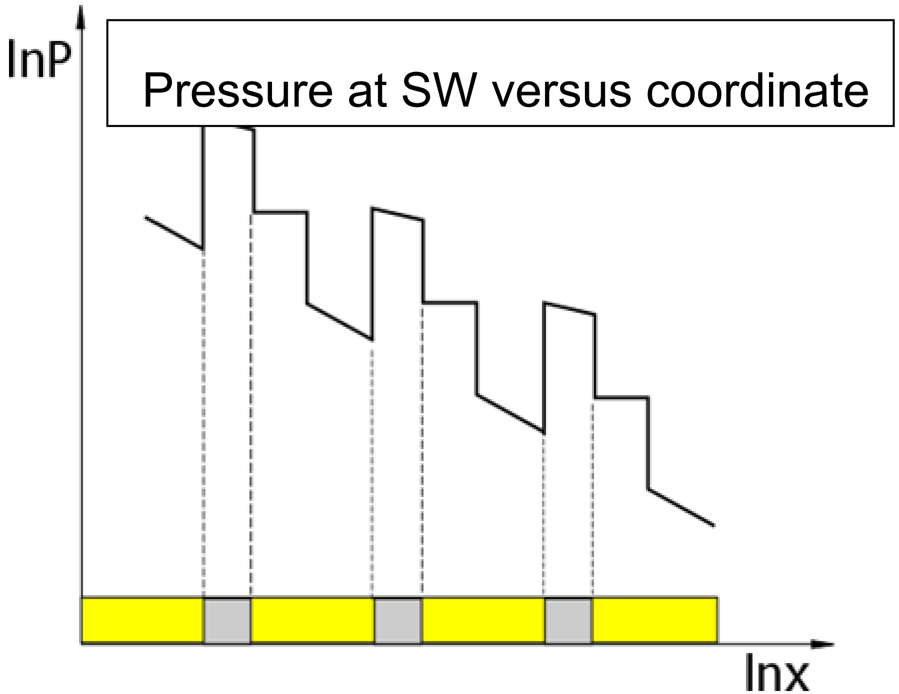


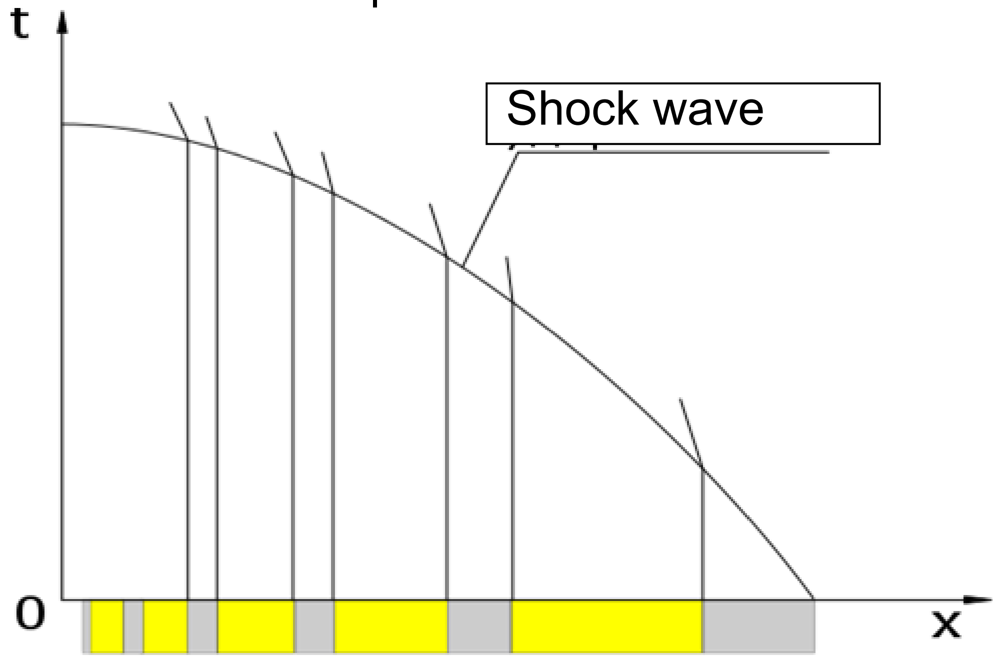






Parameters: γ=5/3; density ratio =25; $\varepsilon_{т}$ = 0,1
$\varepsilon_{л}$ = 0,2.
Result: $p = 1/x^n$ n = 0,23.

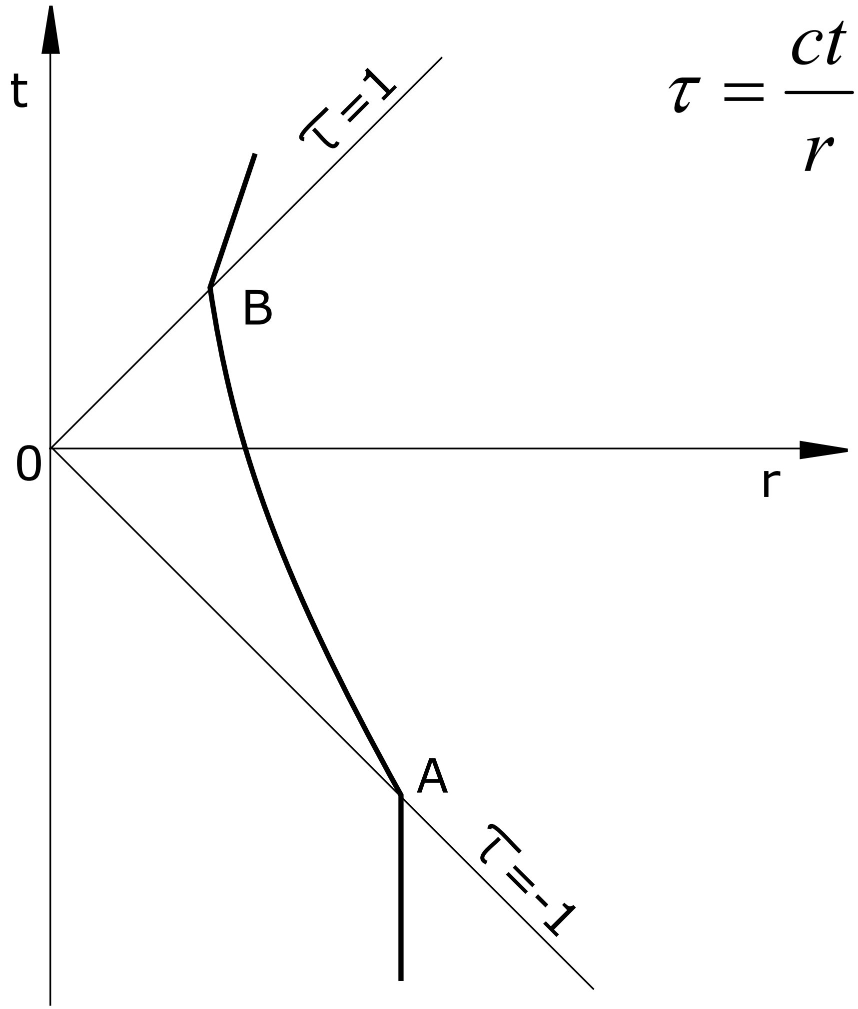




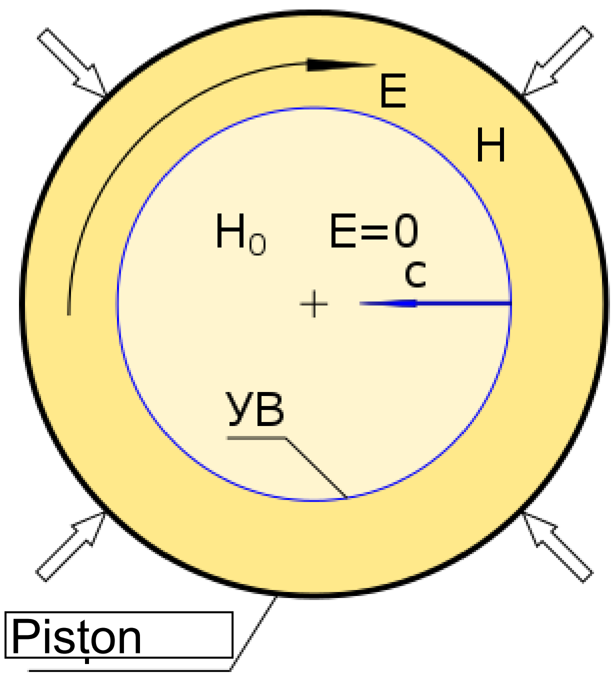
Shock waves of electromagnetic field

E. I. Zababakhin, M.N. Nechaev (1957)

Converging cylindrical wave



$$\tau = \frac{ct}{r}$$



This is acoustic wave therefore amplitude will grows as

$$\frac{1}{\sqrt{r}}\ .$$

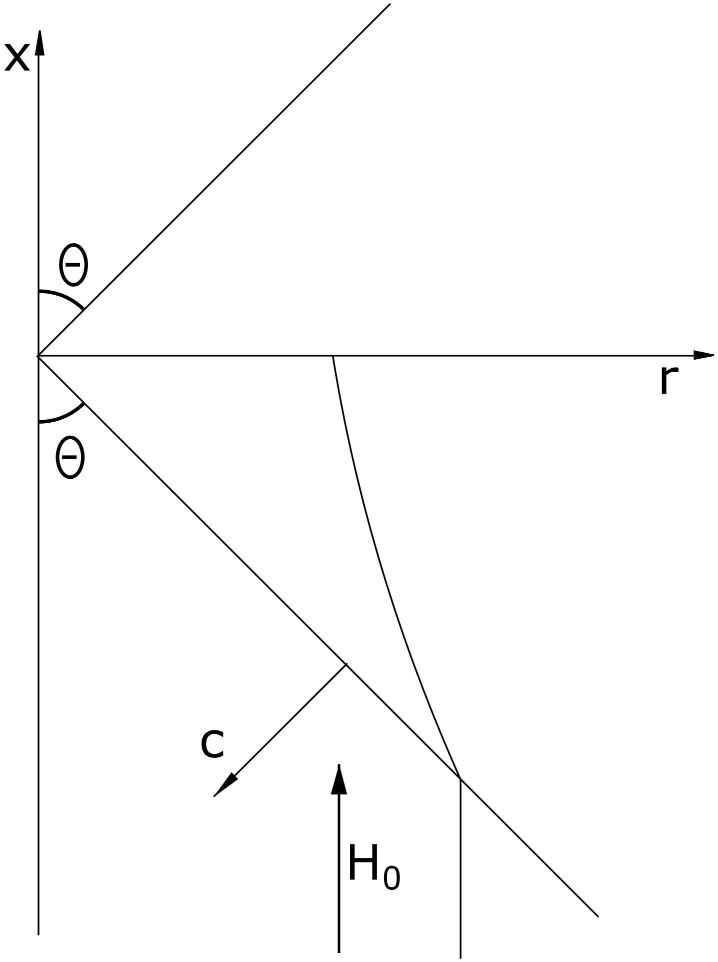
Motion is self-similar, variable is $\tau = \dfrac{ct}{r}$

$$E = E_0\sqrt{\frac{r_0}{r}}e(\tau) \qquad H = H_0\sqrt{\frac{r_0}{r}}h(\tau)$$

Solution: $e(\tau) = 1 - \sum_{n=1}^{\infty}\dfrac{(2n+1)!!(2n-3)!!}{2^{3n}(n!)^2}(1+\tau)n \qquad h(\tau) = \sum_{n=0}^{\infty}\left[\dfrac{(2n-1)!!}{n!}\right]^2\dfrac{(1+\tau)^n}{2^{3n}}$

Series converge at $\tau < 1$ and diverge at $\tau \to 1$. This means that reflection from the axis gives a diverging shock wave, which amplitude is unlimitedly large not only near the cylinder axis but also at a finite distance from it.
This is a qualitatively new type of cumulation.

Converging conic wave



Converging conic shock wave of the field.
At the reflected wave, intensities E and H are infinite everywhere.
Here cumulation is stationary (the only case).
E. I. Zababakhin, B.P. Mordvinov.

Conclusions

The monograph discusses phenomena of cumulation. The last chapter proves instability of unlimited cumulation. However, unlimited cumulation is a useful idealization allowing exact solutions and indicating how to approach it in practice.

It appears in phenomena different in terms of physics and spatial configuration. Examples were developed of instant, stationary and static unlimited cumulation.

Deviation from idealization and account for real features do not destroy the unlimited cumulation in principle.

As a rule, of interest is maximal cumulation, however, in practice it can be harmful, e.g. cavitation.