公开★ 北京应用物理与计算数学研究所
IAPCM Institute of Applied Physics and Computational Mathematics

XVI Zababakhin Scientific Talks
第十六届扎巴巴辛科学论坛

Modelling stochastic radiative transfer in random mixtures

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Outline

□ Introduction

- ✓ What is random mixture?
- ✓ Why does the radiation transport matter?
- ✓ Where does it involve with applications?
- ✓ How is the progress in the field?
- ✓ Experimental verifications?
- ✓ What's old, but important?

□ Method and Results

- ✓ an 1-dimensional problem
- ✓ a 3-dimensional problem

□ Conclusion

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1. Introduction

1.1 What is random mixture?

Basic features:

✓Two or more different materials

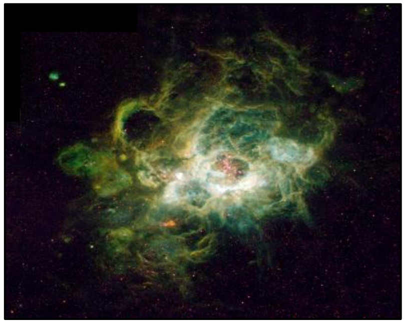
✓Mixed materials are immiscible

✓Randomly distributed spatially

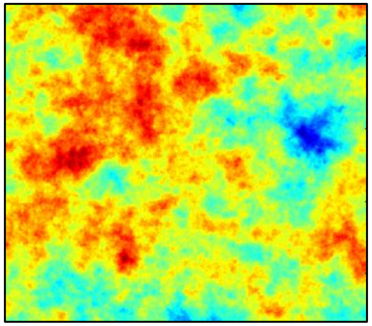
✓Statistical properties

the composition of random media is only known statistically by the mixing probability (or volume fraction) at a specified position at any time

Some examples



interstellar molecular clouds



turbulence

terrestrial clouds

1. Introduction

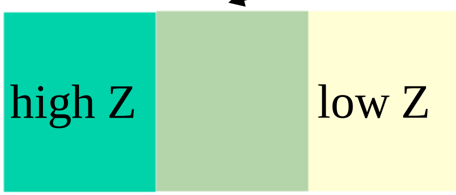
1.2 Why does the radiation transport matter?

Radiation transport describes substantial photons interaction with the material, which is closely related to **the property of the material**.

random mixture

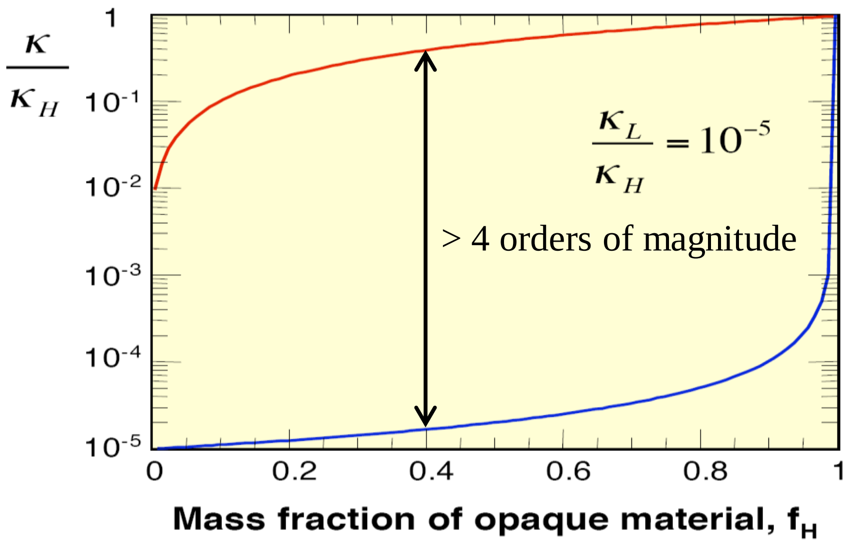
Mixed high Z materials is atomic

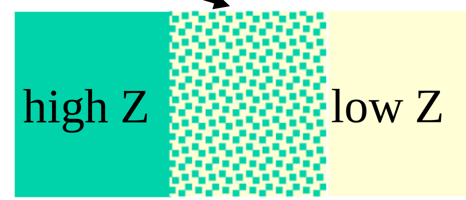
Mixed high Z materials is chunky



Atomically mixed material favors most opaque species

$$\kappa = f_H \, \kappa_H + f_L \, \kappa_L$$





Heterogeneously mixed material favors least opaque species

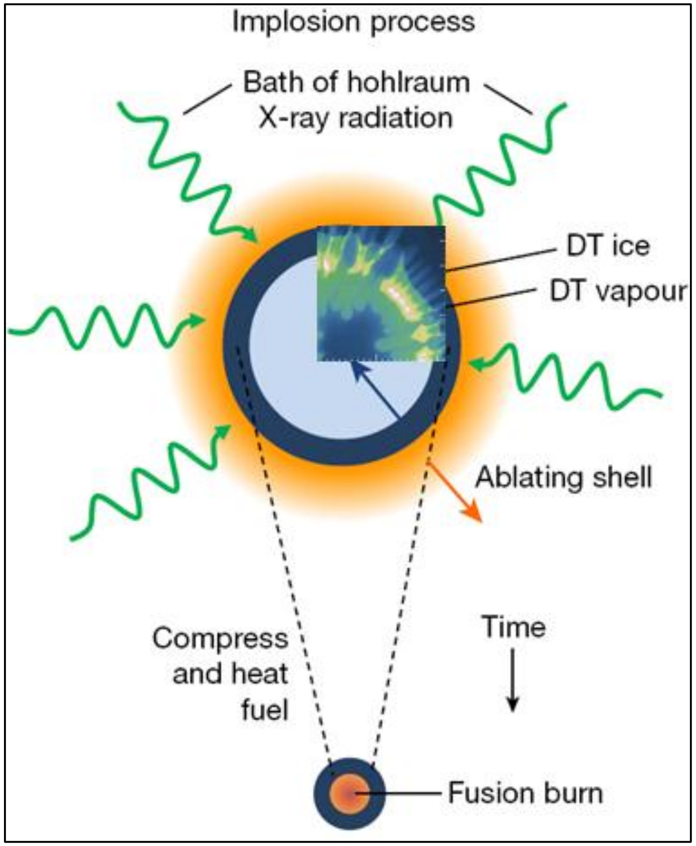
$$\frac{1}{\kappa} = \frac{f_H}{\kappa_H} + \frac{f_L}{\kappa_L}$$

The opacity of random mixture strongly depends on the mixing, whose uncertainty greatly exceeds that of species opacity (~30 %) for (ρ, T).

1. Introduction

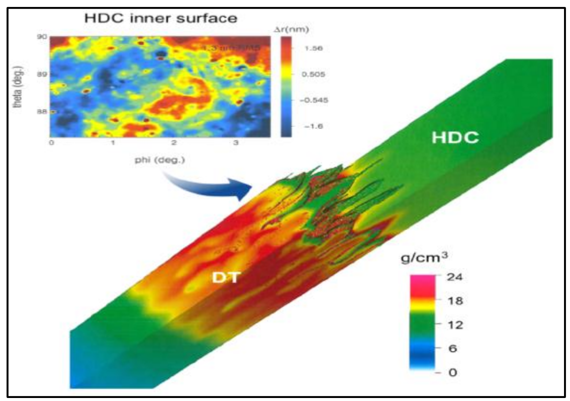
1.3 Where does it involve with applications?

In inertial confinement fusion (ICF), **the mixture of the fuel and shell** initiated by the hydrodynamic instability [e.g., Rayleigh-Taylor (RT) instability] was experimentally observed during the implosion and compression process, and the energy transported through the stochastic fuel-shell mixtures is believed to play a role in the performance of the fusion pellet.



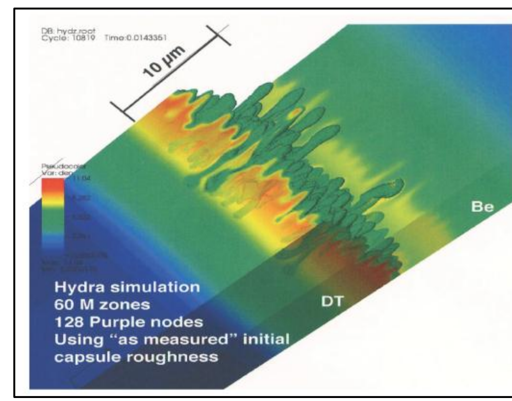
Schematic of the indirect-drive ICF

Nature 601, 542(2022)



Interface mixing

PoP26, 050601 (2019)



Interface mixing

Dimonte, LANL

1. Introduction

1.4 How is the progress in the field?

Journal of Mathematical Physics

Linear transport theory in a random medium

C. D. Levermore
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(Received 17 April 1986; accepted for publication 11 June 1986)

$$\frac{d\Psi(s)}{ds} + \sigma(s)\Psi(s) = S(s).$$

$$\Psi(0) = \Psi_0,$$

Pioneering work by Levermore and his coauthors using a projection operator technique derived an analytical result in the form of two exponentials.

$$\langle\Psi(s)\rangle = \Psi_0\left(\frac{r_+ - \tilde{\sigma}}{r_+ - r_-}\right)e^{-r_+ s} + \left(\frac{\tilde{\sigma} - r_-}{r_+ - r_-}\right)e^{-r_- s},$$

$$r_\pm = \tfrac{1}{2}\{\langle\sigma\rangle + \tilde{\sigma} \pm [(\langle\sigma\rangle - \tilde{\sigma})^2 + 4\beta]^{1/2}\}.$$

1. Introduction

Radiation transport in random mixtures is an unfashionable subject, but its interest is growing in many fields. There are four main concerns:

(i) for simple 1D geometry

JQSRT**36**, 557(1986); JQSRT**40**, 479(1988); JQSRT**42**, 253(1989); JQSRT**50**, 211(1993); JQSRT**51**, 689(1994); JQSRT**112**, 599(2011); PRE**102**, 022111(2020); PRE**105**, 014131(2022)

(ii) going beyond simple 1D geometry

PRE61, 6183(2000); JQSRT**104**, 86(2007); JQSRT**113**, 325(2012); JQSRT**89**, 133(2017)

(iii) developing deterministic models or more accurate closures

JQSRT**42**, 253(1989); JQSRT**51**, 893(1994); JQSRT**168**, 57(2016)

(iv) developing efficient numerical algorithms

Trans. Am. Nucl. Soc.**105**, 498(2011); JQSRT**148**, 127(2014); JQSRT**196**, 270(2017)

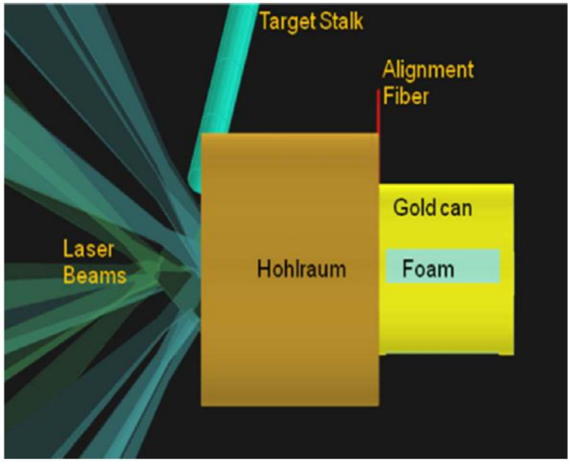
1. Introduction

1.5 Experimental verifications

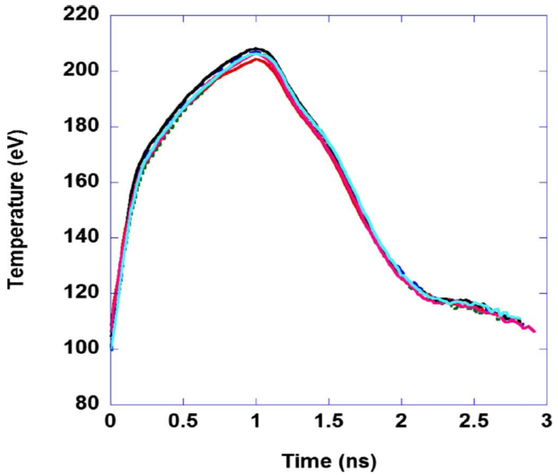
Goal: provide data to help test some of these radiation transport models.

Targets: Au-loaded triacrylate $C_{15}H_{20}O_6$ foam

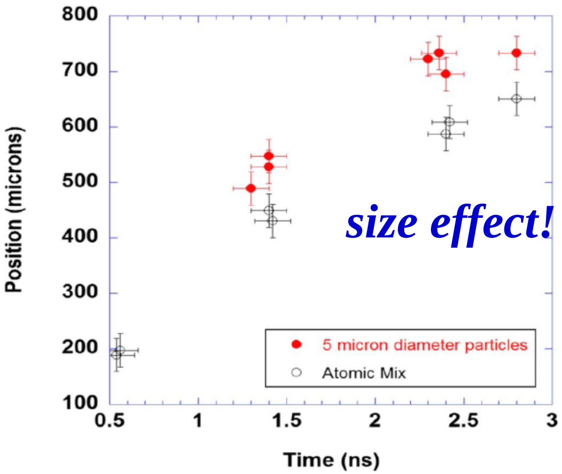
Laser: OMEGA facility at the University of Rochester, USA



A picture of experimental platform



Temperature drive



Temperature front positions

An inhomogeneous transport model must be applied to explain radiation transport in foams loaded with 5 μm diameter gold particles.

1. Introduction

1.6 What's old, *but* important?

Understanding the impact of random media on the radiative transfer

Two questions: How is the impact and Why?

The description of radiation transport is given by

$$\frac{1}{c}\frac{\partial}{\partial t}\psi\left(\vec{r},E,\vec{\Omega},t\right)+\vec{\Omega}\bullet\nabla\psi\left(\vec{r},E,\vec{\Omega},t\right)+\sigma_t\psi\left(\vec{r},E,\vec{\Omega},t\right)=\frac{1}{4\pi}c\sigma_t aT^4\left(\vec{r},t\right)+\frac{1}{4\pi}\sigma_s\int\psi d\vec{\Omega}'$$

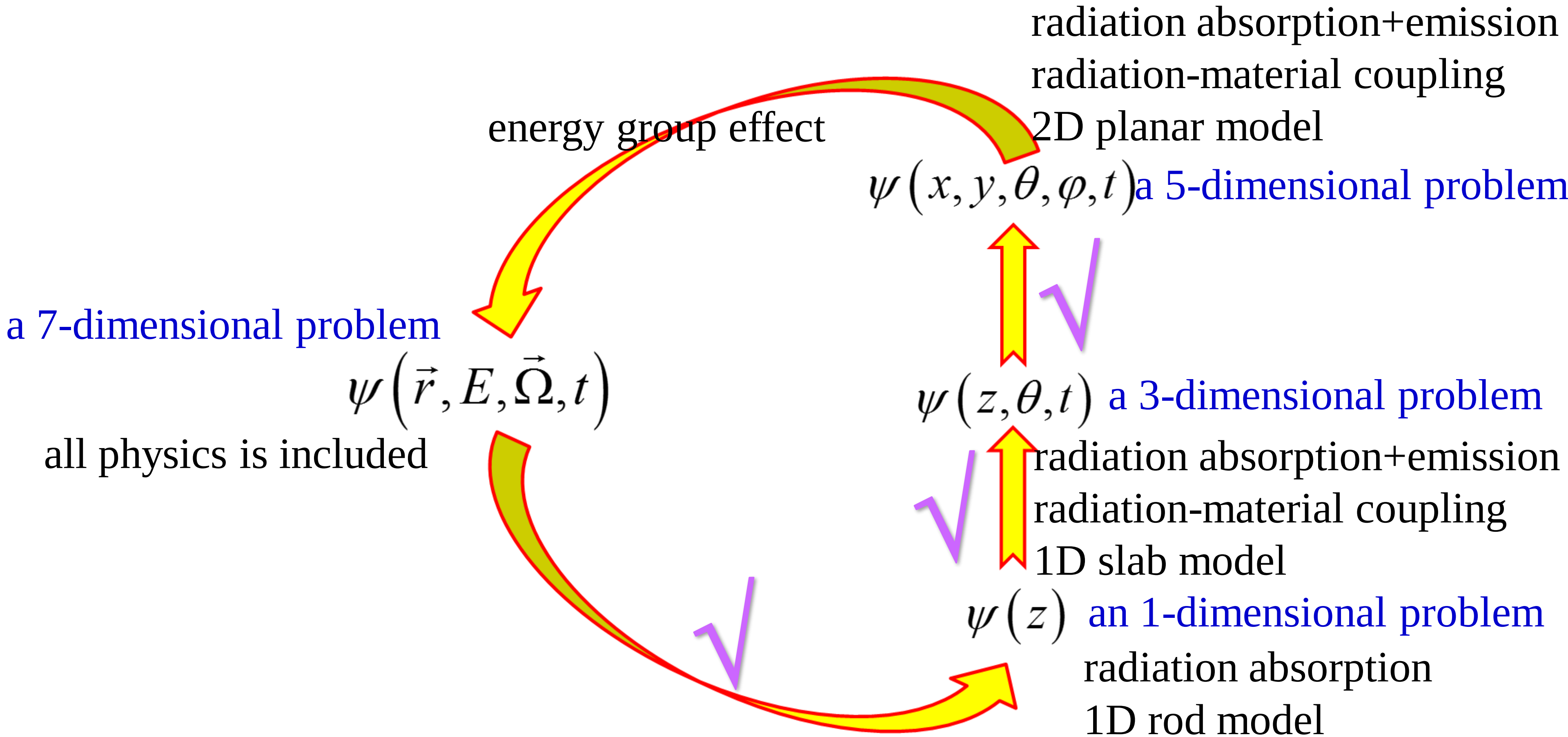
$$\rho C_v\frac{\partial}{\partial t}T\left(\vec{r},t\right)=-c\sigma_t aT^4\left(\vec{r},t\right)+\left(\sigma_t-\sigma_s\right)\int\psi d\vec{\Omega}'$$

ψ : specific radiation intensity, T:material temperature

A **7-dimensional** (3 spatial + 2 directional variables + 1 energy + 1 time variable), **strong non-linearity problem**. *Hard to solve!*
Need to know the mixture's detailed configuration.

2. Method and Results

To make it tractable, we began with the simplest physical scenario and add the complexity step-by-step, and obtained new insights of stochastic radiative transfer at various levels.



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2. Method and Results

2.1 An 1-dimensional problem

In 1D rod model, radiation transport equation is written as

$$\frac{dI(x)}{dx} + \sigma(x)I(x) = S(x)$$

$\sigma(x)$ → random value, σ_1(low Z) or σ_2(high Z)

the analytical solution is $I(x;\tau) = \Gamma e^{-\tau} + \int_0^x dx' S(x') e^{-(\tau-\tau')}$

For random mixtures, we concentrate on the **ensemble-averaged radiation intensity**

$$\langle I(x)\rangle = \int_0^\infty d\tau I(x;\tau) f(\tau;x),$$

$f(\tau;x)$: the PDF for random variable τ at the distance x →

$$\mathcal{C}(\tau;x) = \begin{cases} 0, & \tau < \sigma_1 x \\ \int_0^\tau d\xi f(\xi;x), & \sigma_1 x \leqslant \tau \leqslant \sigma_2 x \\ 1, & \tau > \sigma_2 x. \end{cases}$$

With multiple manipulations, we obtain

$$\langle I(x)\rangle = \Gamma\left\{e^{-\sigma_2 x} + \int_{\sigma_1 x}^{\sigma_2 x} d\tau \mathcal{C}(\tau;x) e^{-\tau}\right\},$$

$e^{-\sigma_2 x}$: deterministic; $\int_{\sigma_1 x}^{\sigma_2 x} d\tau \mathcal{C}(\tau;x) e^{-\tau}$: stochastic

suitable for any random mixtures, but how to make calculations?

2. Method and Results

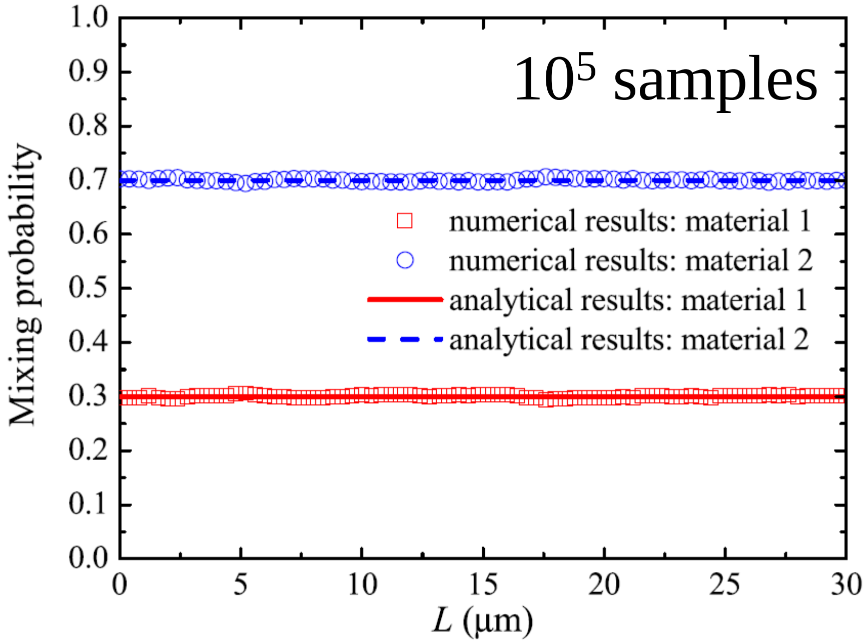
Sampling of random mixtures

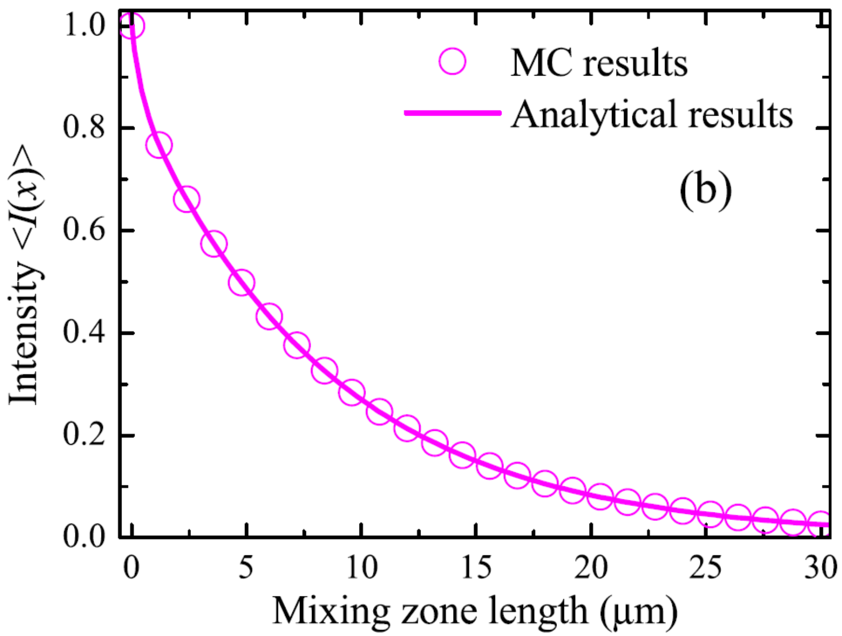
We focus on the binary random mixture with homogeneous statistics, i.e., $p_i(x) = p_i$. The length ζ of a segment of material i is described by the chord length distribution (CLD) $f_i(\zeta)$, and the mean chord length of material i is denoted as λ_i.

1. At the origin (x=0), select the material statistically according to p_1 or p_2.
2. Determine the distance from x=0 to the first material interface.
3. Sample from $f_j(\zeta)$ to determine the length of the first segment of material j.
4. Sample from $f_i(\zeta)$ to determine the second segment of material i's length.
5. Repeat steps 3, 4 until the accumulative length exceeding the total length L.
6. Numerically calculate the optical depth τ for generated physical realization.
7. Redo the above steps for N times, and evaluate the $C(\tau; x)$ for random optical depth. The ensemble-averaged results are thus computed.

2. Method and Results

Verifications

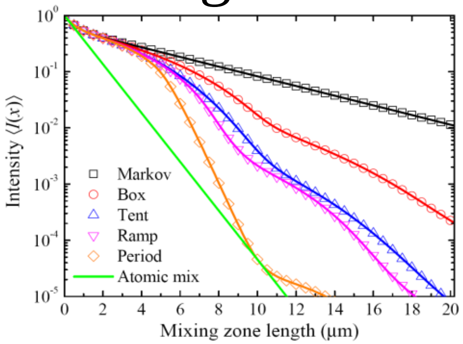


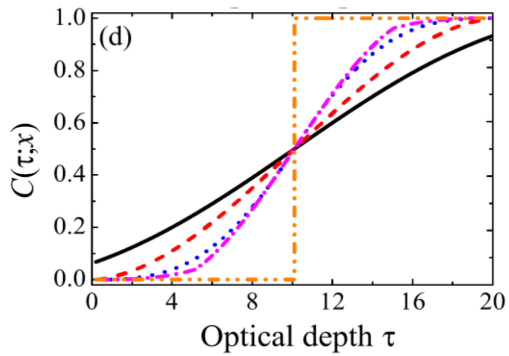


1. The present procedure is robust to yield homogeneous binary random mixture.
2. Simulated results agree perfectly with the analytical curve in the considered range.

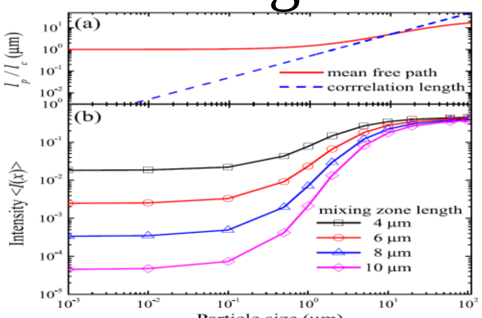
2. Method and Results

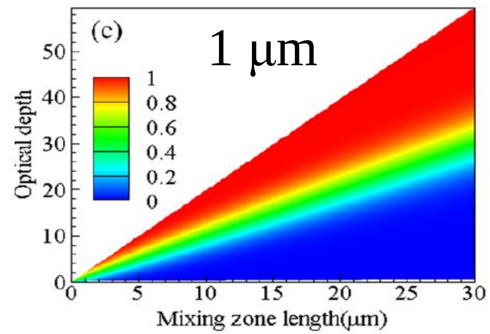
mixing distribution



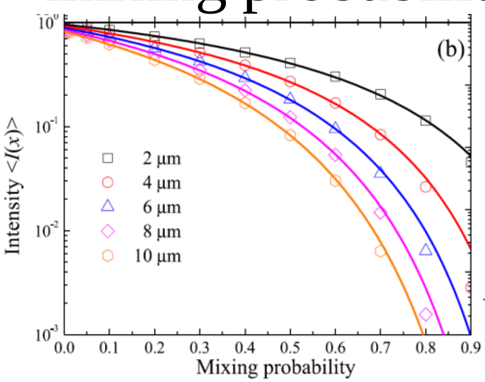


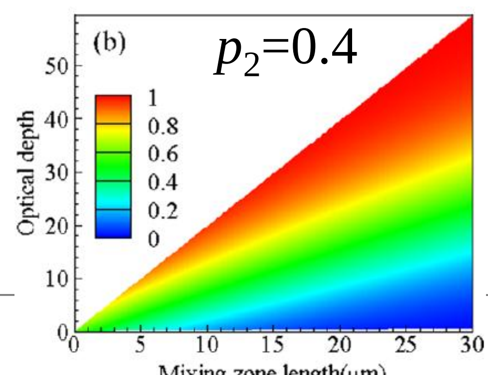
mixing size





mixing probability





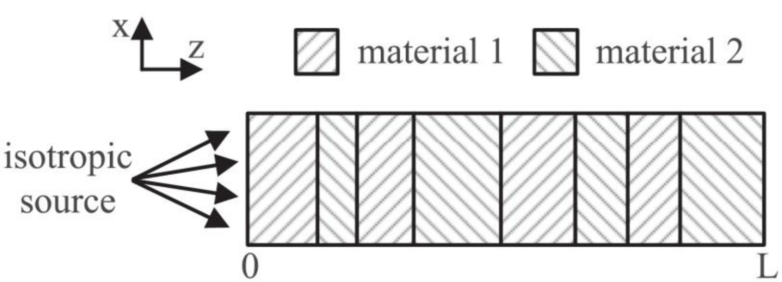
- The impact of random media on the radiative transfer is significant, spanning over several orders of magnitude.
- For the mixing distribution, various mixing statistics can basically generate varied $C(\tau)$. Large L allows τ to distribute over a wide interval, the intensity is mostly associated with the probabilities towards smaller τ.
- For the mixing size, it is sensitive when $l_p \sim l_c$. The extension of the transition region of the cumulative PDF.
- For the mixing probability, substantial reduction of intensity is due to negligible small-optical-depth probabilities for large mixing probabilities.

2. Method and Results

2.2 A 3-dimensional problem

We assume the binary random mixture with homogeneous statistics. Material i follows a chord length distribution (CLD) and has the mean chord length λ_i. Its mixing probability is defined as $\lambda_i/(\lambda_i+\lambda_j)$.

1. Generate a large ensemble of random mixtures
2. Solve the radiation-material coupling equations
3. Collect all calculated physical quantities and make averaging.



Schematic of a physical realization of a binary random medium in the planar geometry.

$$\frac{1}{c}\frac{\partial}{\partial t}\psi(t,z,\mu)+\mu\frac{\partial}{\partial z}\psi(t,z,\mu)+\sigma_a(z)\psi(t,z,\mu)=\frac{c\sigma_a(z)}{4\pi}\phi(t,z)$$

$$\frac{\partial}{\partial t}\phi(t,z)=-c\sigma_a(z)\phi(t,z)+2\pi\sigma_a(z)\int_{-1}^{1}d\mu'\psi(t,z,\mu')$$

radiation-material coupling equations

transmission flux $\langle J(t,z)\rangle=\frac{1}{M}\sum_{i=1}^{M}\int_0^1\mu\psi_i(t,z,\mu)d\mu$

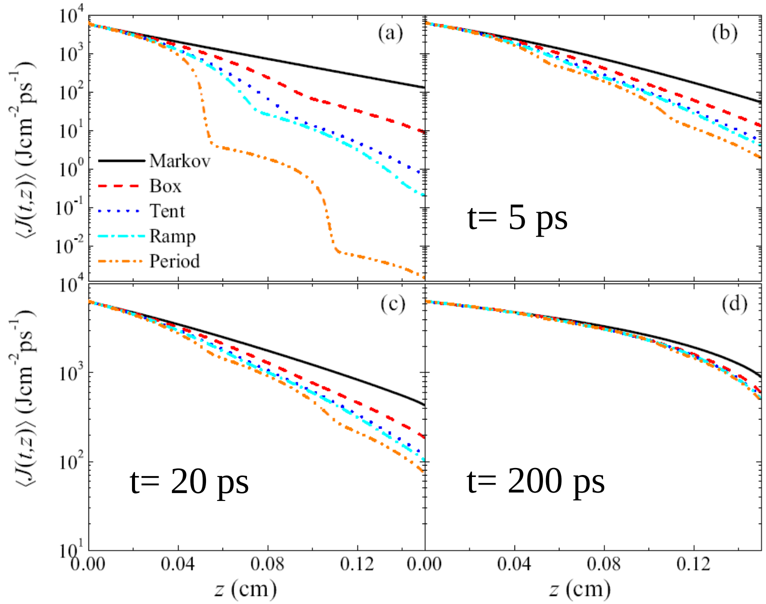
2. Method and Results

We have developed a home-made code RAREBIT1D (RadiAtive tRansfEr in Binary stochastIc mixTures in One Dimension) to simulate stochastic radiative transfer in random mixtures.

TABLE I. Model parameters of the binary stochastic mixture and numerical parameters used in simulations.

Label	Parameters
Material 1	$\lambda_1 = 5.6 \times 10^{-3}$ cm, $p_1 = 0.1$, $\sigma_{a,1} = 10^3\ \mathrm{cm}^{-1}$
Material 2	$\lambda_2 = 5.0 \times 10^{-2}$ cm, $p_2 = 0.9$, $\sigma_{a,2} = 5\ \mathrm{cm}^{-1}$
Total length	$L = 0.15$ cm
Initial temperature	$T_0^m = 0.005$ keV
Radiation source	$T_0^r = 0.5$ keV
Angular direction	$N = 32$
Simulation time	$\tau = 500$ ps
Time step	$\Delta\tau = 1$ ps
Cell coefficient	$f = 0.2$
Convergent value	$\epsilon = 10^{-6}$
Realizations	$M = 10^5$

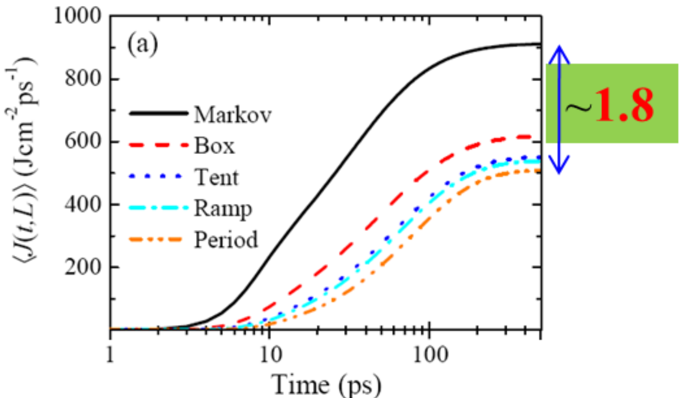
Nonparticipating random mixture



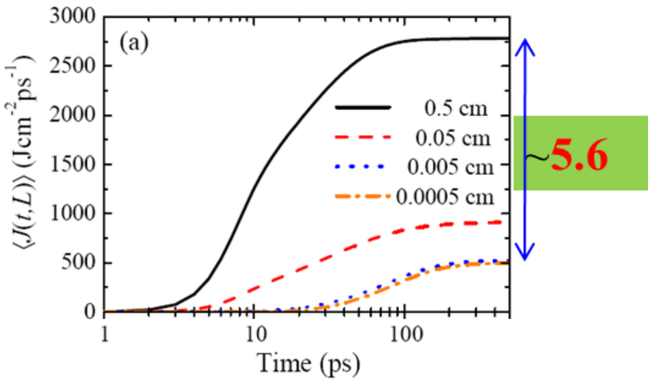
material temperature significantly suppresses the effect of the mixing distribution.

2. Method and Results

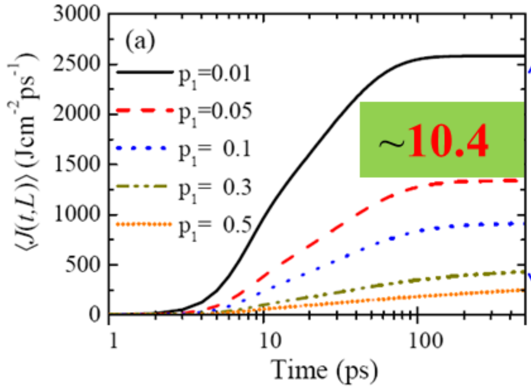
We concern the transmission at the outgoing edge, i.e., $z=L$.



mixing distribution



mixing size



mixing probability

By considering the feedback of material, i.e., thermal radiation emission, it is found that **the mixing probability still affects the transmission by about an order of magnitude, then is the mixing size, the least is the mixing distribution**.

One can make a detailed analysis of energy channels, but it does **NOT** answer why we observed such a dependence of radiation transport on the random mixture's property.

2. Method and Results

Consider 1D, single-group, steady-state, and no scattering radiation transport,

$$\mu \frac{\partial}{\partial z}\psi(z,\mu) + \underline{\sigma_a(z)}\psi(z,\mu) = \underline{S(z)}$$

related to the material

We first weaken the dependence of the photon's direction

$$\frac{\partial}{\partial z}\psi(z,\mu) + \sigma_a'(z)\psi(z,\mu) = S'(z) \ ,$$

$$\sigma_a'(z) = \frac{\sigma_a(z)}{\mu}, \quad S'(z) = \frac{S(z)}{\mu} = \sigma_a'(z)B(T)$$

$$\Rightarrow \quad \frac{d\langle\psi\rangle}{dz} + \langle\sigma_a\rangle\langle\psi\rangle + \gamma\chi = \langle S\rangle$$

$$\frac{d\chi}{dz} + \gamma\langle\psi\rangle + \hat{\sigma}_a\chi = T$$

$$\chi = \sqrt{p_1 p_2}(\psi_1 - \psi_2)$$

The ensemble-averaged intensity can be analytically written as

$$\langle\psi(z)\rangle = \langle\psi(z)\rangle_{\mathrm{homo}} + \langle\psi(z)\rangle_{\mathrm{nonhomo}}$$

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2. Method and Results

With several mathematical manipulations, the ensemble-averaged specific radiation intensity for the direction μ can be analytically solved as

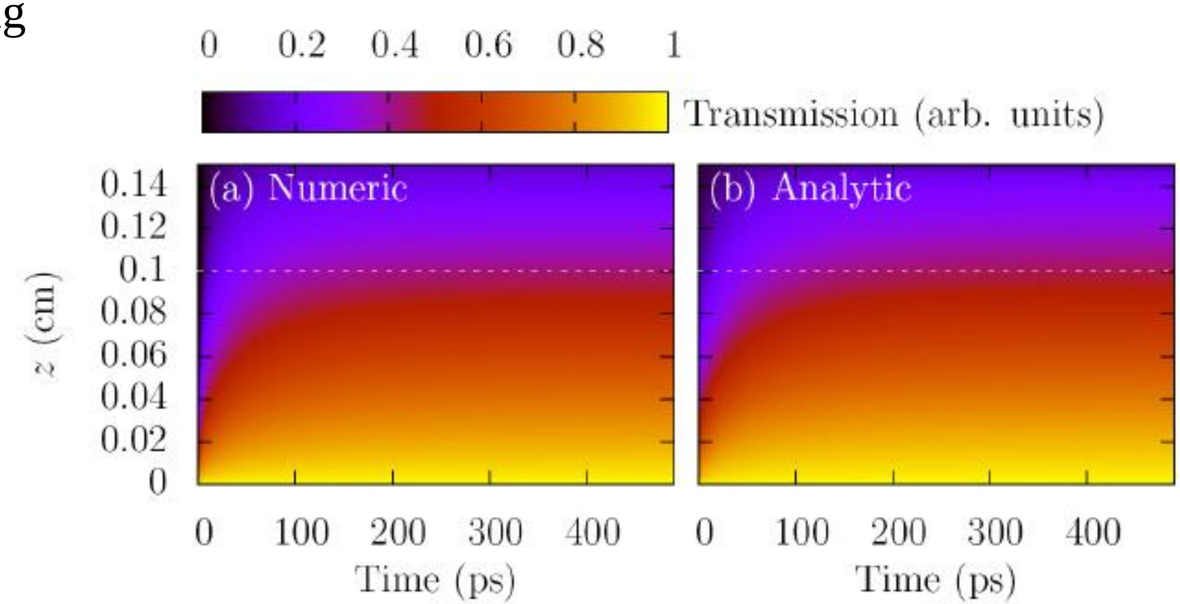
$$\underbrace{\langle\psi_\mu(z)\rangle}_{\text{I}} \approx \underbrace{\psi_0\left[we^{\varepsilon_+ z}+(1-w)e^{\varepsilon_- z}\right]}_{\text{II}} + \underbrace{\langle B(z)\rangle \frac{1+\langle\sigma_a(z)B(z)\rangle/\sigma_{a,1}\sigma_{a,2}\lambda_c\langle B(z)\rangle}{1+\langle\sigma_a(z)\rangle/\sigma_{a,1}\sigma_{a,2}\lambda_c}\left\{1-\left[we^{\varepsilon_+ z}+(1-w)\,e^{\varepsilon_- z}\right]\right\}}_{\text{III}},$$

radiation transport through nonparticipating random mixture

thermal emissions

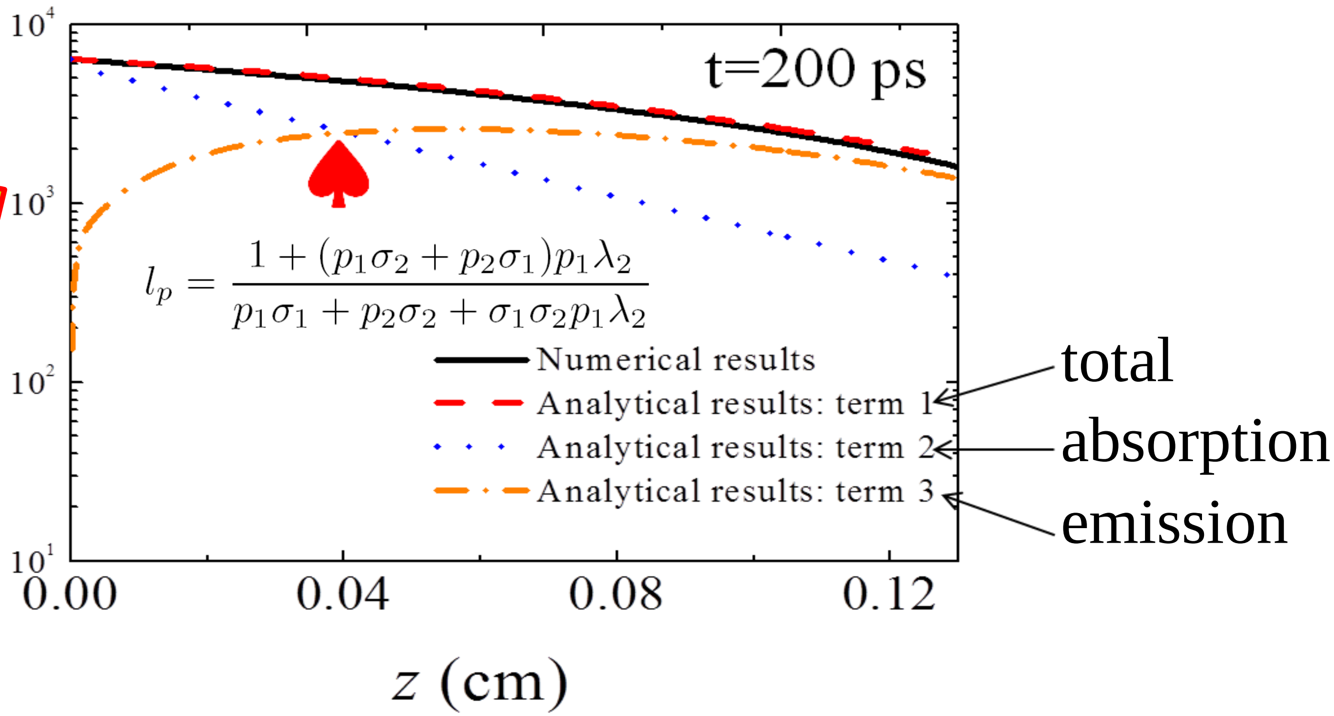
transmission flux

$$\langle J(z)\rangle = \int_0^1 \mu\langle\psi_\mu(z)\rangle d\mu$$



The analytical results agree satisfactorily with simulated data

2. Method and Results



theoretical criterion

$$\frac{l_p}{L} \sim 1$$

1) if $l_p < L$, the material temperature plays a key role, and a pronounced dependence of the transmission on the random mixtures may result from the variation of the value of l_p.

2) if $l_p \geq L$, it is dominated by pure absorption of radiation, which does not depend on the properties of random mixtures.

2. Method and Results

Interpretation of numerical results

Mixing distribution

By varying the mixing distributions considered, l_p is varied from 0.04 cm (Markov) to 0.02 cm (Period), which are always much smaller than L =0.15 cm. In this case, the transmission is dominated by material temperatures, which do not have significant deviations between Markov and Period mixing distributions, since l_p is quite close in both cases. This can explain a weak dependence on the mixing statistics.

Mixing size

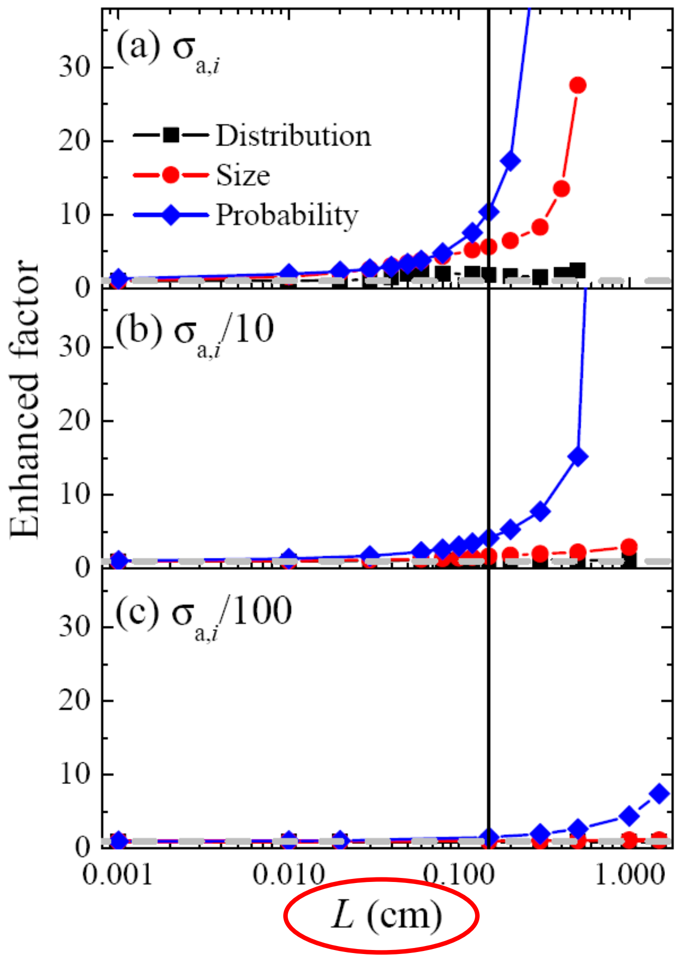
By decreasing the mixing sizes used, l_p is tuned from 0.13 cm to 0.01 cm, which ranges from the region comparable to L to that much lower than L. In this situation, the transmission of large mixing sizes (high l_p) is dominated by pure radiation absorption, while for small mixing sizes (low l_p) material temperature effect is remarkable. These all together result in a moderate dependence on the mixing size.

Mixing probability

When increasing the mixing probability, l_p is changed from 0.09 cm to 0.02 cm, which are smaller than L. In all cases, material temperatures are the essential factor. As the redistribution of thermal radiations decays exponentially with increasing the distance, it is not surprising that the mixing probability acquires the most noticeable l_p on the transmission.

2. Method and Results

Corroborating the theoretical criterion

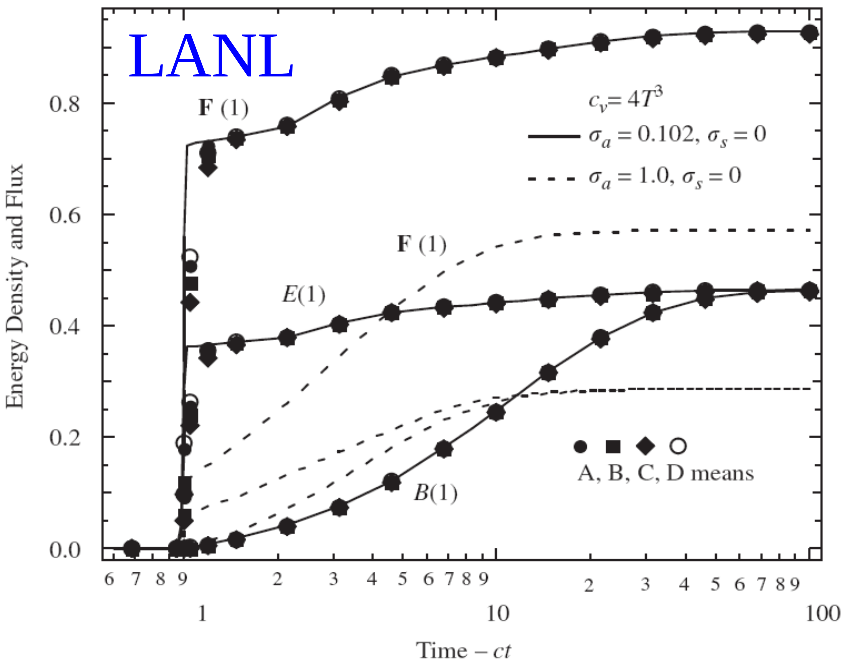


Enhanced factor *v.s.* mixing width

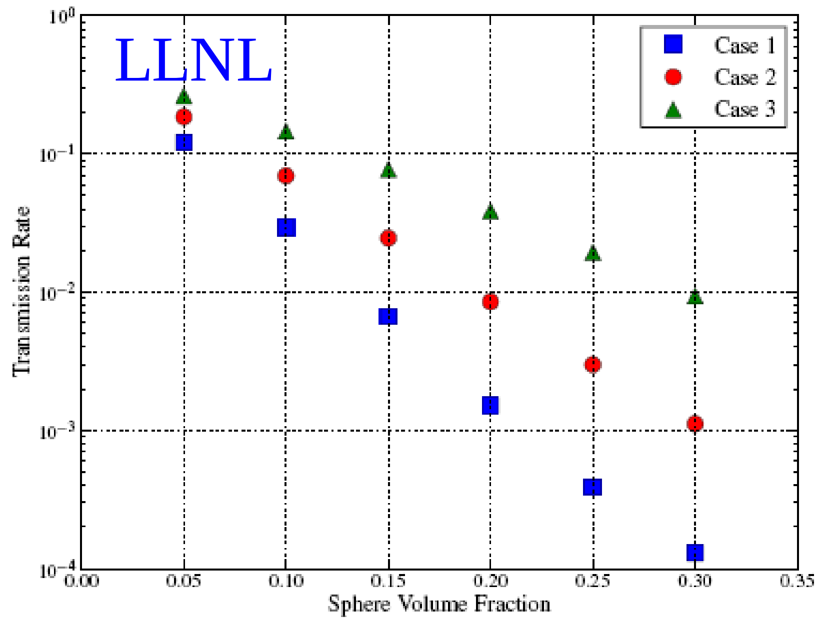
- When l_p ≤0.01 cm, radiation transmission flux is independent of random mixtures. Increasing *L*, transmission flux depends sensitively on the random mixtures.
- When the opacity is decreased by an order of magnitude, the dependence of radiation transmission flux at *L*=0.15 cm on the random mixture is weakened.
- When the opacity is decreased by two orders of magnitude, for *L*=0.15 cm, the relation is $l_p >> L$, radiation transmission flux does not depend on the random mixture.

2. Method and Results

Understanding the disputable problem



JQSRT **104**, 86 (2007)



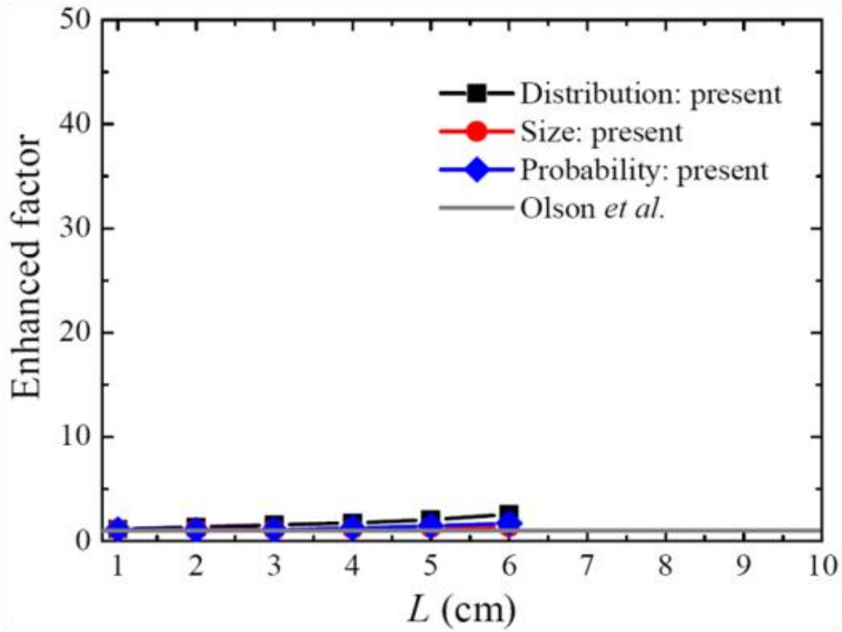
Tech. Rep. (LLNL, CA (United States), 2011)

Different mixing distributions and sizes **do not impact** the transmission flux and energy density.

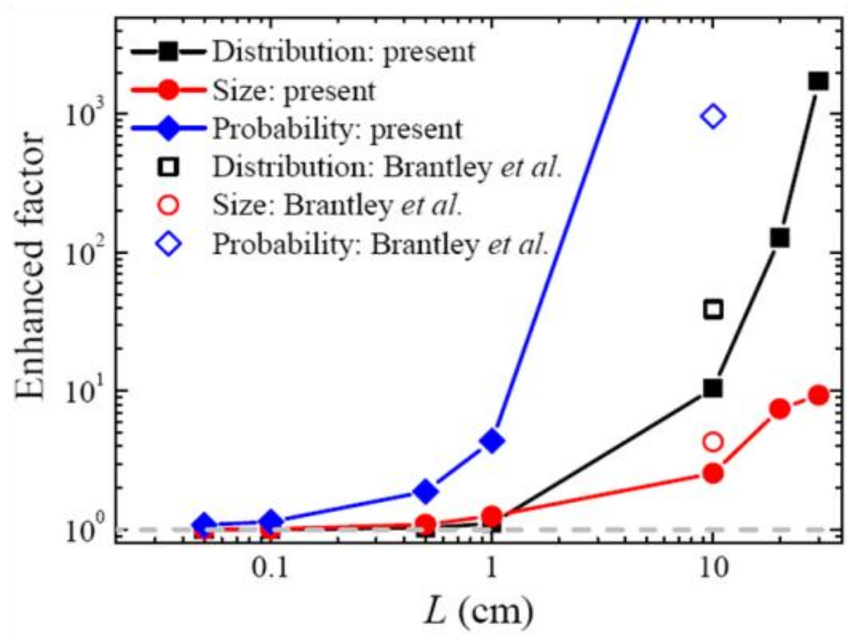
Different mixing sizes **strongly affect** the transmission flux and energy density.

Our results seem to support the LLNL, but why not the LANL?

2. Method and Results



We have performed 1D calculations using identical parameters as the LANL group. We find that present enhanced factors for L = 1.0 nearly approaches 1.0 by varying the properties of random media. For the parameters investigated, l_p is restricted to 1.0-2.03, which invariably fulfills the relation of $l_p \geq L$.



We made 1D calculations based on the parameters used by the LLNL group. It is found that 1D results generally agrees with previous 3D results. For the parameters considered, l_p is ranged from 0.44 to 3.41, which satisfies the condition l_p <<L=10.0.

3. Conclusion

We have modeled stochastic radiative transfer in random mixtures for different physical scenarios:

1. For the 1-dimensional problem, a pure absorption process

- Derived an analytical model for the ensemble-averaged transmission;
- Observed the influence of random mixtures remarkable.

2. For the 3-dimensional problem, an absorption-emission process.

- Analyzed material temperature effect.
- Developed a steady-state stochastic transport model
- Proposed a theoretical criterion
- Resolved the existing disputable issue in previous works.

We have not yet finished the job! Ongoing topic!

Thanks for your attention!

fire Questions at me?